

Modelling of the Interrelationships Between Population Sex Ratio and Food Content and Ecological Niches in the Marine Lampreys

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Abstract. In this paper, a model related to sex ratio and food content was developed and solved. Through logistic function fitting, it was found that the sex ratio of the Marine lampreys was affected by environmental conditions, with approximately 56% males when food was plentiful and up to 78% when conditions changed. A correlation function model between sex ratio and food content was developed and the normality and absence of autocorrelation of the model was verified by residual analysis (Shapiro-Wilk and Durbin-Watson tests). In order to further explore the interrelationships between sea sevendill eels and their ecological niches, this paper extends the classical Lotka-Volterra model to include competitor and predator equations, and analyses the changes of the various populations under the initial conditions by numerical simulations. Through Spearman correlation analysis, it was found that sex ratio significantly affects the number of predators, competitors and prey in the ecosystem, where an increase in the proportion of males suppresses the growth of the number of prey and competitors, and promotes the increase of the number of predators.

Keywords: Sea sevendill Lampreys; Sex Ratio; Ecological Modelling.

1. Introduction

The sea lamprey is an ancient and unique jawless fish widely distributed in the North Atlantic and its tributaries. Its unique ecological habits and complex life cycle make it important in the study of ecosystem dynamics. However, the sex ratio and food availability status of sea lamprey populations have been changing in response to environmental changes and human activities, and these changes have had far-reaching impacts on their population dynamics and ecosystem balance. In order to better understand and manage this important species, it becomes particularly crucial to study the relationship between its sex ratio and food content [1-2].

Among the existing studies, there is still a lack of systematic quantitative analyses on the mechanisms by which the sex ratio of the sea hagfish is affected by environmental factors. In addition, although the Lotka-Volterra model has been widely used in describing predator-prey relationships, the extension and applicability of the classical model for complex ecosystems containing sex ratio variables and multiple interacting factors such as competitors and predators still need to be further explored [3]. Therefore, this paper aims to fill this research gap by establishing and extending the classical Lotka-Volterra model, combined with residual and Spearman correlation analyses, to systematically study the sex ratio, food content and their interactions with ecological niches in the sea lamprey [4].

Specifically, in this paper, we first determined the correlation function model between sex ratio and food content by logistic function fitting, and verified the fit and reliability of the model. Subsequently, the classical Lotka-Volterra model was extended to include competitor and predator equations, and the dynamics of the population under different initial conditions were analysed by numerical simulation. Finally, Spearman correlation analysis was applied to explore the effects of sex ratio on the number of predators, competitors and prey. The study in this paper not only reveals the complex relationship between sex ratios and food content in sea lamprey populations, but also provides a



comprehensive ecological model that can better predict and manage the population dynamics of this species in changing environments. Through this study, we expect to provide a scientific basis for the conservation and management of the sea lamprey, as well as new perspectives and methods for ecosystem studies [5-6].

2. Establishment and solution

2.1. Sex ratio and food content function model

2.1.1. logistics Function fit.

The sex ratio of lampreys is influenced by environmental conditions. When there's an ample food supply, about 56% are males, but this can shift to about 78% under different conditions.

$$\begin{aligned} \lambda_{\min} &= 0.56 \\ \lambda_{\max} &= 0.78 \end{aligned} \tag{1}$$

Based on the data we've gathered and analyzed, the sex ratio and food content of sea lampreys align closely with what you'd expect from a logistic function. Now, let's establish a correlation between the lamprey population and the number of their prey [7].

After debugging, we have established a function model that correlates the sex ratio with food content in a sea lamprey population of around 1000 individuals. Graph of sex ratio and food content is shown in figure 1.

$$\lambda = 0.56 + \frac{0.22}{1 + e^{0.02(x_2 - 750)}} \tag{2}$$

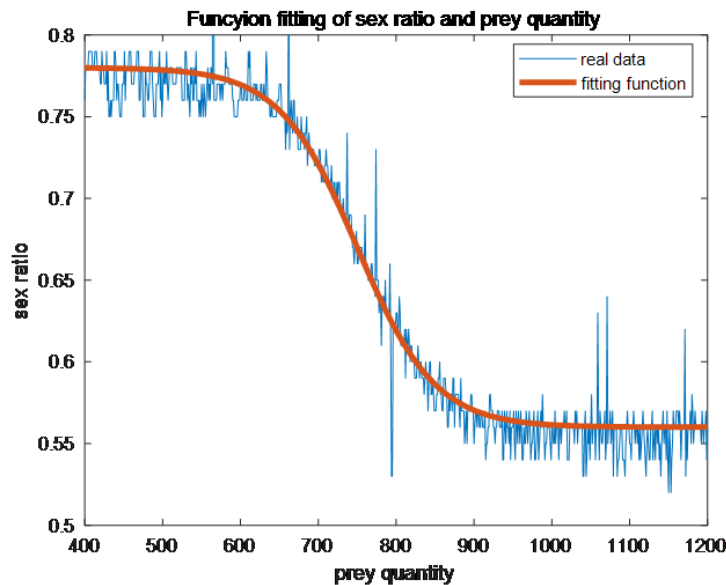


Figure 1. Graph of sex ratio and food content

2.1.2. Residual analysis.

(1) Shapiro-Wilk Test:

Assumption:

H₀ (null hypothesis): The sample data is normally distributed.

H₁ (alternative hypothesis): The sample data is not normally distributed.

If the p-value is less than the predetermined significance level (0.05), the null hypothesis is rejected, indicating that the data does not exhibit a normal distribution. If the p-value is greater than or equal to the significance level, the null hypothesis cannot be rejected, suggesting that the data may conform to a normal distribution [8].

Upon testing: $p=0.12$

The residuals of the fitted model pass the test for normality, indicating a favorable model fit.

(2) Durbin-Watson Test:

Assumption:

H_0 (null hypothesis): There is no autocorrelation in the residuals.

H_1 (alternative hypothesis): There is autocorrelation in the residuals.

If the D value is close to 2, it suggests that there is no significant autocorrelation in the residuals, and thus the null hypothesis (H_0) cannot be rejected. If the D value is significantly smaller than 2, it indicates the presence of positive autocorrelation, leading to the rejection of the null hypothesis (H_0) in favor of the alternative hypothesis (H_1). Similarly, if the D value is significantly larger than 2, it indicates the presence of negative autocorrelation, also resulting in the rejection of the null hypothesis (H_0) in favor of the alternative hypothesis (H_1).

Upon conducting the test: $D=0.796$

The residuals of the fitted model show no significant autocorrelation, indicating a satisfactory model fit.

2.2. The Lotka-Volterra model and model expansion of sea lamprey and each ecological niche

2.2.1. The classic Lotka-Volterra model.

$$\begin{cases} \frac{dx_2}{dt} = r_2x_2 - \alpha x_1x_2 \\ \frac{dx_1}{dt} = -\beta_1x_1 - \eta_2x_1x_2 \end{cases} \quad (3)$$

Among: α is Rate of Decline for the Prey after Being Captured;

β_1 is The natural mortality rate of lampreys;

η_1 is The growth rate of lampreys due to predation.

The classic Lotka-Volterra model elucidates the interplay between lampreys and their prey. To comprehensively investigate the ecological niches within the lamprey-inhabited ecosystem and explore the correlation between gender variations in the lamprey population, noteworthy enhancements have been incorporated into the conventional Lotka-Volterra model. These enhancements encompass:

- To add the lamprey competitor equation.

$$\frac{dx_3}{dt} = r_3x_3 \left(1 - \frac{x_3}{x_{3m}} - S \frac{x_1}{x_{1m}}\right) \quad (4)$$

Among: x_{3m} and x_{1m} are the maximum environmental capacity of the lamprey and its competitors;

S is the ratio of the same number of competitors to the resources consumed by the lamprey.

- To add the lamprey predator equation.

$$\frac{dx_4}{dt} = -\beta_2 x_4 - \eta_2 x_4 x_1 \quad (5)$$

Smong: B_2 is the natural mortality rate of sea lamprey;

η_2 is lamprey rate of rise due to predation.

So, combining the Lotka-Volterra model with the competitor model provides the following model.

$$\begin{cases} \frac{dx_1}{dt} = -\beta_1 x_1 - \eta_2 x_1 x_2 \\ \frac{dx_2}{dt} = r_2 x_2 - \alpha x_1 x_2 \\ \frac{dx_3}{dt} = r_3 x_3 \left(1 - \frac{x_3}{x_{3m}} - S \frac{x_1}{x_{1m}}\right) \\ \frac{dx_4}{dt} = -\beta_2 x_4 - \eta_2 x_4 x_1 \end{cases} \quad (6)$$

2.2.2. Numerical modelling.

When the initial number of lampreys is 1,000, their prey is 2,000, their competitors are 1,000, and their predators are 500, the numerical simulation results are as follows in the figure 2.

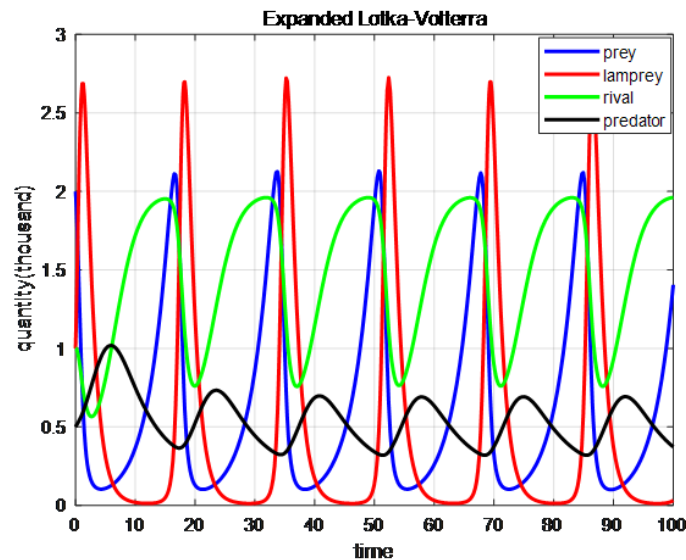


Figure 2. Population change of lampreys and their prey, competitors, and predators over time

According to table 1, the following data on the prey changes over time are available from the Expanded Lotka-Volterra model [9-10].

Table 1. Table of prey values of lampreys over time

time	0	1	99	100
Prey	2000	1372	928	1268

According to table 2, the following data on the rival changes over time are available from the Expanded Lotka-Volterra model.

Table 2. Table of competitors of lampreys over time

time	0	1	99	100
Rival	1000	924	1943	1960

According to table 3, the following data on the predator changes over time are available from the Expanded Lotka-Volterra model.

Table 3. Numerical table of lamprey predators over time

time	0	1	99	100
Predator	500	560	566	560

According to table 4, the following data on the lamprey sex ratio over time can be obtained from the logistics fitting function.

Table 4. Sex ratio of lampreys over time

time	0	1	99	100
Sex ratio	0.52	0.53	0.57	0.56

2.3. Spearman's correlation analysis

Spearman's rank correlation coefficient is a non-parametric correlation measure used to assess the degree of correlation between two variables. Unlike the Pearson correlation coefficient, the Spearman correlation analysis does not assume the data as normally distributed, and it is based on the rank of the variable values rather than the actual values.

$$\rho = \frac{\frac{1}{n} \sum_{i=1}^n (R(x_i) - \bar{R}(x))(R(\lambda_i) - \bar{R}(\lambda))}{\sqrt{(\frac{1}{n} \sum_{i=1}^n (R(x_i) - \bar{R}(x))^2)(\frac{1}{n} \sum_{i=1}^n (R(\lambda_i) - \bar{R}(\lambda))^2)}} \quad (7)$$

The value of the Spearman correlation coefficient is between -1 and 1, where 1 indicates a complete positive correlation, -1 indicates a complete negative correlation, and 0 indicates no correlation.

Upon testing:

Table 5. Spearman's rank correlation coefficient

	Prey	Rival	Predator
Sex ratio	-0.858	-0.453	0.940

According to Table 6, The sex ratio of lamprey significantly affected predators, competitors and hunters in the ecosystem, The proportion of males in the lamprey population is significantly negatively correlated with the number of prey population and competitor population, and significantly positively correlated with the number of predator population.

3. Conclusion

In this paper, we investigated the sex ratio, food content and their interrelationships with ecological niches of the sea sevendill eel population by establishing and extending the classical Lotka-Volterra model. Firstly, the correlation function model between sex ratio and food content was determined by logistic function fitting, and the fit and reliability of the model were verified. The results of residual analyses showed that the model was well fitted with no significant autocorrelation. In extending the

classical Lotka-Volterra model, we added competitor and predator equations and analysed the dynamics of the population under different initial conditions by numerical simulation. The simulation results showed that the population trends of the various populations were as expected under changes in sex ratio, prey, competitors and predators in initial numbers. We further explored the effect of sex ratio on the number of predators, competitors and prey through Spearman correlation analysis. The results showed that sex ratio was significantly negatively correlated with the number of prey and competitors, while it was significantly positively correlated with the number of predators. This suggests that increasing the male sex ratio can inhibit the growth of prey and competitors while promoting an increase in the number of predators. In summary, the study in this paper not only reveals the complex relationship between sex ratio and food content in sea lamprey populations, but also proposes a comprehensive ecological model that provides a scientific basis for predicting and managing the dynamics of sea lamprey populations in a changing environment. This study provides theoretical support for the conservation and management of sea lampreys and offers new perspectives and methods for ecosystem studies.

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