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# Evaluation of Uncertainty in Flatness Measurement of Coordinate Measuring Machines Based on Monte Carlo

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**Abstract.** Based on the measurement method of coordinate measuring machine flatness, establish a black box evaluation model for CMM flatness measurement uncertainty, and analyze the sources of each uncertainty. Combining measurement examples, this paper elaborates on the evaluation process of CMM flatness measurement uncertainty based on Monte Carlo, calculates the included interval and expanded uncertainty of CMM flatness, and provides reference for the evaluation of CMM flatness measurement uncertainty.

**Key words**: metrology; coordinate measuring machine; flatness; uncertainty in measurement; Monte Carlo.

## 1. Introduction

Coordinate Measuring Machine (CMM) is an efficient and multifunctional precision geometric measurement instrument that can quickly and accurately measure the dimensions, shapes, positions, and other geometric quantities of components with complex shapes. Due to the diversity and complexity of CMM measurement objects, the evaluation of measurement uncertainty is relatively difficult. The main reason is that there are multiple sources of CMM measurement uncertainty, the construction of uncertainty models is complex, and the uncertainty components are difficult to accurately quantify. Generally speaking, when evaluating the uncertainty of CMM measurement, it is usually aimed at specific measurement tasks, that is, it is necessary to specify a specific measurement object, as well as relevant measurement environment elements, detection strategies, and other influencing conditions. In order to address the assessment of CMM measurement uncertainty for specific tasks, relevant scholars have conducted research. Reference[2] studied the uncertainty evaluation method for CMM size measurement and established a black box model for measurement uncertainty evaluation. Using the measurement system analysis method, quantitatively analyze the uncertainty components introduced by the six characteristic indicators of quantity values. Reference[3] uses measurement system analysis method to evaluate the uncertainty of coordinate measuring machines in product inspection based on quantity characteristic indicators. Reference[4] studied the uncertainty evaluation problem of coordinate measuring machine (CMM) shape measurement tasks, achieving rapid and reliable evaluation of CMM shape measurement uncertainty. To further standardize and guide the evaluation of measurement uncertainty in CMM, theChinese standard[5] GB/T 24635-2020 provides technical guidelines for determining measurement uncertainty in CMM. On the basis of the aforementioned research, this article conducts a study on the uncertainty evaluation of CMM flatness measurement for flatness measurement tasks.

## 2. Measurement model

Flatness belongs to the form error in geometric errors, which refers to the distance between the actual measured plane and the ideal plane. The evaluation methods for flatness include the minimum containment area method, the least squares plane method, the diagonal plane method, and the three distant point plane method[6]. When using CMM for flatness measurement, there are two modeling methods for measurement uncertainty, transparent box and black box, based on the representation of flatness measurement input and output[2]. In the transparent box model, the measured parameters are obtained by measuring other parameters and converting them into functional relationships. The black box model considers complex measurement methods as simple unknown black boxes with excitation source inputs, and the final measurement results are output by this black box, with input and output quantities in the same unit of dimension[7]. The following will construct a CMM flatness measurement uncertainty evaluation model based on these two modeling methods.

Flatness refers to the difference between the maximum and minimum deviation (which can be negative) from the reference plane among all measured points as the flatness error. According to this definition, the measurement model of flatness f can be calculated by the following equation

$$f = F + \sum \delta_i = d_H - d_L + \sum \delta_i = \frac{(z_H - z_L) - a(x_H - x_L) - (y_H - y_L)}{\sqrt{1 + a^2 + b^2}} + \sum \delta_i \tag{1}$$

where, $f$ is the flatness measurement result, $F$ is the estimated value of flatness, $\sum\delta_i$ is the comprehensive impact of various sources of uncertainty on the measurement results, $d_H$ is the maximum distance from the reference plane in the measurement point, $d_L$ is the minimum distance from the reference plane in the measurement point, ($x_H$, $y_H$, $z_H$), ($x_L$, $y_L$, $z_L$) are the coordinate values of the maximum and minimum distance measurement points, respectively, $a$ and $b$ are the coefficients of the reference plane fitting equation.

From the transparent box measurement model with equation(1) as flatness $f$, it can be seen that this model is related to the coordinate data of each measurement point on the measured plane. When using this model for subsequent evaluation, if sensitivity analysis method (referred to as GUM method) is used for analysis, it is necessary to calculate the sensitivity coefficients of all influencing factors in the model based on the uncertainty propagation law in GUM, and consider their correlation. Obviously, the difficulty of implementing this analysis is relatively high. To achieve this, the model of equation (1) can be simplified to obtain a black box measurement model for flatness f as follows

$$f = F + \sum \delta_i \tag{2}$$

Comparing the two types of models, it can be seen that the black box measurement model for flatness f starts directly from the measurement results, avoiding the complexity of the relationship between the sources of uncertainty and the transmission law of CMM measurement errors[2]. Although the black box model does not directly reflect the impact on measurement uncertainty, it can quantify the impact of various sources of uncertainty on measurement results by using the measurement systems analysis method(MSA), thereby achieving uncertainty evaluation of measurement results. The six main indicators of quantity characteristics are repeatability, reproducibility, resolution, stability, offset, and linearity. The relevant research conclusions indicate that[4], when evaluating the uncertainty of CMM measurement, the three main influencing factors of quantity characteristics are CMM detection error, measurement repeatability, and measurement reproducibility. Therefore, the black box measurement model for flatness f can be further represented as

$$f = F + \delta_1 + \delta_2 + \delta_3 \tag{3}$$

In equation (3), $\delta_1$ is the impact of CMM detection error on measurement results, $\delta_2$ is the impact of flatness measurement repeatability on the measurement results, $\delta_3$ is the impact of flatness measurement reproducibility on measurement results. $\delta_1$ is a uniform distribution with an expected value of 0, and the standard deviation is the uncertainty $u_1$ introduced by the CMM detection error. $\delta_2$ is a normal distribution with an expected value of 0, and the standard deviation is the uncertainty $u_2$ introduced by measurement repeatability. $\delta_3$ is a normal distribution with an expected value of 0, and the standard deviation is the uncertainty $u_3$ introduced by measurement reproducibility.

## 3. Evaluation of measurement uncertainty

### 3.1. Measurement example

Measure the flatness of a certain part using CMM under laboratory environmental conditions, as shown in Figure 1. The laboratory environment temperature is (20±0.5)℃, and the measurement range of the CMM used is (900×1300 ×700) mm, with a maximum allowable error of ±(0.4μm+1.2×$10^{-6}L$). Select appropriate measuring needles and rods according to the measurement requirements. In this example, a measuring head with a diameter of 8mm, a measuring rod with a length of 114mm, a contact measuring force of 150mN, and the normal direction of the measured plane is consistent with the Z direction of the CMM. Uniformly distribute several measuring points (25 measuring points) on the measured plane for contact detection, fit the measured plane using the least squares method, and evaluate the flatness. Repeat three measurements and take the average of the three measurements as the measurement result. The flatness measurement result is 3.24 μm.

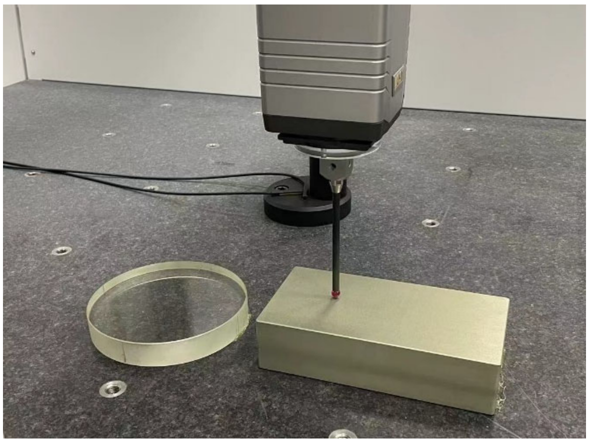

**Figure 1.** Schematic diagram of CMM flatness measurement process

### 3.2. Quantization of uncertainty components

The above analysis shows that the uncertainty components of CMM flatness measurement mainly include the uncertainty u1 introduced by CMM detection error, the uncertainty $u_2$ introduced by measurement repeatability, and the uncertainty $u_3$ introduced by measurement reproducibility. The following analysis and calculation will be conducted for the quantification of each component.

In reference[4], the CMM detection error directly refers to the detection error MPEP in the CMM traceability certificate. However, according to the CMM calibration specification[8], it is known that the MPEP is calculated by coordinate sphere detection. The tested object in this example is a plane, so this article uses a traceable standard plane to obtain the plane detection error of CMM. At the same position of the tested workpiece, fix the flat crystal in the same way, and repeat the measurement of the flat crystal surface three times using the same measurement strategy. Calculate the detection error $MPE_P$ is 0.28μm for CMM flatness measurement. Assuming a uniform distribution, the uncertainty $u_1$ introduced by CMM detection error is

$$u_1 = \frac{0.28\mu\text{m}}{\sqrt{3}} = 0.16\mu\text{m} \tag{4}$$

Under repeatability conditions, repeat the measurement of the measured plane 10 times and calculate the standard deviation $s$ of the repeated measurement experiment to be 0.05μm. Therefore, the uncertainty $u_2$ introduced by the best estimate of flatness through repeated measurement is

$$u_2 = \frac{0.05\mu\text{m}}{\sqrt{3}} = 0.03\mu\text{m} \tag{5}$$

Regarding the impact of measurement reproducibility on measurement results, considering that the CMM measurement program operates automatically, flatness is a small error, and environmental temperature changes have little impact on it, this article sets the reproducibility difference measurement settings to include changes in probe diameter, rod length, and measurement force, ignoring the influence of factors such as measurement personnel and measurement environment. Perform three measurements on the measured plane under each reproducibility condition, and take the average of the three measurements as the measured value under that condition, as shown in Table 1.

**Table 1.** Flatness measurement results under reproducibility difference measurement settings

| No. | Repeatability measurement settings (Measuring head diameter/measuring rod length/measuring force) | Measured value(μm) |
|---|---|---|
| 1 | 8mm / 114mm / 150mN | 3.24 |
| 2 | 8mm / 114mm / 100mN | 3.25 |
| 3 | 8mm / 100mm / 150mN | 3.29 |
| 4 | 8mm / 100mm / 100mN | 3.31 |
| 5 | 5mm / 75mm / 150mN | 3.34 |
| 6 | 5mm / 75mm / 100mN | 3.39 |
| 7 | 5mm / 135mm / 150mN | 3.31 |
| 8 | 5mm / 135mm / 100mN | 3.26 |
| 9 | 3mm / 58mm / 150mN | 3.46 |
| 10 | 3mm / 58mm / 100mN | 3.37 |
| 11 | 3mm / 130mm / 150mN | 3.14 |
| 12 | 3mm / 130mm / 100mN | 3.15 |

According to the measurement results in Table 1, the uncertainty $u_3$ introduced by the reproducibility of flatness measurement can be calculated using the Bessel formula as follows

$$u_3 = 0.09\mu\text{m} \tag{6}$$

### 3.3. Input data processing based on Monte Carlo

Based on the above analysis, the impact of CMM detection error on measurement results $\delta_1$ is a uniform distribution, with an expected value of 0 and a standard deviation of $u_1$ is 0.16μm. The Influence of flatness measurement repeatability on degree measurement Results $\delta_2$ is a normal distribution with an expected value of 0 and a standard deviation of $u_2$ is 0.03μm. The impact of flatness measurement reproducibility on measurement results $\delta_3$ is a normal distribution with an expected value of 0 and a standard deviation of $u_3$ is 0.09μm. According to $\delta_1$, $\delta_2$, $\delta_3$ probability distribution features are sampled with a sample size of $M$ is $10^6$, and the flatness measurement model's values at each sample are calculated[9]. Calculate the average of $10^6$ flatness sample values obtained as the estimated value $F$ of flatness $f$. The standard deviation of $10^6$ flatness sample values calculated by the Bessel formula is the standard uncertainty $u(f)$ of flatness $f$. The calculation results are as follows

$$F = 3.24\mu\text{m} \tag{7}$$

$$u(f) = 0.13\mu\text{m} \tag{8}$$

### 3.4. The inclusion interval and extended uncertainty of the output

Draw a distribution histogram of the $10^6$ flatness sample values obtained, and obtain the frequency distribution of flatness $f$, as shown in Figure 2. It can be seen that the probability density function of flatness $f$ is symmetrically distributed with respect to its estimated value. Sorting $10^6$ flatness sample values from small to large, when the inclusion probability $p$=95%, the inclusion interval of flatness $f$ can be calculated as

$$[f_{25000}, f_{975000}] = [2.98, 3.50]\mu m \tag{9}$$

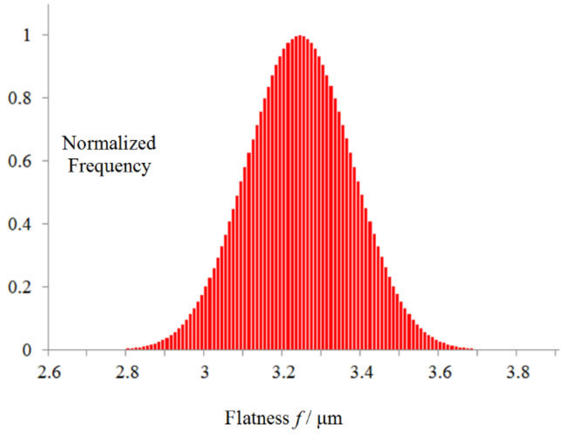


**Figure 2.** Normalized frequency statistical histogram of flatness measurement results based on MCM

Meanwhile, due to the symmetric distribution of the inclusion interval and the probability of inclusion *p* is 95%, the extended uncertainty $U_{95}$ of flatness *f* can be calculated as

$$U_{95} = f_{975000} - f_{25000} = 0.52\mu m \tag{10}$$

## 4. Conclusion

Based on the measurement method of CMM flatness, a black box evaluation model for CMM flatness measurement uncertainty was established. The measurement system analysis method was used to analyze three key characteristic values and quantify the sources of uncertainty. Combining measurement examples, this paper elaborates on the evaluation process of CMM flatness measurement uncertainty based on Monte Carlo, calculates the CMM flatness inclusion interval and its extended uncertainty, and provides reference for the evaluation of CMM flatness measurement uncertainty.

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