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Optimal Crop Planting Scheme Based on Mixed Integer Programming

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Abstract. In order to achieve sustainable development of rural economy, make full use of limited cultivated land resources, improve production efficiency, reduce planting risks, and consider the actual rural situation and crop yield per mu affected by climate, market and other factors. It is necessary to comprehensively consider the expected sales volume, yield per mu, uncertainty of planting cost and selling price of various crops and potential risks, and give the optimal planting plan of crops. First, the maximum expected benefit is taken as the objective function, the planting area of crops planted in each plot in each season of the year is taken as the decision variable, and the influence of uncertain factors such as climate and market conditions is comprehensively considered. The parameter change model of each situation in which the uncertain factors are represented is established, and the objective function, constraints and uncertainties are comprehensively considered. Using Monte Carlo simulation optimization algorithm, the Monte Carlo optimal benefit model is established. Through programming solution, we obtained that the planting plots of similar crops were concentrated, and most of them only planted one crop in one plot, or at most two crops in one plot. In this paper, the Monte Carlo simulation algorithm is innovatively adopted to fully consider the uncertainties of crop sales volume, price, per mu yield and planting cost, making the decision more scientific and reasonable.

Keywords: Monte Carlo Simulation Optimization Algorithm; Optimal Decision; Parameter Change Model.

1. Introduction

According to the instability [1] of the future sales market, we predict the sales volume and price fluctuation of crops, and establish a parameter change model [2]. The compound annual growth rate model is used to obtain the expected sales volume of wheat and corn, the planting cost and selling price of crops, and the random change model is used to obtain the yield per mu of crops. Secondly, for the consideration of uncertainty [3], we conducted Monte Carlo simulation of different variables to obtain the values of variables under different parameters. Finally, we considered the sensitivity analysis under different parameters to obtain the planting scheme with the greatest total benefit. Therefore, this paper aims to establish a Monte Carlo optimal benefit model [4] which can deal with uncertainty by introducing Monte Carlo simulation optimization algorithm. The model will help us more fully assess how various planting strategies perform in a complex and changing environment, so that we can develop more robust and efficient crop planting plans [5]. The disadvantage of the Monte Carlo algorithm is that it requires the construction of stochastic models in advance and relies on accurate estimates of the model parameters. If the model is not constructed properly or the parameter estimation is not accurate, the accuracy and reliability of the simulation results will be directly affected. Therefore, in this paper, the mixed integer algorithm and Catmonlo algorithm can bypass the direct processing of these complex constraints to a certain extent, and approximate the optimal solution or feasible solution through a large number of random experiments.

2. Correlation theory

2.1. Monte Carlo method

Monte Carlo method [6], as a powerful computing tool, its core lies in the use of random sampling and statistical simulation to approximate the solution of complex problems. By constructing a probabilistic model or a stochastic process framework closely related to the problem to be solved, this method subtly avoids the complexity of direct solution, and turns to the generation and analysis of a large number of random samples to estimate the values of required parameters or features, so as to achieve approximate solution [7] of the problem.

The basic process refinement of the Monte Carlo process is outlined below:

In this paper, we use Monte Carlo method to solve the maximum expected return. The basic idea of the Monte Carlo method is to transform the integral into an expected value problem related to a random variable. Specifically, we can construct a random variable X that is in the interval $[a,b]$. The probability density function of a normal distribution is. Then, we consider the expected value $f(x)$ of the function $E(f(x))$.

The functions in this article are:

$$f(X)=\sum_{t=2024}^{2030}\sum_{i=1}^{41}\sum_{j=1}^{54}\sum_{k=1}^{2}\left(\min\left(x_{i,j,t,k}\cdot y_{i,j,k},z_{i,t}\right)\cdot p_{i,t}-x_{i,j,t,k}\cdot c_{i,j,t,k}\right) \tag{1}$$

Let $\theta=\int_a^b f(x)dx$

According to the definition of expected value and the probability density function, we have:

$$E[f(X)]=\int_a^b f(x)p(x)dx \tag{2}$$

Because $p(x)$ follows a normal distribution. That's $p(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Then the above formula becomes:

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Let $\theta=\int_a^b f(x)dx$

According to the definition of expected value and the probability density function, we have:

$$E[f(X)] = \int_a^b f(x)p(x)dx \tag{4}$$

Because $p(x)$ follows a normal distribution. That's $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Then the above formula becomes:

$$E[f(X)] = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \int_a^b f(x)dx = \frac{\theta}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{5}$$

To estimate θ, we can take N independent, identically distributed samples $X_1, X_2, X_3, ..., X_N$ from the distribution $p(x)$ and calculate the average value of the function corresponding to these samples, that is, the sample mean:

$$\overline{f} = \frac{1}{N} \sum_{i=1}^{N} f(X_i) \tag{6}$$

According to the Law of large numbers, when $N \to \infty$, the sample mean $\overline{f}$ almost necessarily converges to the expected value $E[(f(X)]$. Therefore, we can use $\overline{f}$ to estimate $E[(f(X)]$, and thus θ.

For the case of a normal distribution, we have:

$$\theta \approx \frac{\sqrt{2\pi}\sigma}{e^{-\frac{(x-\mu)^2}{2\sigma^2}}} \overline{f} = \frac{\sqrt{2\pi}\sigma}{e^{-\frac{(x-\mu)^2}{2\sigma^2}}} \frac{1}{N} \sum_{i=1}^{N} f(X_i) \tag{7}$$

The detailed flow chart is shown in Figure 1

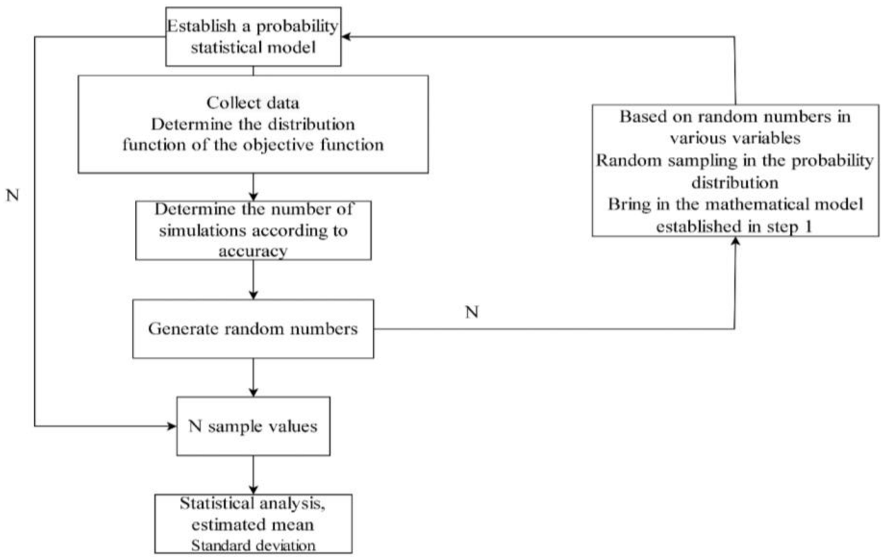

Figure 1. Detail chart

2.2. Advantages of Monte Carlo simulation

Monte Carlo method [8] is different from traditional exhaustive method with its unique random search mechanism, the latter follows fixed rules for ergodic search, while the former explores the solution space through random sampling. Although it cannot guarantee to find the absolute optimal solution, the quality of its solution will gradually approach the optimal state with the increase of sample size, which reflects the strategic advantage of "winning by quantity".

3. Experiment

3.1. Deterministic objective function

We treat excess sales as slow volume, with the goal of maximizing expected earnings under uncertain conditions for the 2024-2030 period. Therefore, the parameter of the income function is a random variable. Therefore, the objective function [9] for this problem should be:

$$\text{Max } Z_3 = \text{Max E}\left[\sum_{t=2024}^{2030}\sum_{i=1}^{41}\sum_{j=1}^{54}\sum_{k=1}^{2}\left(\min\left(x_{i,j,t,k} \cdot y_{i,j,k}, z_{i,t}\right) \cdot p_{i,t} - x_{i,j,t,k} \cdot c_{i,j,t,k}\right)\right] \tag{8}$$

3.2. Deterministic constraint

Cultivated land area limit: for each piece of land, the planting area can not exceed its actual area.

$$\sum_{\text{i}} x_{i,j,t,k} \le S_j, \forall \text{j,t,k} \tag{9}$$

Crop rotation requirements: Each plot of land is planted with pulses at least once in three years.

$$\sum_{\text{t}=T}^{T+2}\sum_{\text{i}} \text{x}_{\text{i,j,t,k}} \ge 1, \forall j, T \in [1,6] \tag{10}$$

Planting crops without repetition: The same crop cannot be grown in successive seasons on the same plot of land.

For single-season plants, the same crop cannot be grown for 2 consecutive years:

$$\text{W}_{\text{i,j,t,1}} + W_{\text{i,j,t+1,1}} \le 1, \forall j, i \in \{1,2,3,\cdots,34\}, t < T \tag{11}$$

For double-cropping plants, the two seasons of the same year cannot plant the same crop, and the second season of one year cannot plant the same crop with the first season of the following year:

$$W_{\text{i,j,t,1}} + W_{\text{i,j,t,2}} \le 1, \forall j, i \in \{27,28,\cdots,54\}, t < T \tag{12}$$

$$W_{\text{i,j,t,2}} + W_{\text{i,j,t+1,1}} \le 1, \forall j, i \in \{27,28,\cdots,54\}, t < T \tag{13}$$

$W_{i,j,t,k}$ whether to plant i crop in plot j in the k season of year

The constraint of non-scattered planting area: avoid the planting area of each crop in each quarter of the year is too scattered.

$$\sum_{j} W_{i,j,t,k} \leq N, \quad \forall \mathrm{i,t,k} \tag{14}$$

Non-negative constraint: the planting area, total production and total sales of each field of each numbered crop in each quarter of the year cannot be negative.

$$\mathrm{x}_{\mathrm{i,j,t,k}}, y_{i,j,k}, z_{i,t} \geq 0, \forall i, j, t, k \tag{15}$$

The area of each crop planted in a single plot should not be too small.

$$W_{i,j,t,k} \cdot S \leq x_{i,j,t,k} \leq W_{i,j,t,k} \cdot M \tag{16}$$

S is the smallest area and M is infinite

Restrictions on planting any one crop per season on each plot of land:

$$W_{\mathrm{i,j,t,k}} \leq E_{i,j,k}, \forall i \in I, j \in J, t \in T, k \in \{1,2\} \tag{17}$$

3.3. Expected Compound Annual Growth rate (CAGR) model

Wheat and maize:

$$z_{i,t} = z_{i,2023} \times \left(1 + r_{i,t}\right)^{t-2023}, \forall i \in \{6,7\}, r \sim N\left(0.075, \frac{1}{144}\right) \tag{18}$$

Other crops:

$$z_{i,t} = z_{i,2023} \times \left(1 + \vartheta_{i,t}\right)^{t-2023}, \forall i \in \{1 \leq i \leq 5 或 8 \leq \mathrm{i} \leq 41\}, \vartheta_{i,t} \sim N\left(0, \frac{1}{3600}\right) \tag{19}$$

Planting cost compound annual growth rate model:

$$c_{i,j,t,k} = c_{i,j,t2023,k} \times (1.05)^{t-2023}, \forall i \in \{1, \cdots, 41\}, t \in \{2024, \cdots, 2030\} \tag{20}$$

Sales price compound annual growth rate model:

For grain:

$$p_{i,t} = p_{i,2023}, \forall i \in \{17, \cdots, 37\} \tag{21}$$

For vegetables:

$$p_{i,t} = p_{i,2023} \times (1.05)^{\text{t-2023}}, \forall \text{i} \in \{17, \cdots, 37\} \tag{22}$$

For edible fungi:

$$p_{i,t} = p_{i,2023} \times (1-\eta_{i,t})^{t-2023}, i \in \{38, \cdots, 41\}, \eta_{i,t} \sim N\left(0.03, \frac{1}{22500}\right) \tag{23}$$

For morels:

$$p_{i,t} = p_{i,2023} \times (0.95)^{t-2023}, i = 41 \tag{24}$$

Random change model of mu yield:

$$y_{i,j,k} = y_{i,j,k} \times (1 + r_{i,j,k}), \forall i \in \{1, 2, \cdots, 41\}, j \in \{1, \cdots, 54\}, k \in \{1, 2\}, r_{i,j,k} \sim N\left(0, \frac{1}{900}\right) \tag{25}$$

3.4. Monte Carlo optimal benefit model

$$\text{Max } Z_3 = \text{Max E}\left[\sum_{t=2024}^{2030} \sum_{i=1}^{41} \sum_{j=1}^{54} \sum_{k=1}^{2} \left(\min\left(x_{i,j,t,k} \cdot y_{i,j,k}, z_{i,t}\right) \cdot p_{i,t} - x_{i,j,t,k} \cdot c_{i,j,t,k}\right)\right] \tag{26}$$

$$\text{s.t.}\begin{cases} \sum_{\text{i}} x_{i,j,t,k} \le S_j, \forall \text{j,t,k} \\ \sum_{\text{t}=T}^{T+2} \sum_{\text{i}} \text{x}_{\text{i,j,t,k}} \ge 1, \forall j, T \in [1,6] \\ \text{W}_{\text{i,j,t,1}} + W_{\text{i,j,t+1,1}} \le 1, \forall j, i \in \{1, 2, 3, \cdots, 34\}, t < T \\ W_{\text{i,j,t,1}} + W_{\text{i,j,t,2}} \le 1, \forall j, i \in \{27, 28, \cdots, 54\}, t < T \\ W_{\text{i,j,t,2}} + W_{\text{i,j,t+1,1}} \le 1, \forall j, i \in \{27, 28, \cdots, 54\}, t < T \\ \sum_{\text{j}} W_{i,j,t,k} \le N, \quad \forall \text{i,t,k} \\ \text{x}_{\text{i,j,t,k}}, y_{i,j,k}, z_{i,t} \ge 0, \forall i, j, t, k \\ W_{i,j,t,k} \cdot S \le x_{i,j,t,k} \le W_{i,j,t,k} \cdot M \\ W_{\text{i,j,t,k}} \le E_{i,j,k}, \forall i \in I, j \in J, t \in T, k \in \{1, 2\} \end{cases} \tag{27}$$

4. Result

Based on the probability model and Monte Carlo optimal benefit model, we use python to solve the following results. When solving the results, we took economic benefit as the main goal, and concentrated planting plan and single plant species in a single plot as the secondary goal, considering the diversity of crops to solve. At the same time, we considered the future planting and cultivation costs and other factors, we set the minimum planting unit to 0.1 as far as possible. The difficulty of implementing the scheme is reduced, and the usability of the scheme is improved. The specific data of the best planting plan[10] in the first quarter are shown in Table 1-6 below

A total of 20 crops were planted in the first quarter, spread over 54 plots:

Table 1. Part result 1

Soya Bean	Soya Bean	Soya Bean	Soya Bean	Soya Bean	Soya Bean	Black Soya Bean	Black Soya Bean	Ormosia Hosiei	Ormosia Hosiei
B11	B12	B13	B14	C1	C2	B10	B11	B7	B8
32.4	45	35	20	15	7	20.7	27.6	17.2	44

Table 2. Part result 2

Ormosia Hosiei	Urad	Urad	Climbing Beans	Climbing Beans	Wheat	Wheat	Wheat	Wheat	Wheat
B19	B6	B7	C5	C6	A1	A2	A3	A4	A5
1.8	63	37.8	7.3	18.9	80	55	35	72	2.2

Table 3. Part result 3

Corn	Corn	Corn	Corn	Corn	Millet	Millet	Millet	Millet	Sword Bean
B2	B3	B4	B5	B6	A5	A6	B1	B2	D8
32.5	40	28	25	23	65.8	55	60	13.5	13.8

Table 4. Part result 4

Gaoliang	Gaoliang	Glutinous Broom Born	Glutinous Broom Corn	Buckwheat	Pumpkin	Pumpkin	Sweet Potato	Rice	Rice
B9	B10	C3	C4	C6	C4	C5	C5	D1	D2
48.2	4.3	15	5.2	1.1	12.8	0.8	18.9	15	10

Table 5. Part result 5

Rice	Rice	Cowpea	Cowpea	Cowpea	Potato	Potato	Potato	Potato	Potato
D3	D4	D4	D5	D6	E1	E2	E3	E4	E5
14	5.1	0.9	10	1.4	0.6	0.6	0.6	0.6	0.6

Table 6. Part result 6

Potato	Potato	Potato	Potato	Potato	Potato	Potato	Potato	Potato	Potato
E6	E7	E8	E9	E10	E11	E12	E13	E14	E15
0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

Table 7. Part result 7

Potato	Potato	Potato	Potato	Potato	Tomato	Tomato	Eggplant	Brassica Chinensis	Brassica Chinensis	Cucumber
E16	F1	F2	F3	F4	D6	D7	D6	D7	D8	D8
0.6	0.6	0.6	0.6	0.6	3.4	13.2	7.2	9.8	1.6	4.6

The specific data of the best planting plan in the second quarter are shown in Table 7 to 9 below.

A total of 8 crops were planted in the second quarter, spread over 25 plots:

Table 7. Part result 7

Potato	Potato	Potato	Potato	A Bigger Variety Of Chinese Cabbage	A Bigger Variety Of Chinese Cabbage	A Bigger Variety Of Chinese Cabbage	A Bigger Variety Of Chinese Cabbage	Ternip	Ternip
F1	F2	F3	F4	D4	D5	D6	D7	D7	D8
0.6	0.6	0.6	0.6	0.9	10	12	8.6	13.4	12.8

Table 8. Part result 8

Garden Radish	Elm Yellow Mushroom	Elm Yellow Mushroom	Elm Yellow Mushroom	Xianggu	Xianggu	Xianggu	Pleurotus Nebrodensis	Pleurotus Nebrodensis	Pleurotus Nebrodensis
D8	E1	E2	E3	E14	E15	E16	E11	E12	E13
7.2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

Table 9. Part result 9

Toadstool	Toadstool	Toadstool	Toadstool	Toadstool	Toadstool	Toadstool
E4	E5	E6	E7	E8	E9	E10
0.6	0.6	0.6	0.6	0.6	0.6	0.6

In the table above, we present the specific data of our final planting plan in order of plant species. In order to simplify planting, we set the minimum division to 0.1, which can reduce the difficulty of planting. In addition, in order to make small plots better planted and cultivated, we made them only plant one crop, which perfectly reduced the difficulty and cost of cultivation.

5. Conclusion

In this paper, Monte Carlo optimization model is used to solve a rural planting planning problem. This model is based on Monte Carlo simulation algorithm and mixed integer algorithm. By constructing an optimal crop planting model and considering the dynamic changes and uncertainties of crop sales volume, price, per mu yield and planting cost, the optimal crop planting plan from 2024 to 2030 is formulated for a village in the mountainous area of North China. The model effectively dealt with the market changes and potential risks, realized the maximum expected benefits of crop cultivation, and provided a scientific basis for the sustainable development of rural economy Compared with traditional crop planting planning, this method can dynamically adjust the planting plan in many aspects and obtain the best planting plan in real time. Considering the relatively concentrated crop types on the planting plot, it is not only easy to manage, but also helps to improve land use efficiency and crop yield.

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