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Data Reconstruction of Wireless Sensor Network Based on Graph Signal

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Abstract. The environmental and other factors can cause data missing in power systems; thus, data reconstruction is of great significance. In this paper, we model the observed signal as time-varying signal based on graph signal processing (GSP) and establish an optimization problem with the objective of minimizing the error between the true signal and the reconstructed signal at the sampling points and improving the smoothness of the reconstructed signal. To solve the optimization problem, Taylor series expansion is performed on the Hessian inverse matrix of the objective function, and truncated Taylor series is used as an approximation of the Hessian inverse matrix. In the simulation, the algorithm proposed in this paper is compared with the gradient descent algorithm, and the result shows that the proposed algorithm converges faster and the reconstructed signal is more accurate.

Keywords: Data reconstruction, graph signal processing, smoothness, Taylor series.

1. Introduction

With rapid growth of population and expansion of power grid coverage, the number of electrical equipment connected to the power grid has rapidly increased, bringing heavy pressure on the power system. In order to maintain the normal operation of the power system, it is necessary to collect and monitor real-time data to reduce the impact of power equipment malfunction on the entire power system.

In the process of data collection, certain special circumstances can lead to data missing. For example, some collecting nodes in the monitoring network are located in harsh environments, which can easily cause the failure or damage of collection equipment. Similarly, when the data transmitted by the collecting node to the data center passes through areas with high-intensity noise interference, it can easily cause data missing. Therefore, data reconstruction has become an urgent and important issue to infer missing data from collected data.

There are many methods for data reconstruction, and graph signal processing is one of the important methods. Graph Signal Processing (GSP) is a new paradigm in the field of signal processing. It extends traditional signal processing theory to irregular domains represented by graph networks. The GSP has a certain network topology structure, and graph signals is collected through nodes in the network. To address the missing graph signal in the network, interpolation is used to infer the values of all graph signals from the collected partial graph signals. However, due to the possibility that missing data may be located at critical nodes in the network, special reconstruction algorithms need to be designed.

The data reconstruction based on graph signals has received increasing attention from scholars. To reconstruct bandlimited graph signal from sampled data, two local-set-based iterative methods were proposed, where one of the proposed methods reweighted the sampled residuals for different vertices, while the other propagated the sampled residuals in their respective local sets [1]. A graph as a mathematical representation of the smart grids was generated, and reconstruction of graph Laplacian and Fourier transform calculation for signals had been executed to obtain the local properties and behaviors [2]. The authors formulated the signal recovery task as a convex optimization problem that minimized the total variation of the graph signal while controlling its global or node-wise empirical error, and proposed a first-order primal-dual algorithm to solve the optimization problem [3]. The authors gave sampling design criteria to mitigate the effect of noise and model mismatching, and proposed algorithms and optimal sampling strategies for graph signal reconstruction, where both sampling set and signal values were allowed to vary with time [4]. To deal with blind reconstruction problem where the vertex defects occur randomly over the graph, the authors formulated the blind reconstruction problem as Mixed-Integer Nonlinear Programming, and proposed a Joint Detection and Reconstruction method to simultaneously detect the vertices' working states and reconstruct the bandlimited signal [5]. A reconstruction algorithm based on the statistics of the local smoothness of the graph signals along with the global smoothness of the graph signals was proposed to reconstruct the unobservable states in power systems [6].

In this paper, we introduce the graph signal processing into the data reconstruction with the objective of minimizing reconstructed error and improving the smoothness of the reconstructed signal. The optimization problem is transformed into a convex optimization problem by using the truncated Taylor series to obtain the approximation of the Hessian inverse matrix of the objective function.

2. System model

In large power network, it includes numerous power equipment, such as generators, transmission lines, and distribution equipment. In order to obtain the operating status of the equipment, sensors are installed in the system to collect data.

2.1. Model for the power network

The power network can be modeled as a graph, and the sensor in the power network is the node with arbitrary spatial distribution. The graph is a nonlinear data structure composed of a set of node and edge which connects nodes, and it can be denoted as $G=(V,E,W)$ where V is the node set, E is the edge set. $W \in R^{N\times N}$ is the weighted adjacency matrix, and its weight value w_{ij} represents the association between node v_i and node v_j. The higher the association between nodes, the greater the weight value. In addition, node v_i is associated with node v_j, then $w_{ij}>0$. If node v_i is connected to node v_j, then $w_{ij}=1$. Otherwise $w_{ij}=0$.

Given the graph with N nodes, a graph signal is defined as the map on graph nodes that assigns the real number set R to the node set V. If the node order of a graph is determined in a certain way, the graph signal can be abbreviated as an N-dimensional vector $\mathbf{x}=[x_1,x_2,...,x_N]^T \in R^N$, where x_i is the signal value of node v_i. The graph signal that changes over time during a period of time T is a time-varying graph signal, and it is represented as a matrix $\mathbf{X}=[\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_T]\in R^{N\times T}$ which is composed of a set of graph signal vectors at each time point.

2.1. Smoothness of graph Signal

Considering the difference in signal values between nodes, the smoothness of a graph signal is introduced. Generally, the smoothness of a graph signal can be measured by the graph Laplacian Quadratic form $x^T Lx$. According to the definition of graphic Laplacian matrix, the Quadratic form of Laplacian can be rewritten as

$$x^T Lx=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\left(x_i-x_j\right)^2 \quad (1)$$

Based on the positional relationship between nodes in a wireless sensor network, the nearest node is regarded as the neighbor of the node, and the wireless sensor network is modeled as an undirected weighted graph. Generally, the data measured by sensor nodes with closer distances is more similar. Therefore, the weight of the edges is set as a Gaussian kernel function based on the geographical distance between nodes, and it is negative correlation with the distance between nodes as follows:

$$[W]_{ij} = w_{ij} = e^{-\frac{\|p_i - p_j\|^2}{\sigma^2}} \tag{2}$$

where p_i and p_j is the position coordinate of node v_i and v_j.

It can be seen from the above formula that the smaller the difference between the signal values of the two nodes connected by the edge with larger weight is, the smaller the value of the Laplacian Quadratic form is, and the smoother the graph signal is.

2.2. Graph Laplacians

Graph Laplacians is widely used in graph theory [7]. For undirected and unweighted graph signal networks, the graph Laplacians is defined as

$$L = D - W \tag{3}$$

where D is the degree matrix of the graph, whose element satisfies $d_{ii} = \sum_{j=1}^{N} w_{ij}$.

Based on the weight values of the edges, we can construct the degree matrix D, weighted adjacency matrix W, and Laplace matrix L corresponding to the graph. Since the Laplace matrix is a real symmetric matrix, and it satisfies the following orthogonal diagonalization as

$$L = U \Lambda U^T \tag{4}$$

where U is the graph Fourier transformation matrix, the eigenvalues of Λ are the graph frequency, and T is the transpose operation.

2.3. Data Collection

The incomplete signal is abstracted as the time-varying graph signal sampled in a period of time. The sampling observation model for time-varying graph signals is given as

$$y_t = S_t x_t + n_t, \quad t = 1, 2, \ldots, T \tag{5}$$

where y_t is the observation value of the graph signal at time t, x_t is the true value of the graph signal, n_t is the observation noise with independent identical distribution, S_t is the sampling operation, and it is a diagonal matrix.

The diagonal matrix S_t in Equation (5) satisfies

$$[S_t]_{ii} = \begin{cases} 1, & \text{node is sampled} \\ 0, & \text{node is not sampled} \end{cases} \tag{6}$$

2.4. Graph Signal Reconstruction

After the sampling operation, the observation value of the graph signal has zero element, that is, the observation data is missing. The reconstruction model can be used to reconstruct incomplete signals as follows:

$$\min_{x_t} \frac{1}{2} \|S_t x_t - y_t\|_2^2 + \frac{\theta}{2} x_t^T L x_t \tag{7}$$

where the first term of the objective function is the data matching term, which aims to reconstruct the signal which approximate to the true value at the sampling point, while the second term is the smoothing regularization term, which is used to improve the smoothness of the reconstructed signal.

3. Improved Graph Signal Reconstruction

The solving algorithm for optimization problem in Equation (7) is a first-order algorithm, which has relatively slow convergence speed and is easily affected by the condition number of the Hessian matrix. In this regard, the problem of data reconstruction is reduced to a convex optimization problem, and the truncated Taylor order is used to approximate the Hessian matrix of the original objective function.

3.1. Graph Signal Reconstruction Base on Convex Optimization Model

There are two types of correlations between time-varying graph signals. First, the adjacent graph signals in space are similar at a certain moment, which reflects the space-domain correlation. Second, the graph signal of a certain node changes slowly over time, which reflects the time-domain correlation.

The model in Equation (7) used for the reconstruction task of spatiotemporal signals only considers the smoothness of the graph signal in space-domain, and does not fully utilize the time-domain correlation. Therefore, the reconstruction error is large [8]. That means it is not enough to simply use the quadratic form of the Laplacian matrix $(x_t)^T Lx_t$ to describe the smoothness of the graph signal.

Here, we use the following differential quadratic form of Laplacian matrix to describe the smoothness of the time-varying graph signal

$$(x_t - x_{t-1})^T L(x_t - x_{t-1}) \tag{8}$$

Further the reconstruction problem of the graph signal in Equation (7) can be transformed into the unconstrained least squares optimization problem as

$$\min_{x_t} \frac{1}{2}\|S_t x_t - y_t\|_2^2 + \frac{\lambda}{2}(x_t - \tilde{x}_{t-1}) L (x_t - \tilde{x}_{t-1}) \tag{9}$$

where $\tilde{x}_{t-1}$ is the reconstructed signal at time $t-1$, and λ is an adjustable positive parameter.

3.2. Hessian Matrix Decomposition

The Hessian matrix of the objective function can be obtained by calculating the second-order derivative of the objective function as follows:

$$H_t = S_t + \lambda L \tag{10}$$

where H_t is the second-order information of the objective function, which is a positive definite matrix that reflects change rate of the objective function gradient.

The Hessian matrix in Equation (10) can be decomposed as

$$\begin{cases} H_t = S_t + \lambda L = K_t - \lambda W \\ K_t = S_t + \lambda D \\ L = D - W \end{cases} \tag{11}$$

where K_t is the positive definite diagonal matrix obtained through decomposition.

The decomposition is according to the matrix series as follows:

$$(I - X)^{-1} = \sum_{i=0}^{\infty} X^i, radius(X) < 1 \tag{12}$$

where I is the identity matrix.

The Hessian inverse matrix can be expanded by Taylor series as

$$
\begin{aligned}
H_t^{-1} &= \left(K_t - \lambda W\right)^{-1} = \left[K_t^{\frac{1}{2}}\left(I - \lambda K_t^{-\frac{1}{2}} W K_t^{-\frac{1}{2}}\right) K_t^{\frac{1}{2}}\right]^{-1} \\
&= K_t^{-\frac{1}{2}}\left(I - \lambda K_t^{-\frac{1}{2}} W K_t^{-\frac{1}{2}}\right)^{-1} K_t^{-\frac{1}{2}} = K_t^{-\frac{1}{2}}\left[\sum_{m=0}^{\infty}\left(\lambda K_t^{-\frac{1}{2}} W K_t^{-\frac{1}{2}}\right)^m\right] K_t^{-\frac{1}{2}}
\end{aligned} \tag{13}
$$

3.3. Solving Convex Optimization Problem

The truncated Taylor series is used as an approximation of the Hessian inverse matrix, and it is substituted into the Newton method iteration formula to solve the optimization problem until the iteration termination condition is met. The Hessian inverse matrix is approximation as

$$
\begin{aligned}
H_t^{-1} \approx P_t^{(M)} &= K_t^{-\frac{1}{2}}\left[\sum_{m=0}^{M}\left(\lambda K_t^{-\frac{1}{2}} W K_t^{-\frac{1}{2}}\right)^m\right] K_t^{-\frac{1}{2}} \\
&= \left[\sum_{m=0}^{M}\left(\lambda K_t^{-1} W\right)^m\right] K_t^{-1}
\end{aligned} \tag{14}
$$

where M is the order of the Taylor series, and it reflects the approximation degree between the matrix $P_t^{(M)}$ and H_t^{-1}.

After solving the optimization problem, the finally reconstruction signal $\tilde{x}_t$ can be obtained, and the iteration formula is given as

$$
x_t(n+1) = x_t(n) + d_t^{(M)}(n) = x_t(n) - P_t^{(M)} g_t(n) \tag{15}
$$

where $d_t^{(M)}(n)$ is the approximate descent direction of the objective function at time t, and n is iterations number.

3.4. Algorithm Flow

The algorithm flow is given in Table 1.

Table 1: The Algorithm Flow

Obtain $[S_t]_{ii}$, $[L]_{ij}$, $[\tilde{x}_{t-1}]_i$, λ, M **Input:** $[y_t]_i$ **Output:** $[\tilde{x}_t]_i$
1. Calculate $[K_t]_{ii} = [S_t]_{ii} + \lambda [D]_{ii}$ 2. initialize $[x_t(0)]_{ii} = [\tilde{x}_{t-1}]_{ii}$, $n = 0$ **3.** Calculate $[v_t(n)]_i = [x_t(n)]_i - [\tilde{x}_{t-1}]_{ii}$ 4. Exchange data with neighboring nodes $[v_t(n)]_i$, initialize $m = 0$, and calculate

$$\left[g_t(n)\right]_i = \left[S_t\right]_{ii}\left[x_t(n)\right]_i - \left[y_t\right]_{ii} + \lambda\sum_{j\in N_t}[W]_{ij}\left(\left[v_t(n)\right]_i - \left[v_t(n)\right]_j\right)$$

$$\left[d_t^{(0)}(n)\right]_i = -\left[g_t(n)\right] / \left[K_t\right]_{ii}$$

5. Determine if m is less than M ? If so, exchange data $\left[d_t^{(m)}(n)\right]_i$ with neighboring nodes and calculate

$$\left[d_t^{(m+1)}(n)\right]_i = \left[d_t^{(0)}(n)\right]_i + \lambda\sum_{j\in N_t}[W]_{ij}\left[d_t^{(m)}(n)\right]_j \Big/ \left[K_t\right]_{ii},$$

then $m = m+1$ and return to Step 5. Otherwise, go to next Step 6.

6. Calculate $\left[x_t(n+1)\right]_i = \left[x_t(n)\right]_i + \left[d_t^{(M)}(n)\right]_i$

7. Determine if the iteration termination condition is met? If so, the algorithm ends. Otherwise, $n = n+1$ and return to step 3

4. Simulation

To verify the performance of the improved algorithm proposed in this paper, we compare it with gradient descent algorithm. The improved algorithm takes first and second order approximations with M=1 and M=2, and step size of the gradient descent algorithm is set as 0.6. With sufficient maximum iteration K, the performance evaluation index is set to relative error as

$$RE = \frac{\left\|x_t(n) - x_t^*\right\|^2}{\left\|x_t(0) - x_t^*\right\|^2} \tag{16}$$

Due to the fact that the maximum iterations number within each shorter signal observation interval of the algorithms are usually limited, it is necessary to compare the performance of the algorithms under limited iterations number N . Then, the following cumulative error can be used as the performance evaluation index

$$CE = \frac{1}{T\sqrt{N}}\sum_{t=1}^{T}\left\|\tilde{x}_t - x_t^O\right\|_2 \tag{17}$$

where the iteration termination condition is set as $N = 1500$ or $RE < 10^{-4}$

Figure 1 and Figure 2 shows the comparison of relative error and cumulative error between the two algorithms. In the simulation, the conditional number of the Hessian matrix of the objective function is 1283. It can be seen from Figure 1 and Figure 2 that when the conditional number is large, the convergence speed of the gradient descent algorithm has greatly decreased. However, the convergence speed of the improved algorithm is still fast, and it is not sensitive to the conditional number.

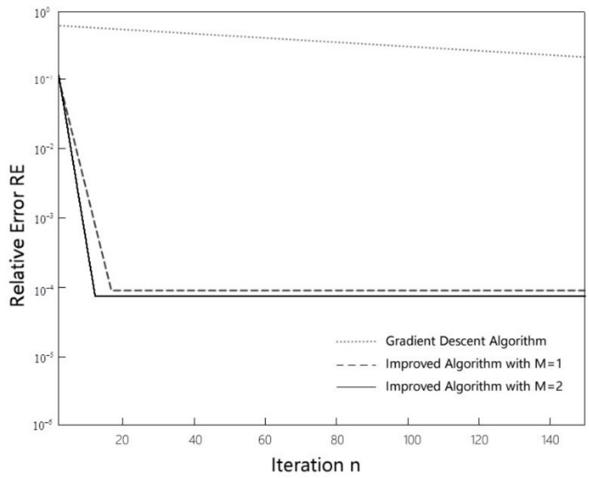

Figure 1: Comparison of relative error between different algorithms

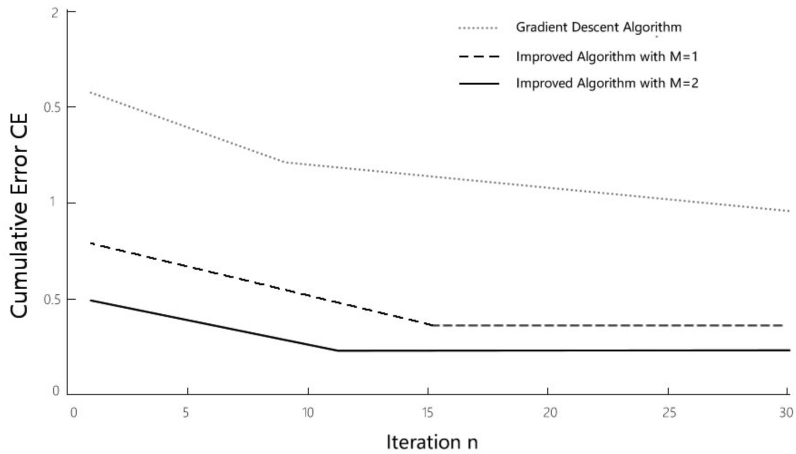

Figure 2: Comparison of cumulative error between different algorithms

Figure 3 shows the comparison of the total communication times of network nodes to achieve the same relative error index between the two algorithms. When the conditional number is large, the iteration number of the gradient descent algorithm to obtain the target accuracy is too large, thus the number of node communication is much higher than that of the improved algorithm. Due to the slow convergence speed, the gradient descent algorithm cannot obtain reconstruction signals with effective accuracy in finite iterations, while the improved algorithm still has good reconstruction performance.

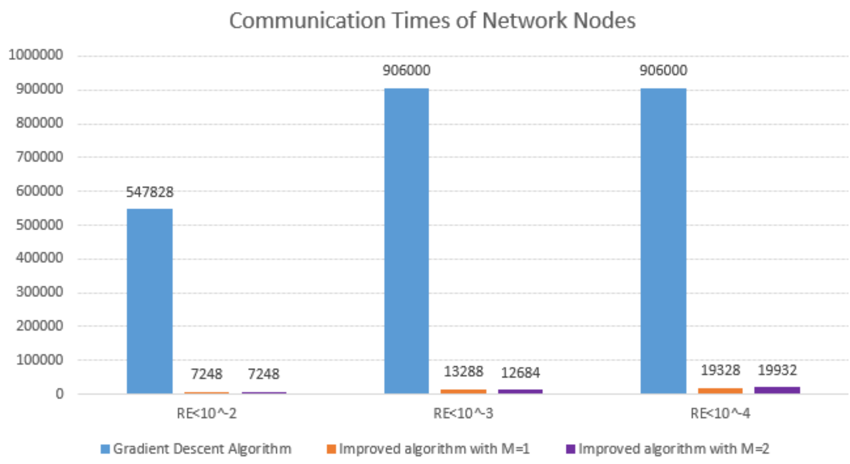

Figure 3: Comparison of communication times between different algorithms

5. Conclusion

Regarding the issue of data missing in the power system, the graph signal processing is introduced into the data reconstruction in this paper. The collected and observed signal are regarded as time-varying signals, and one object is to minimize the error between the reconstructed signal and the true signal, the other is to make the reconstructed signal as smooth as possible by introducing the concept of signal smoothness. To solve the optimization problems, the Hessian inverse matrix of the objective function is expanded by Taylor series, and the truncated Taylor series is used as an approximation of the Hessian inverse matrix to transform the optimization problem into a convex optimization problem. In order to verify the performance of the proposed algorithm, it was compared with the gradient descent method, and the results showed that the proposed algorithm had better performance than the gradient descent method.

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