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Survival Relationships and Ecosystems of the lampreys on the Basis of Several Models

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Abstract. The population of lampreys, one of the only remaining jawless vertebrates in the world, has survived on Earth for hundreds of millions of years, predating the dinosaurs. This remarkable endurance has made lampreys a significant topic of discussion among biologists, particularly for their relevance to the study of biological evolution and ecological stability. In this paper, the predation and competition relationship between lampreys and their natural enemies is described using the Lotka-Volterra model, and the cyclic amplitude law of the male ratio in lampreys and their predators is resolved. Additionally, an Agent-Based Model is employed to simulate the dynamics of the lamprey population, revealing a pattern of "Peak - Sharp Decline - Tendency Toward Equilibrium." Finally, the stability of the lamprey's internal ecology is analyzed using the Jacobi matrix and eigenvalues, demonstrating the model's optimality presented in this paper and providing theoretical support and reference for future lamprey research.

Keywords: Lamprey; Lotka-Volterra Model; Agent-Based Model; Ecological Stabilization.

1. Introduction

Suryanarayana, Shreyas M., and his team discovered potential similarities between the forebrain structure of the lamprey and other vertebrates, suggesting that vertebrate evolution may date back half a billion years [1]. This study, through an in-depth analysis of the lamprey's forebrain structure, revealed its complexity in neuroanatomy, indicating an evolutionary path for early vertebrates' brain structures. By comparing the brain structures of lampreys and other vertebrates, Suryanarayana et al. provided new evidence supporting vertebrate evolution theories, highlighting the importance of the lamprey as a living fossil in understanding the early evolution of vertebrates. Meanwhile, Feixiang Wu's team provided a detailed description of the lamprey's feeding structures, life cycle, and population habitats [2]. Their research focused on revealing the ecological habits of lampreys at different stages of their life cycle, including their feeding mechanisms and reproductive behaviors. By investigating the distribution and population dynamics of lampreys in various habitats, Wu's team further deepened the understanding of the lamprey's ecological role, offering valuable data for the conservation and management of lamprey populations. In contrast, Sean A. Lewandoski proposed a method to prevent and control the invasive spread of lampreys, discussing their impact on ecological balance [3]. Lewandoski's research pointed out that lampreys, as an invasive species, have significantly negative impacts on the ecosystems they invade, including predation pressure and competitive effects on native fish populations. He proposed a series of management strategies aimed at controlling the spread and reproduction of lampreys through ecological regulation and physical control measures to protect the health and stability of native ecosystems.

This paper innovatively examines the sex ratio of lampreys, using both the Lotka-Volterra model [4] and an Agent-Based model [5] to simulate population dynamics among lampreys, their prey, and natural enemies with MATLAB software, revealing potential cyclic patterns. The Lotka-Volterra model, as a classic ecological model, is often used to describe predator-prey relationships between two species, while the Agent-Based model provides a more detailed analysis of population dynamics through simulating individual behaviors and interactions. By combining these two models, this study thoroughly analyzed the dynamic changes in lamprey populations under different ecological conditions, revealing the possible mechanisms of their population's periodic fluctuations. Additionally, to verify the stability of the lamprey's ecosystem, this paper employs Jacobi matrix and eigenvalue analysis [6]. By mathematically modeling and analyzing the interactions among different populations in the lamprey ecosystem, we assessed the stability and resilience of the ecosystem under various disturbances. The Jacobi matrix, as a tool for linear stability analysis, provides a quantitative assessment of the relationships and interactions among variables in the ecosystem. Eigenvalue analysis further reveals the stability characteristics of the system under different states. The results are largely consistent with existing literature and empirical observations. These findings not only support current ecological theories and models but also provide new perspectives and methods for managing lamprey populations. By conducting an in-depth study of lampreys and their ecosystems, we have gained a better understanding of this ancient species and provided scientific grounds for its conservation and management.

In summary, this paper systematically studies the dynamics and ecological stability of lamprey populations by comprehensively using different ecological models and mathematical analysis methods. The results are significant not only theoretically but also practically, offering strong support for ecological management of lampreys. Future research can further integrate field data and advanced models to explore the complexity of lamprey population dynamics and the mechanisms of ecosystem stability.(Data sources: https://www.comap.com/).

2. Constructing a predator-prey Lotka-Volterra model

The dynamics of the lamprey population is affected by marine resources [7]. Based on the Lotka-Volterra model, this paper analyses the complex interactions between the availability of environmental resources and changes in sex ratios in the lamprey, taking into account the interdependence of various factors in time and space, and thus provides reliable predictions of population changes in the aquatic food web. A differential equation describing the change in sex ratio of the lamprey is presented, taking the predator-prey relationship as an entry point:

$$\frac{dP}{dt} = \alpha P - \beta PQ \tag{1}$$

$$\frac{dQ}{dt} = \delta PQ - \gamma Q \tag{2}$$

Where P and Q denote the population size of prey and predator, respectively, α is the natural growth rate of prey, β is the predation rate, γ is the natural mortality rate of predators, and is the predation success rate, which was adjusted to simulate the effect of resource abundance on population size, and thus to reflect changes in the sex ratio of the lamprey.

Based on equation (1) a differential equation that portrays the interaction relationship between the three groups can be derived:

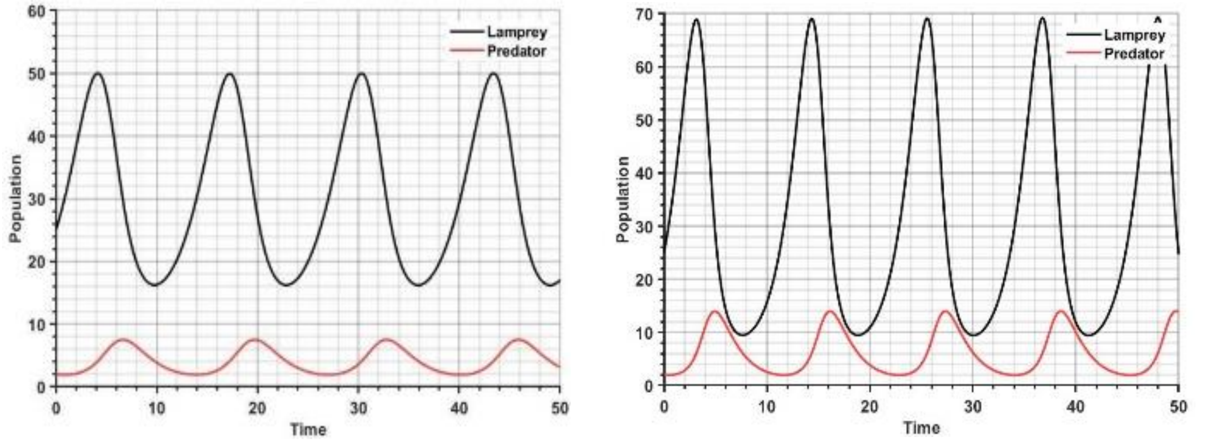
$$\frac{dG}{dt} = \alpha_1 G - \beta_1 GR \tag{3}$$

$$\frac{dR}{dt} = \beta_1 GR - \gamma_1 R - \beta_2 RE \tag{4}$$

$$\frac{dE}{dt} = \delta_1 RE - \gamma_2 E \tag{5}$$

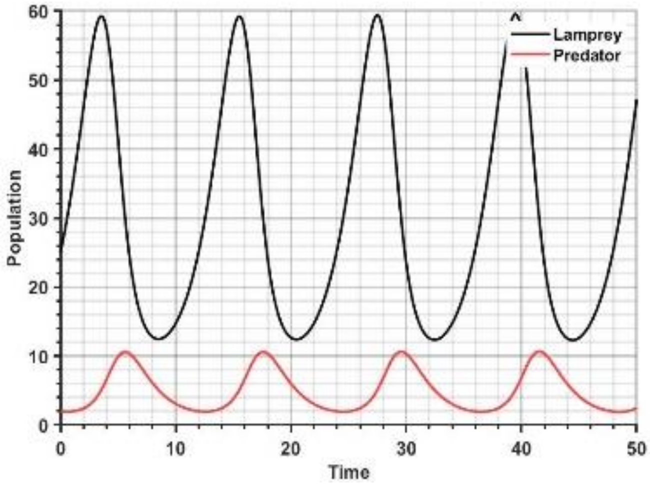
Where G, R and E represent the population size of prey, lampreys and natural enemies of lampreys, respectively, α_1 is the natural growth rate of prey, β_1 is the efficiency of lampreys to feed on prey, γ_1 is the natural mortality rate of lampreys, β_2 is the efficiency of natural enemies to feed on lampreys, δ_1 is the increase in the survival rate of the lampreys after feeding on them by the natural enemies, and γ_2 is the natural mortality rate of the natural enemies.

Fluctuations in population dynamics due to changes in the sex ratio of the lamprey, and potentially disruptive effects on ecosystem stability, are illustrated in Figure 1. An increase in the proportion of males leads to an increase in the amplitude of the population cycle, which means that there is a potential for overfishing of prey, which in turn leads to a scarcity of prey. However, a significant reduction in the number of predators could lead to a surplus of prey, followed by a period of starvation due to overfishing by predators. These cascading effects can result in resource depletion and destabilization of energy flows within food webs, thus threatening the sustainability of ecosystems.



(a) male rate = 0.56

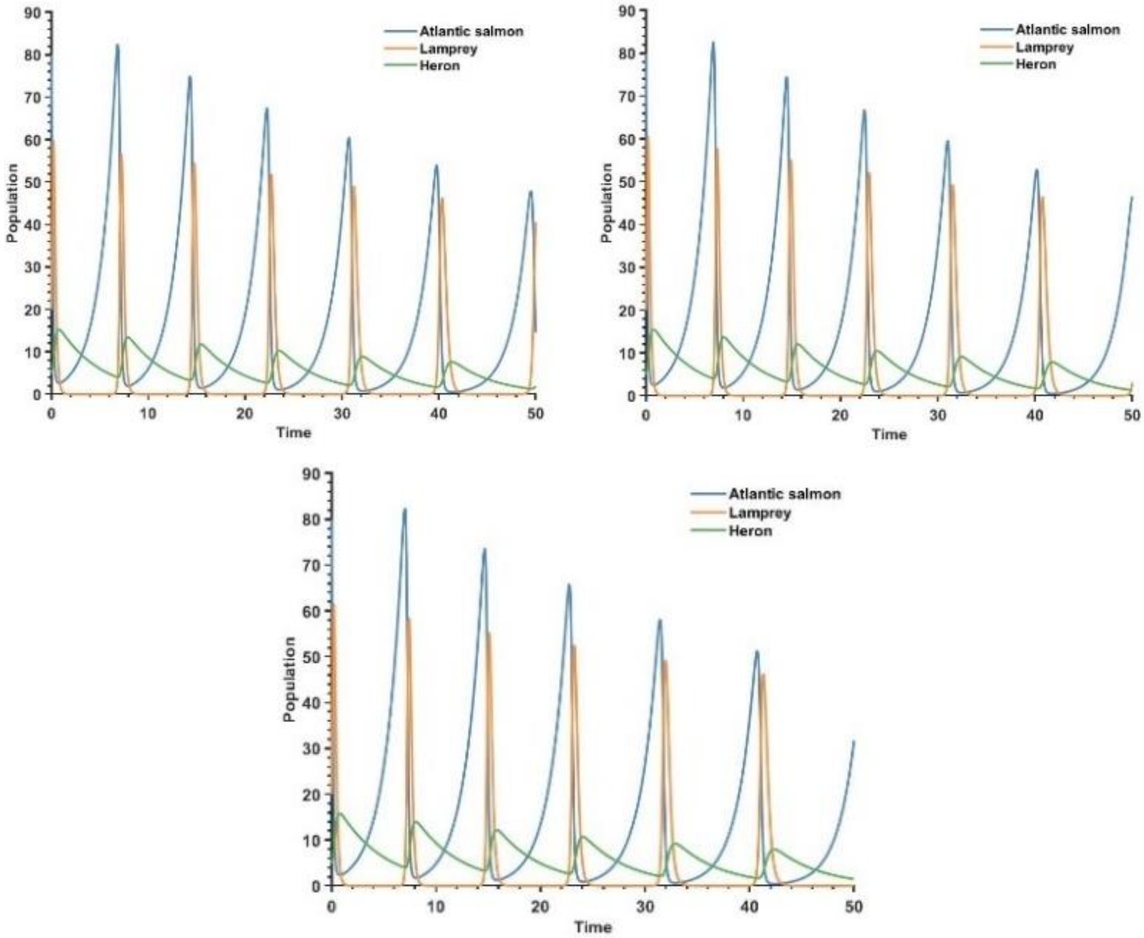
(b) male rate = 0.66



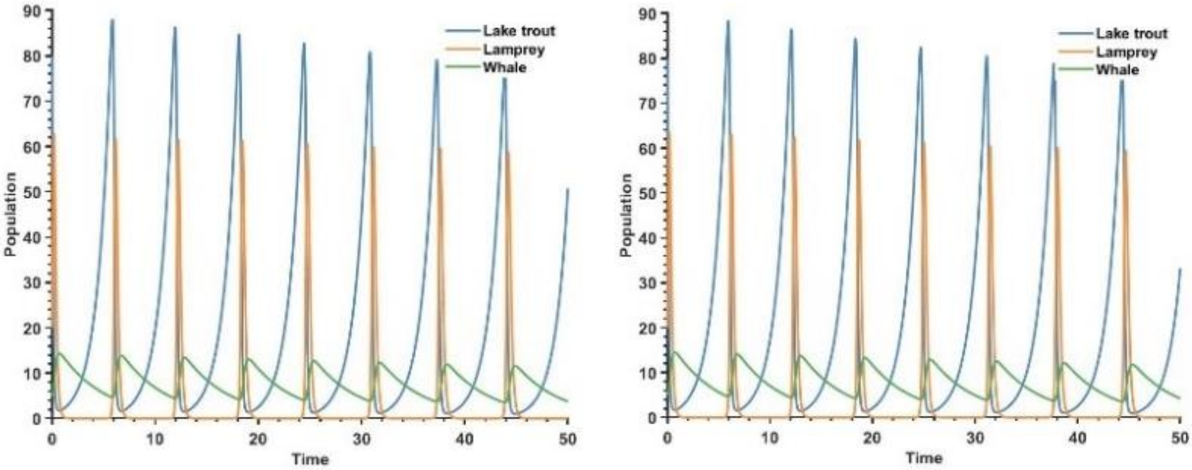
(c) male rate = 0.76

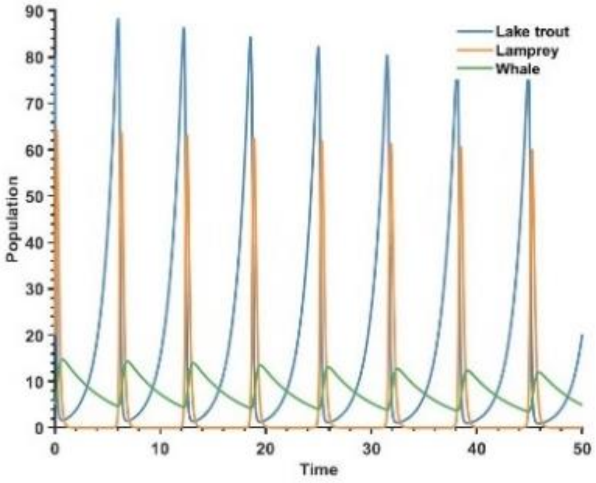
Figure 1. Plot of predator and prey versus proportion of males

Based on equation (2), this paper simplifies the food web into 12 different food chains and randomly shows the effects of sex ratio changes on the three food chains in the same period. In this paper, three different male ratios of 0.56, 0.66 and 0.76 were selected and the dynamics of the number of various populations in the food chains under these ratios were shown, which were found to be significantly correlated with the sex ratio of the lamprey. The greater magnitude of population peaks in Atlantic salmon with increasing male ratios is attributed to lower reproductive rates at higher lamprey male ratios, resulting in reduced predation pressure on the population. In contrast, the peak in the population of herons feeding on lampreys was less pronounced, possibly due to reduced food availability, as shown in Figure 2.

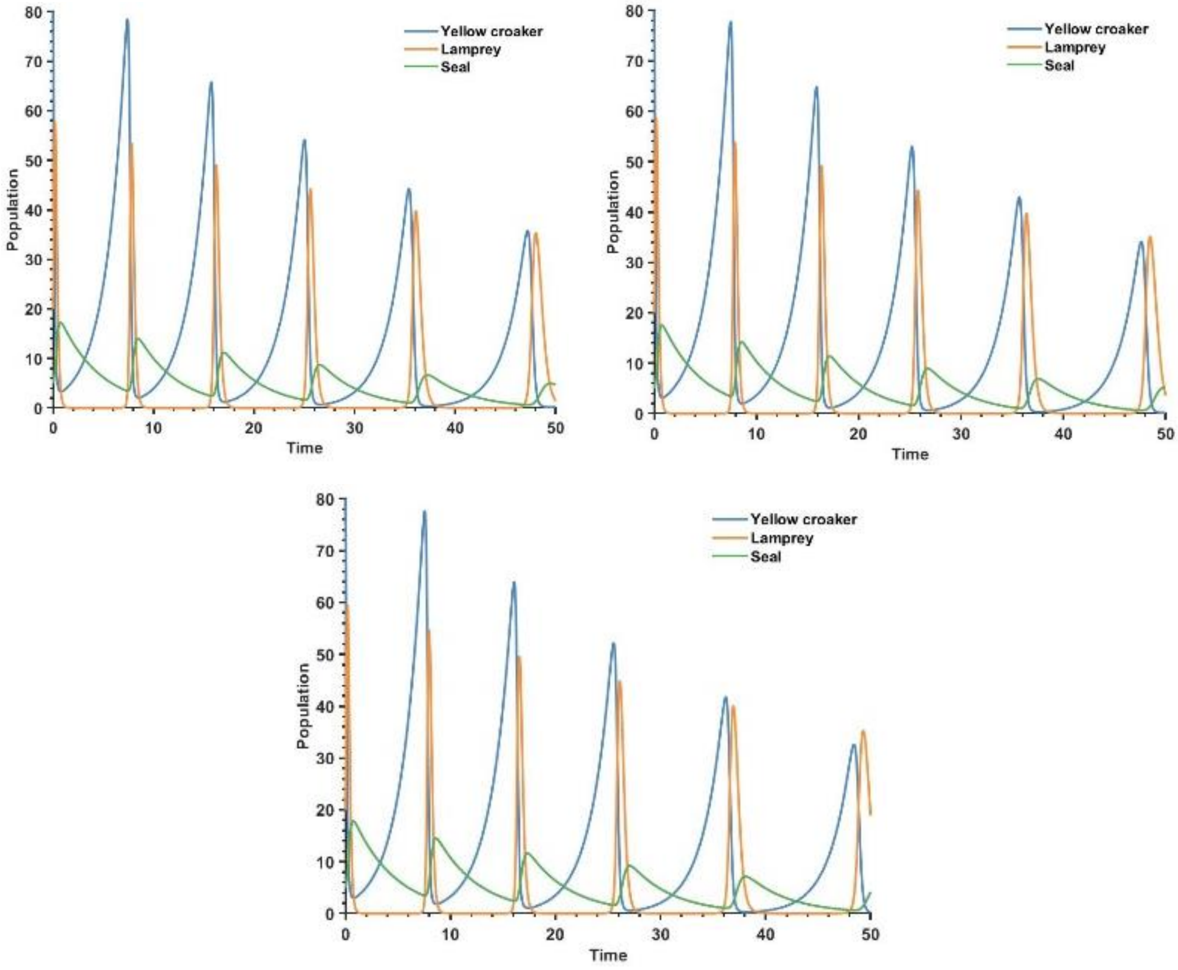


(a) Food Chain: Atlantic salmon - Lamprey - Heron





(b) Food Chain: Lake trout – Lamprey – Whale



(c) Food Chain: Yellow croaker - Lamprey – Seal

Figure 2. Time-lapse plots of various groups of the food chain

3. Modelling the "Population and Behavioral Dynamics" of the Lampreys

Using the Population and Behavioural Dynamics Model (IPBDM), the aim of this paper is to analyse the effects of changes in the sex ratio of lamprey populations from a complementary perspective. On the one hand a modified one, incorporating sex-specificity, was used to assess the dynamic effects of sex ratio on populations, such as reproduction and mortality. On the other hand an Agent-Based model is used, where each agent in the model represents an individual lamprey, endowed with specific behavioural parameters [8, 9], which determine how they perceive their environment, forage, reproduce, and interact with conspecifics Lotka-Volterra model. This complementary approach allows for a comprehensive exploration of the aggregate and individual factors affecting lamprey populations.

3.1. The dynamic Lotka-Volterra model

Using the Lotka-Volterra equation, this paper sets the interval of change of the male proportion in [0.56, 0.78], and adjusts the intrinsic growth rate and predation probability based on this interval. As shown in Figure 3, by setting the model initiation parameters of predators and prey, this paper simulates the dynamic changes of the population in a specified time period, demonstrating how the number of lampreys, their food sources, and the number of predators fluctuates with the changes in the proportion of males.

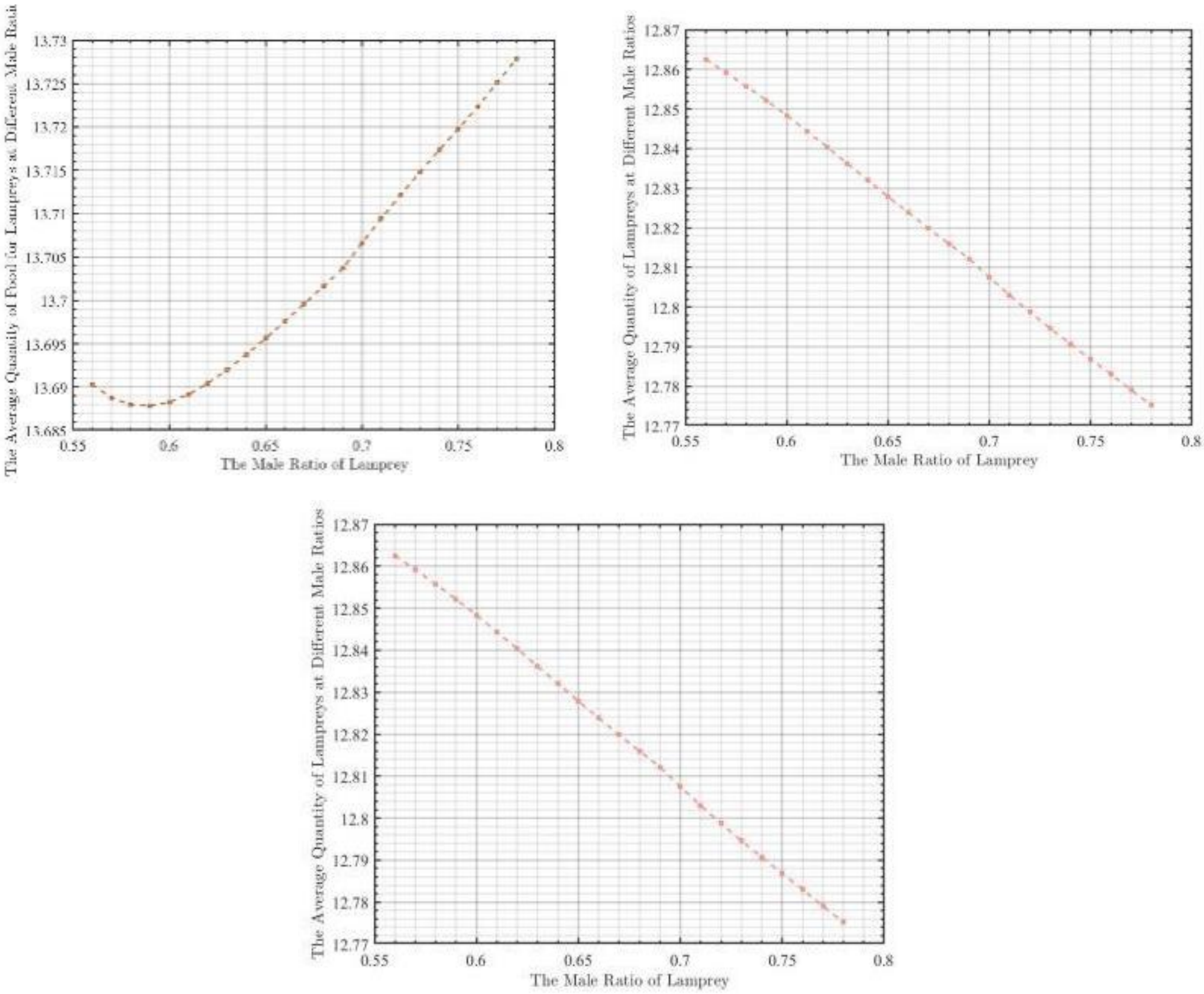


Figure 3. The Average Quantity of Three Populations at Different Male Ratios

Population declines were correlated with an increase in the proportion of males in lampreys, suggesting potential reproductive or survival challenges. The correlation between higher male proportions and lower food availability suggests greater competition for resources. In addition, predator numbers peaked at moderate levels, suggesting a complex predator-prey dynamic influenced by the sex composition of prey populations. The population dynamics in Figure 3 reflect the complex relationships within aquatic ecosystems and the importance of balanced sex ratios for the stability of lamprey populations.

3.2. Constructing an Agent-Based Model of the Lampreys

In this Agent-Based model, the paper is set up as follows: the initial male ratio is 0.78, the agent startup parameter is 1000, the total run step size is 100, the various populations undergo processes affected by food shortages and life stages (including reproduction at 10 years of age and maximum lifespan at 50 years of age), the survival and reproduction of the agents depend on food availability, and the likelihood of males finding food is Higher. Stochastic events such as mortality or changes in sex ratio are allowed to occur, and agents may migrate within a certain radius.

As shown in Figure 4, the proportion of males in lampreys declined dramatically over time to an equilibrium level. This trend may indicate that over a span of 100 time units, factors in the model are driving male proportions towards a new equilibrium. The rapid decline suggests that there may be excessive competition for resources or high male mortality. Levelling off of proportions reflects adaptive processes in the population or a new equilibrium attributed to factors such as mating strategies, sex-specific survival rates or human-induced changes in the ecosystem. The stabilisation of the proportion of males at low levels is broadly consistent with the phenomenon observed in natural populations, where this trend is also essential for the sustainability of the species.

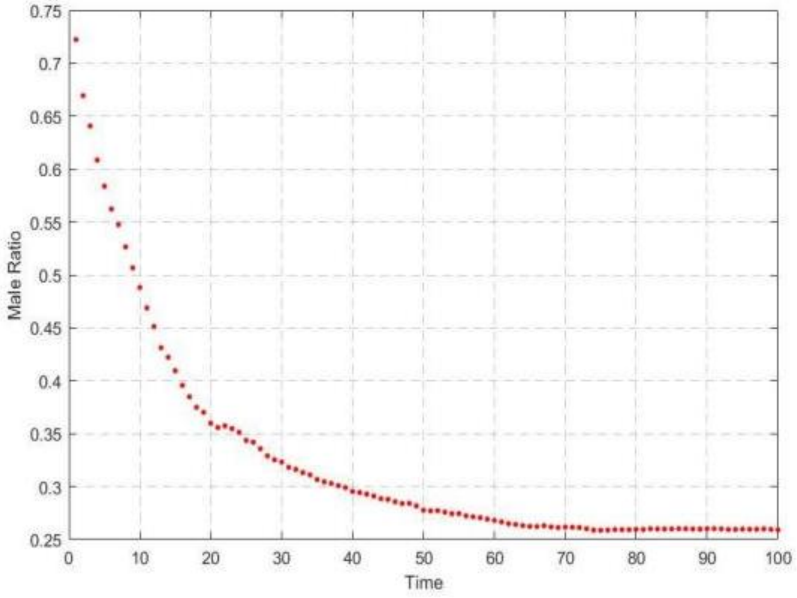


Figure 4. Time-lapse plot of the proportion of males

4. Exploring the ecological stability of the lampreys

The Lotka-Volterra model demonstrates the dynamic relationship between two populations, prey and predator, through two differential equations [10], while the stability point analysis reveals the conditions under which prey and predator reach equilibrium in the system. In this paper, the stability of the system is analysed , based on the eigenvalues of the Jacobi matrix at the stability points [11]. If the real part of all eigenvalues is less than zero, the system is stable at that point; on the contrary, if the real part of eigenvalues is greater than zero, the system is unstable at that point.

4.1. Modelling prey and predators

Taking the lamprey population as an example, the Lotka-Volterra based differential equation is reconstructed in this paper:

$$\frac{dx}{dt} = \alpha_2 x - \beta_2 xy \tag{6}$$

$$\frac{dy}{dt} = \delta_2 xy - \gamma_2 y \tag{7}$$

Where x is the number of prey, α_2 is the natural growth rate of prey, y is the number of predators, and β_2 is the probability that a predator will prey on the bait. δ_2 is the growth rate of the predator after successful predation, γ_2 is the natural mortality rate of the predator.

4.2. Jacobi matrix solution and stability analysis

The stabilisation points can be expressed as $\left(x^*, y^*\right)$, where $* dx/dt = 0$ and $dy/dt = 0$. Usually, solving the post-differential equation yields the origin $(0,0)$ and $\left(\gamma_2/\delta_2, \alpha_2/\beta_2\right)$ two stable points.

The Jacobi matrix J at the stabilisation point $\left(x^*, y^*\right)$ is defined as:

$$J = \begin{bmatrix} \frac{\partial}{\partial x}\text{big}\left(\alpha_2 x - \beta_2 xy\,\text{big}\right) & \frac{\partial}{\partial y}\text{big}\left(\alpha_2 x - \beta_2 xy\,\text{big}\right) \\ \\ \frac{\partial}{\partial x}\text{big}\left(\delta_2 xy - \gamma_2 y\,\text{big}\right) & \frac{\partial}{\partial y}\text{big}\left(\delta_2 xy - \gamma_2 y\,\text{big}\right) \end{bmatrix}_{\left(x^*, y^*\right)} \tag{8}$$

New Jacobi matrices J_1 can be constructed after extending the Lotka-Volterra model to a three-dimensional space containing prey (G), lampreys (R) and natural enemies (E), which are equal at the points(G, R, E):

$$J_1 = \begin{bmatrix} \frac{\partial}{\partial G}(\alpha G - \beta_1 GR) & \frac{\partial}{\partial R}(\alpha G - \beta_1 GR) & \frac{\partial}{\partial E}(\alpha G - \beta_1 GR) \\ \\ \frac{\partial}{\partial G}(\beta_1 GR - \gamma_1 R - \beta_2 RE) & \frac{\partial}{\partial R}(\beta_1 GR - \gamma_1 R - \beta_2 RE) & \frac{\partial}{\partial E}(\beta_1 GR - \gamma_1 R - \beta_2 RE) \\ \\ \frac{\partial}{\partial G}(\delta RE - \gamma_2 E) & \frac{\partial}{\partial R}(\delta RE - \gamma_2 E) & \frac{\partial}{\partial E}(\delta RE - \gamma_2 E) \end{bmatrix} \tag{9}$$

In this matrix, each element represents the effect of the rate of change of the system at a given point on the number of various populations at that point. By calculating the eigenvalues of the Jacobi matrix, this paper is able to understand the local stability of the system at that point. If the real part of all eigenvalues is negative, the point is locally stable; if the real part of any of the eigenvalues is positive, the point is locally unstable.

In order to gain insight into the temporal dynamics of the lamprey population and its interactions with predators, the Jacobi matrix was solved numerically using the ode45 solver of MATLAB. In this paper, a set of initial conditions is established to reflect potential changes in population size in real ecosystems. For each set of initial conditions, numerical trajectories of prey and predator populations were tracked and the data points were connected to form phase trajectories, revealing the behavioural patterns of the system under different conditions.

As shown in Figure 5, a male ratio of 0.56 indicates more stable interactions and less oscillation between prey and predator, suggesting a balanced ecosystem. In contrast, when the male ratio was 0.76, the system experienced great fluctuations, indicating system imbalance. In addition, the phase trajectories did not exhibit closed periodic paths, but rather converged towards a stable point, suggesting that the population system of prey, lampreys, and predators would gradually stabilise without complex dynamics such as limit rings or chaos. The significant differences in male ratio trajectory patterns highlight the critical role of sex ratio in ecological balance, which has important implications for biodiversity conservation and species management strategies.

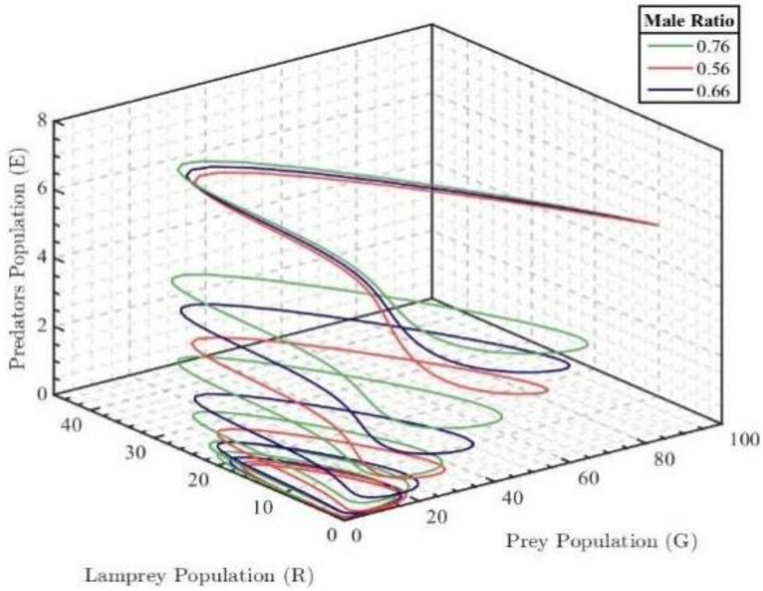


Figure 5. Phase trajectory diagram of the population system

The results of different male ratio simulations show patterns of ecological stability and instability within the system. After calculation, as shown in Table 1 in this paper, it was found that stable conditions could be observed at male ratios of 0.57, 0.58, 0.72, 0.73, 0.77 and 0.78, where all the eigenvalues were negative, suggesting that the system was able to return to equilibrium after disturbance. The other scaling values correspond to eigenvalues that are negative, will the system tends to a sharp increase or decrease in numbers. This ultimately leads to ecological damage.

Table 1. Partial eigenvalue solution for Jacobi matrix

Male Ratio	Eigenvalue 1	Eigenvalue 2	Eigenvalue 3	Stability
0.57	-0.5626+0.2555i	-0.5626-0.2555i	-0.026	Stable
0.58	-0.3479 + 0.1349i	-0.3479 - 0.1349i	-0.1162 + 0.0000i	Stable
0.72	-0.3911 + 0.1953i	-0.3911 - 0.1953i	-0.0736 + 0.0000i	Stable
0.73	-0.408	-0.0338	-0.1442	Stable
0.77	-0.5005	-0.3047	-0.2336	Stable
0.78	0.78	-0.201	-0.2284	Stable

5. Conclusion

The three models used in this paper are robust and credible. The Lotka-Volterra model successfully captures the dynamics of the ecosystem under different proportions of lamprey males, adapts to different parameter settings, and proves its robustness under a range of environmental conditions. In addition, the combination of Lotka-Volterra differential equations and Agent-Based modelling provided a scientifically sound framework for exploring the impacts of changing sex ratios in lamprey populations. The use of Jacobi matrices and eigenvalues to simulate the dynamics of the system in which the lamprey survives, i.e., the support of the data and the visualization of the dynamics of the changes provide a high degree of confidence in the experimental results.

In addition, the reader is encouraged to creatively optimize the model in this paper, e.g. by introducing additional variables such as genetic diversity, morbidity and mortality, and population-specific competitive relationships, which are highly supportive of the complexity and accuracy of the model.

of course, further refinement of the model especially in terms of modelling complex interactions in ecosystems that may affect migration and population dynamics is also a very interesting research direction.

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