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Dynamic Analysis of A Three Dimension Chaotic System

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Abstract. Memristor is a newly realized physical element, and it is a non-linear circuit element with memory function. We constructed a new three-dimensional chaotic system is constructed which contains four parameters and five non-linear terms. The system is analyzed by nonlinear dynamic analysis methods such as theoretical deduction analysis, numerical simulation, lyapunov exponent spectrum, and bifurcation diagram. The dynamic behavior of the system. This paper proposes a new three-dimensional chaotic system. The system contains four parameters, and each equation contains a non-linear product term. Based on theoretical derivation, numerical simulation, Lyapunov exponential spectrum, bifurcation diagram, the basic dynamic characteristics of the chaotic system are analyzed.

Keywords: Three-dimensional system; Lyapunov exponent; Chaotic circuit

1. Introduction

Since 1960s, Lorenz first discovered chaos in the numerical test of weather forecast and proposed the Lorenz system [1]. Much research has been done on chaotic humans in the past few decades. Chaos is often used in dynamic systems that require complexity, such as secure communication, data encryption, nonlinear oscillator circuits [2], network security [3], and robots [4]. In the early 1970s, Professor Cai Shaotang predicted from the completeness of the combination of basic variables that a fourth basis exists in the circuit (Memristive element). Memristive element is a non-linear basic passive two-terminal circuit element with memory function. It can memorize the amount of charge flowing through it and can represent the mathematical relationship between magnetic flux and charge [5]. Then a lot of research has been done on chaotic circuit. After the experts' study, a simple equation chaotic system chaotic system consisting of continuous mixing, can exhibit more chaos in attractors coexist. These attractors have their own independent attracting domains and that the initial conditions change, there may be coexistence of stable periodic attractors and chaotic attractors in the phase plane. Li and Sprott proposed a continuous three-dimensional autonomous chaotic system capable of coexisting a periodic attractor, two point- attractors, and two strange attractors [6].

He Qiling proposed a new three-dimensional autonomous chaotic system containing cubic terms and used linear feedback control methods to achieve the synchronous control of the chaotic system[7].

This paper proposes a new three-dimensional chaotic system which contains four parameters, and each equation contains a non-linear product term. Through theoretical derivation, numerical simulation, Lypunov exponential spectrum, bifurcation diagram, the basic dynamic characteristics of the chaotic system are analyzed.

2. Description of System Model

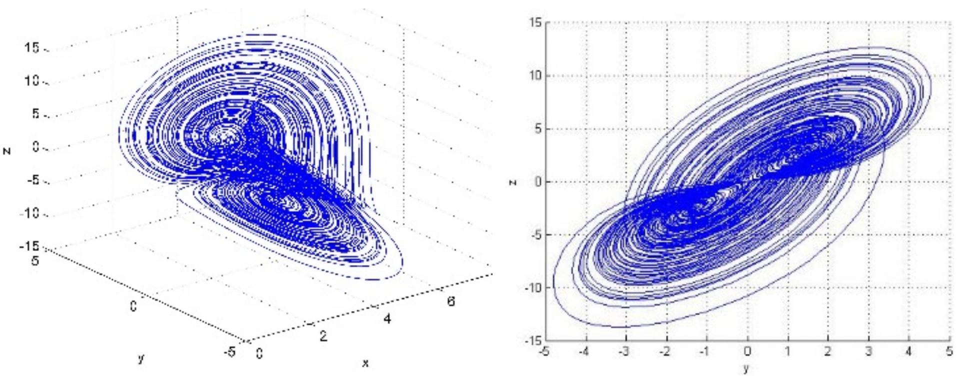
On the basis of He's article, we could modifying the system equation is

$$\begin{cases} \dot{x} = -ax + yz \\ \dot{y} = by - xz + z \\ \dot{z} = -cz + y^3 + dxy \end{cases} \tag{1}$$

Among them, x , y , z are state variables of the system and a , b , c , d are constants .

2.1 System Model

Under the appropriate initial conditions, the system will generate chaos which can observed in phase portaits. When $a = 1.25, b = 4.33, c = 9.66, d = 5.33$, $\begin{pmatrix} x_0 & y_0 & z_0 \end{pmatrix} = \begin{pmatrix} 0.081 & 0.085 & 0.125 \end{pmatrix}$ the system (1) is simulation numerically by MATLAB, and the system phase portraits and attractors are shown in Figure 1. It can be seen that the generated sequence has disorder. Later, it will be verified whether the system is in a chaotic state.



(a) (b)

Fig. 1 Phase portraits (a)x-y-z (b) y-z

Then we will analysis dynamic characteristics from symmetry, invariability, dissipation the existence of attractor, equilibrium point and stability of the system.

2.2 Symmetry and Invariability

We can know that there is a symmetric transformation $S:(x, y, z) \to (x, -y, -z)$ by observation the system model equation. Under S its action, the system has invariance, which means the system is axisymmetric and can holds for all system parameters.

2.3 Dissipation and the Existence of Attractor

The divergence of system is $\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a + b - c$, we can find a set of parameter $a = 1.25, b = 4.33, c = 9.66, d = 5.33$, that can made $\nabla V = -6.58 < 0$, which means the system (1) is dissipative. I.e. the system state changes when selecting the above parameters is bounded, and converges exponentially $\frac{dV}{dt} = e^{-6.58}$.

A volume element with initial volume $V(0)$ shrinks to volume element $V(0)e^{-6.58t}$ at time t. When $t \to \infty$, each small volume element containing the trajectories of the system shrinks exponentially to 0 at an exponential rate of - 6.58, so all trajectories of the system are limited to a set of zero volumes, and the progressive motion is fixed on an attractor.

2.4 Equilibrium Point and Stability

If $a=1.25, b=4.33, c=9.66, d=5.33$ the right side of equation (1) is equal to zero, the equilibrium point of the system can be obtained.

$$p_0=(0,0,0),\ p_1=(3.1957, 1.4232,\ 2.8070),\ p_2=(3.1957,-1.4232,-2.8070),$$

$$p_3=(-16.9882,-9.3923, 2.2611),\ p_4=(-2.6701,-1.6819, 1.9846),$$

$$p_5=(-2.6701, 1.6819,-1.9846),\ p_6=(-16.9882, 9.3923,-2.2611),$$

If the system (1) is linearized at the equilibrium point, its Jacobi matrix can be obtained：

$$J_p=\begin{bmatrix} -a & z & y \\ -z & b & -x+1 \\ d*y & d*x+3*y*y & -c \end{bmatrix},$$

because of $\det[\lambda I-J(0)]=0$, we can get the eigenvalue of Jacobian matrix at the corresponding equilibrium point.

Because p_1/p_2 , p_3/p_6 and p_4/p_5 are symmetrical, the eigenvalues are equal, eigenvalues of each balance point are shown in Table 1 below.

Table 1 Eigenvalues of Each Balance Point

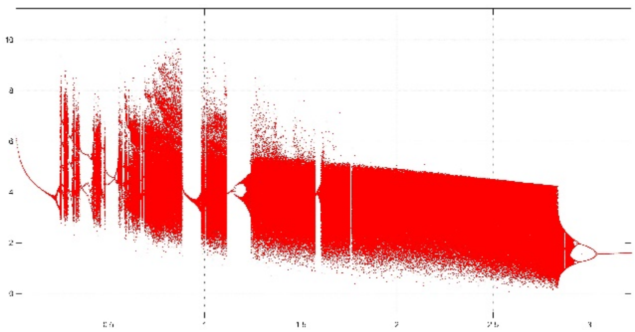
p_0	p_1/p_2	p_3/p_6	p_4/p_5
-1.2501	1.0699+5.4921i	57.5729	-10.4695
4.3305	1.0699 - 5.4921i	-0.9741	1.9435 + 3.4179i
-9.6629	-8.7223	-63.1812	1.9435 - 3.4179i

In order to analyze the dynamic behavior of complex systems in depth, we mainly analyze the influence of parameters on the 3D chaotic system used in this paper. And calculation the bifurcation points, by changing parameters. Lyapunov exponential spectrum analysis system is used to analyze the stability of the system, and the QR decomposition method is used to calculate the Lyapunov spectrum to ensure the accuracy of the calculation. In order to avoid accidental results, we choose 20,000 iteration points. The Lyapunov exponent of the system is LE=[1.0250 0.0007 -7.6082], and the system is a chaotic system.

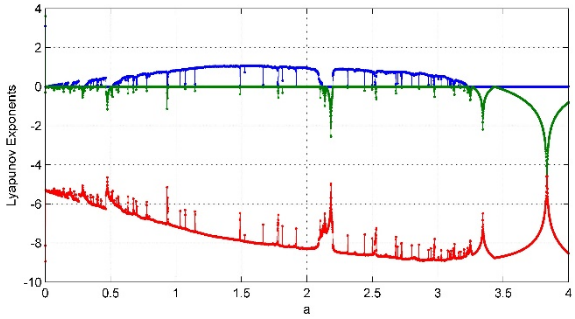
3. Analysis of the chaotic system

When the system parameters change, the stability of the system equilibrium point and the motion state of the system will also change accordingly. By analyzing the bifurcation diagram of the system and the Lyapunov exponential spectrum (LE spectrum), the change of the motion state of the system is studied when the parameters change.

Taking the above system parameters and the initial value $x_0=(0.081\quad 0.085\quad 0.125)$, the bifurcation diagram and LE spectrum of the system (1) with the parameter change can be obtained.



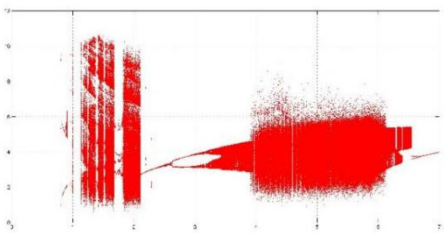
(a)



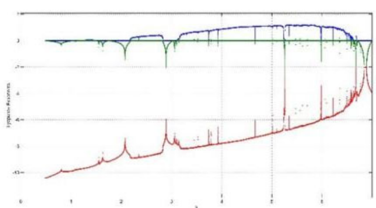
(b)

Fig .2 (a) The bifurcation diagram (b) LE spectrum

When the parameter a ∈ (0,4), the bifurcation of the system about x and the LE spectrum are shown in Fig. 2, where the numerical simulation step size is taken as 0.001, and Fig.2(a) is the bifurcation of the state variable x changing with the parameter a Fig.2(b) is the LE spectrum of the system as a function of parameter a. If the maximum LE of the system (1) is less than zero, then the system is in a periodic motion state; if the maximum LE of the system (1) is greater than zero, then this When a ∈ (0, 0.22) and a ∈ (3.034, 4), the system is in a steady state motion and tends to a stable fixed point. When the maximum LE of the system is less than zero; when a∈(0.526, 0.88) , a ∈ (1.152, 1.579) , a ∈ (1.751, 2.83) and a∈ (2.912, 3.034), the system is in chaotic motion, and the system's Maximum LE is greater than zero.



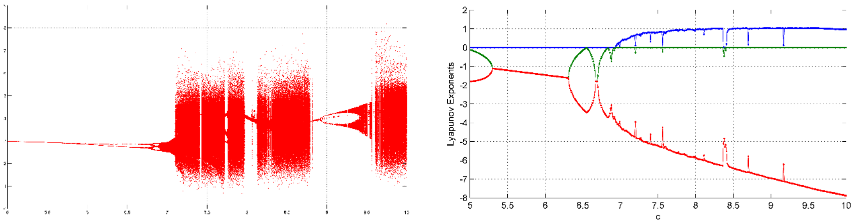
(a)



(b)

Fig.3 (a)The bifurcation diagram-b (b) b-LE spectrum

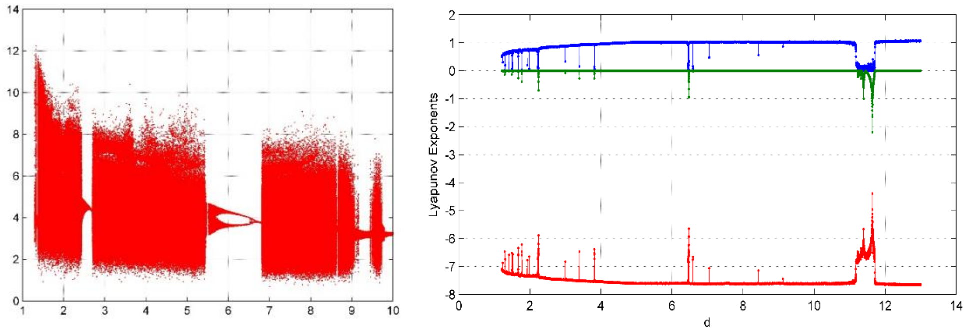
When the parameter b $\in$ (0,7), the bifurcation of the system about x and the LE spectrum are shown in Figure3, The analysis process is similar to the parameter a, but the difference lies in the process of parameter change, the multi-period coexistence occurs.



(a) (b)

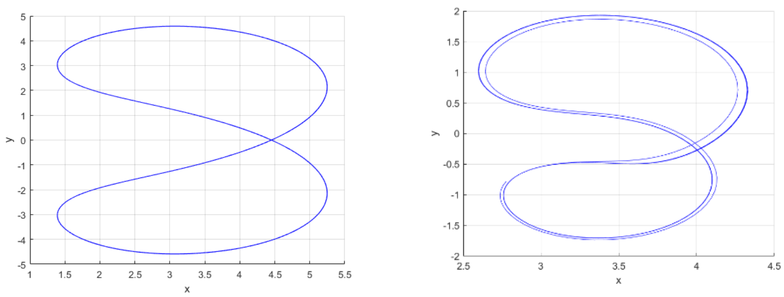
Fig. 4 (a)The bifurcation diagram-c (b) c-LE spectrum

When the parameter c $\in$ (5,10), the bifurcation of the system about x and the LE spectrum are shown in Figure4.When the parameter d $\in$ (0,14), the bifurcation of the system about x and the LE spectrum are shown in Figure5.



(a) (b)

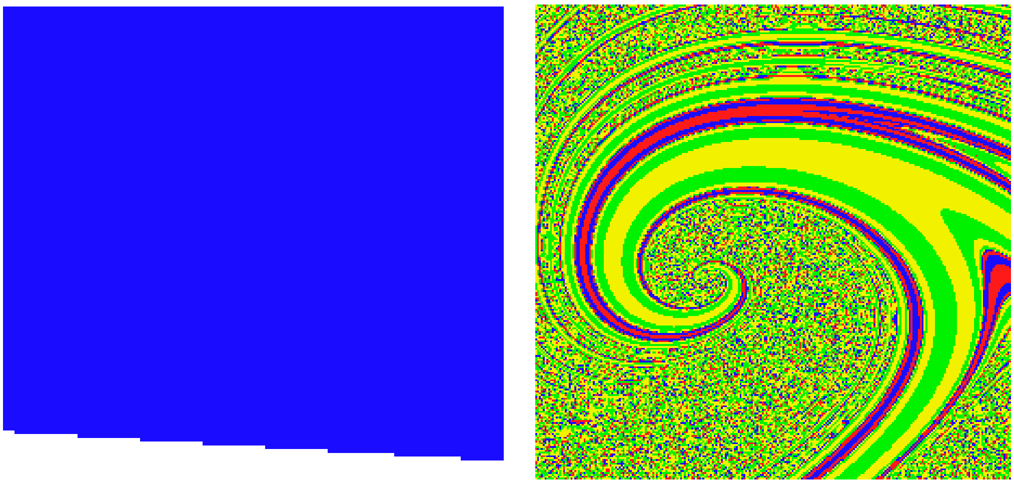
Fig. 5 (a)The bifurcation diagram-d (b) d-LE spectrum



(a) (b)

Fig. 6 Period phase portraits

Several typical phase diagrams with different parameters a, state variables y and x changing with a are shown in Figure 6. (a) and (b) in Fig. 6 are the period 1 limit cycle of the system and the period 2 limit cycle after period-doubling bifurcation, respectively.With the change of parameters, different attractors will appear in the system, and the attractive domain of several typical attractors is calculated by using the continuation method, as shown in Fig. 7.



(a) Domain of attraction of a periodic attractor (b)The Domain of Attraction of Chaos Attractors

Fig. 7 The domain of attraction

In Figure 7(a), the blue area represents that the initial value from this area will be attracted to the stable equilibrium point, and the white area represents that the initial value from this area will be attracted to the corresponding attractors. In Fig. 7(b), different colors represent the corresponding attractors of different periods. As can be seen from the figure above, with the progress of the period-doubling bifurcation, the attraction domains of different attractors will change differently, and the anti-interference ability of the system will also change differently. The larger the attraction domain of the attractors, the stronger the anti-interference ability of the system, and the stronger the chaotic performance of the system in the chaotic system.

4. Summary

In this paper, the dynamic model of a three-dimensional chaotic system is analyzed, the dimensionless mathematical model of the system is established, and the nonlinear dynamics of the parallel memristive chaotic circuit under different circuit parameters is studied by numerical simulation. The simulation results show that the system can generate chaotic attractors, and the system has a wide chaotic state region. And with the change of parameter a, the system will produce transient chaos, chaotic state mutation to periodic state and other rich dynamic phenomena, which has certain potential application value in chaotic secure communication.

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