

Analysis of impacts on larger ecosystems resulting from dynamic sex ratio of lampreys based on Lotka-volterra and Predator model

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Abstract. lampreys exhibit a unique characteristic which it can adjust their sex ratio in response to environmental resource availability. Their population changes in turn affects the environment stability. This study scrutinizes the decline in population numbers and the destabilization of the environment by simulating changes in the male-female ratio of lampreys under nutrient-deficient conditions. To accurately predict environment stability which caused by the male-female ratio of lampreys under nutrient-deficient conditions, combined with the advantages of Lotka-Volterra and predator models, a dynamic sex ratio population prediction model for large data is constructed. then concluded the dynamic sex ratio resulting in an irreversible vicious circle of environment. The paper provides a new vital influence factor and solution idea for future sustainable management of ecological environment, which is regarded as a unique valuable reference for scientifically enhancing environmental stability and safeguarding the ecological environment at both population and environmental levels.

Keywords: Dynamic sex ratios, Differential equation dynamic prediction models, Lotka-Volterra models, Predator-prey models.

1. Introduction

The sex of most species in the world are fundamentally either male or female with a 1:1 sex ratio at birth,^[1] Lampreys are such special animals, playing complex roles in the ecosystem with dynamic sex ratio^[2]. Some are regarded as significant parasites with ecological impacts, while others serve as a food source in some regions, resulting unignorable effects to species and ecosystem^[3].

Current studies suggest that the sex ratio can vary significantly due to external environmental factors^[4], primarily depending on their growth rate during the larval stage^[5]. However, these studies only considered the amount of available food, neglecting the numerous factors that affect nutrient supply. Considering multiple factors and theoretical frameworks, including resource availability theory and ecological theories, among others^[6]. Species can exhibit adaptive behaviors based on resource availability, such as the impact of food supply levels on the growth rate of lamprey larvae, which subsequently influences their probability of becoming male or female. Moreover, changes in sex ratio may have repercussions for population survival rates, reproductive capacity, and competitive strength, thereby affecting the overall stability of ecosystems^[7].

Based on the problem that the stability of ecological environment is closely related to the ratio of male to female in population. A multi-trophic level prediction model for phytoplankton, predators, and lampreys is created to provide insights into the resulting interactions in an ecosystem caused by dynamic sex ratio.

2. The Differential equation Model

2.1. The structure of Differential Equation

Lampreys hold a distinctive position within the ecosystem's trophic web, directly influencing the health and distribution of their host populations, thereby affecting the nutrient cycling and energy flow of the entire aquatic ecosystem. They may assume various roles in the trophic web at different life stages, ranging from filter feeders to apex predators or parasites. Their impact on the physical

characteristics of habitats and their potential regulatory effect on the size and distribution of host populations may play a pivotal role in maintaining the dynamic equilibrium within ecosystems^[8].

Lampreys, through their parasitic lifestyle, potentially regulate the size and distribution of the host population. This regulatory mechanism may play a vital role in maintaining the dynamic balance within the ecosystem. By controlling the number of certain fish hosts, lampreys may indirectly foster the preservation of ecological diversity. Differential equations can be employed to describe discrete time systems, with their general form being:

$$x_{k+1} = ax_k + b, k = 0,1,2,.. \quad (1)$$

Here, by utilizing differential equations separately to characterize the rate of change in the total lamprey population (N), the number of female lampreys (N_F), and the number of male lampreys (N_M) over time.

Since the population size is jointly determined by the birth and death rates of the population, a differential equation can be constructed to depict the dynamic variation of the lamprey population size N over time:

$$\frac{dN}{dt} = \text{Birth rate} - \text{Death rate} \quad (2)$$

The birth rate of female lampreys depends on the food availability level (R) and a parameter α that quantifies the extent of the influence of food supply on the birth rate of female lampreys. Hence, the birth rate for female lampreys is represented as such (1), and when combined with the death rate of females, it yields a differential equation describing the rate of change in the number of female lampreys:

$$\frac{dF}{dt} = \alpha \cdot R - D_F \quad (3)$$

For male lampreys, the rate of change follows a similar pattern to females, where their birth rate also depends on the food availability level (R) and a parameter which characterizes the effect of food supply on the birth rate of male lampreys. Therefore, combining this birth rate with the death rate leads to the differential equation describing the rate of change in the number of male lampreys:

$$\frac{dM}{dt} = \beta \cdot R - D_M \quad (4)$$

Employing the Euler method for numerical solution, a time step dt of 0.01 years was set, and the total simulation duration t_{max} was established as 25 years, thereby calculating a total number of num_steps . Utilizing the Euler approach. Specifically, within every time step, the changes in the male (dM_dt) and female (dF_dt) population numbers were computed based on growth rates and death rates influenced by resource utilization. Consequently, the overall change in the total population (dP_dt) was derived. Concurrently, the rate of change in ecosystem stability (dS_dt) is defined as a contingent upon the variation in the ratio between the female and male population sizes^[9].

$$\begin{cases} dM_dt = \alpha \cdot R_{i-1} \cdot \rho - \beta M_{i-1} \\ dF_dt = \gamma \cdot R_{i-1} \cdot \rho - \beta F_{i-1} \\ dP_dt = dM_dt + dF_dt \\ dS_dt = k \left(\frac{dF_dt}{M_{i-1}} - F_{i-1} \cdot \frac{dM_dt}{M_{i-1}^2} \right) \end{cases} \quad (5)$$

$$\begin{cases} M_i = M_{i-1} + dt \cdot dM_{dt} \\ F_i = F_{i-1} + dt \cdot dF_{dt} \\ P_i = P_{i-1} + dt \cdot dP_{dt} \\ R_i = R_{i-1} \\ S_i = S_{i-1} + dt \cdot dS_{dt} \end{cases} \quad (6)$$

2.2. Population Dynamics Model

The above model elucidates the impact of sex ratio on lamprey population size.

Step 1: Analysis of Relevant Factors in Population Dynamics Models

In accordance with ecological principles, the primary factors to consider include: the growth rate of phytoplankton, the grazing rate of prey on phytoplankton; the reproduction and mortality rates of the prey; as well as the predation, reproduction, and mortality rates of lampreys.

Step 2: Establishment of Difference Equation Models

In this model, the critical variables to be expressed are the population size of phytoplankton (y_1), the prey population (y_2), and the lamprey population (y_3).

The population size of phytoplankton y_1 is determined by its natural growth and consumption by herbivores. The natural growth of phytoplankton can be represented by a variable ry_1 , while the predation of food by prey can be denoted by ay_1y_2 . The differential equation describing the change in the phytoplankton population y_1 with respect to time t is formulated as:

$$\frac{dy_1}{dt} = ry_1 - ay_1y_2 \quad (7)$$

For the prey population y_2 , its dynamics are governed by its reproduction, natural mortality, and death caused by parasitism from lampreys. The prey's natural reproduction can be represented by bay_1y_2 , its natural death rate by $m \cdot y_2$, and death due to predation by cy_2y_3 . The differential equation that describes the variation of the prey population y_2 over time t is constructed as:

$$\frac{dy_2}{dt} = b \cdot a \cdot y_1y_2 - m \cdot y_2 - c \cdot y_2y_3 \quad (8)$$

Finally, the population of lampreys y_3 is influenced by their reproductive success through predation and natural mortality. Reproduction via predation on prey can be denoted by dcy_2y_3 , and natural death by ny_3 . Therefore, the differential equation outlining the change in the lamprey population y_3 over time t is established as:

$$\frac{dy_3}{dt} = dcy_2y_3 - ny_3 \quad (9)$$

Step 3: Solution of Difference Equation Models

To solve this system of differential equations, it needs to specify the initial population sizes for each of the three biological species at the starting time. Assume that at the initial moment, the populations of phytoplankton, prey, and lampreys are all 10 units: $y_0 = [10,10,10]$. Set the time interval as $[0,100]$, which means simulating the changes in the ecosystem from the initial time up to the 100th time unit^[10].

Upon solving the system, visualize the results by plotting the curves showing the variation of each of the three species' population sizes over time. This will enable an intuitive analysis of how changes in sex ratio affect the ecosystem dynamics. Lampreys population data such as birth and death rates come

from High predation of native sea lamprey during spawning migration | Scientific Reports (nature.com) and Growth and annual survival estimates to examine the ecology of larval lamprey and the implications of ageing error in fitting models - PubMed (nih.gov).

3. Results

3.1. The establishment of simulation model

The time series data obtained during the simulation were plotted to create charts that visually represent the temporal trends in the male population, female population, total population, and ecosystem stability. The figure, “Visualization of all variables,” demonstrated how, under the influence of resource availability parameters (indirectly represented by the variable ρ), there is a dynamic interplay between the number of females and males within the lamprey population. As shown in Figure 1. This leads to a notable gradual decline in ecosystem stability over time. Specifically, after approximately 23 years, the ecosystem stability decreased to 0.825 and gradually approached stability. The stability is shown in Figure 2.

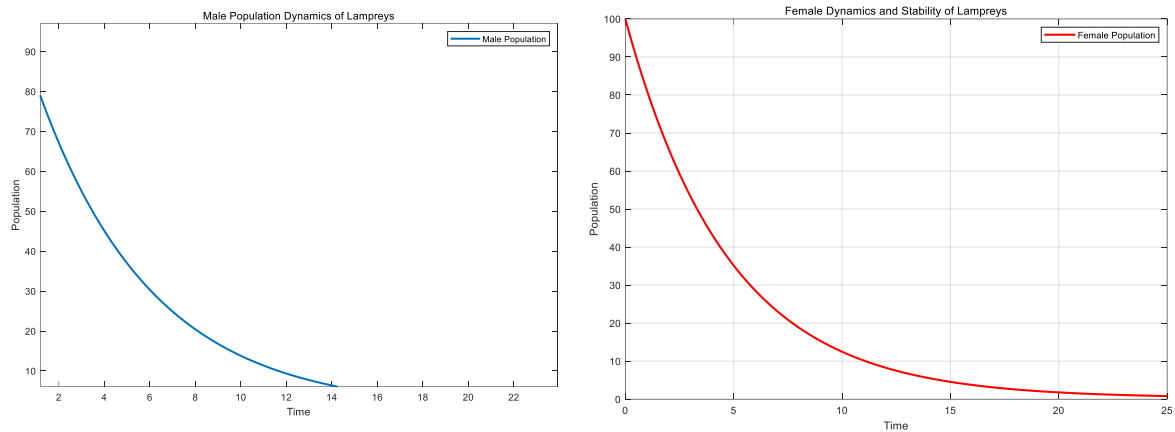


Figure 1 Dynamics of the male and female population

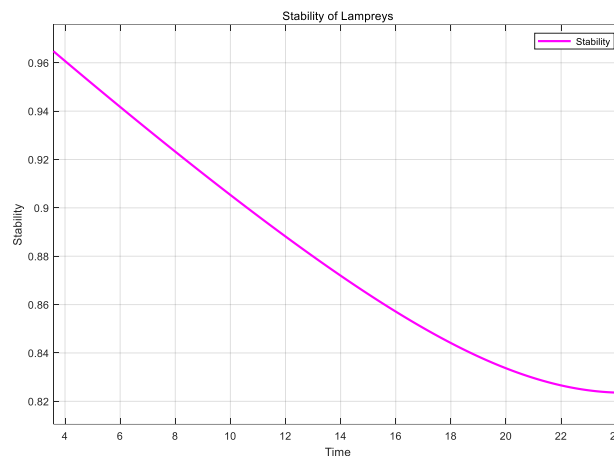


Figure 2 Dynamics of environment stability

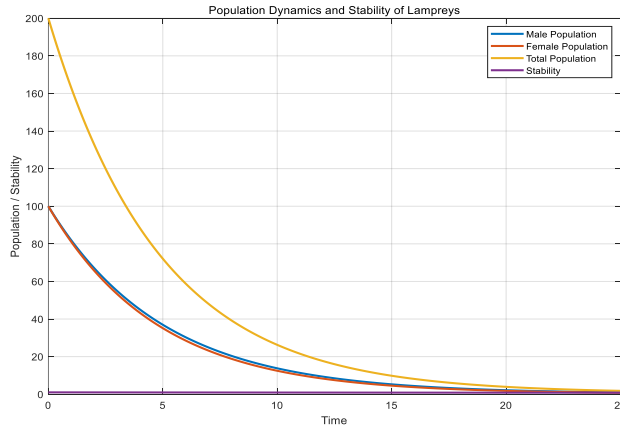


Figure 3 Visualization of all variables

The Figure.3 Visualization of all variables demonstrated how, under the influence of resource availability parameters (indirectly represented by the variable ρ), there is a dynamic interplay between the number of females and males within the lamprey population. This leads to a notable gradual decline in ecosystem stability over time. Specifically, after approximately 23 years, the ecosystem stability decreased to 0.825 and gradually approached stability.

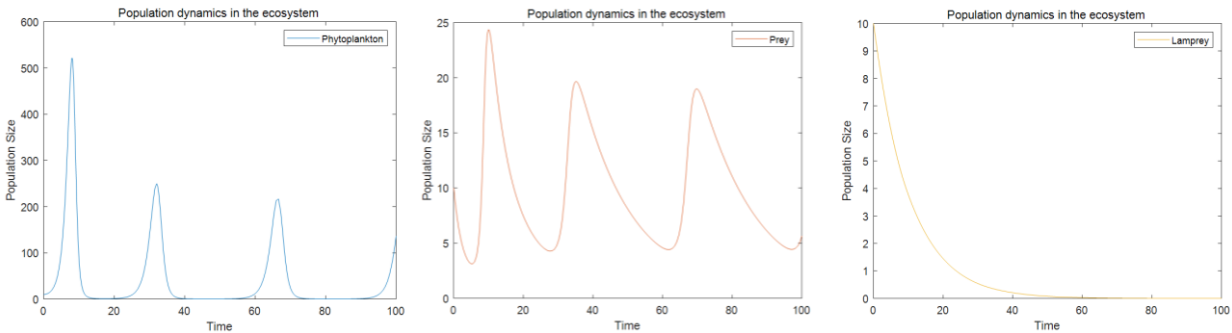


Figure 4: Dynamics of phytoplankton, prey and lamprey populations in ecosystem

Through Figure 4 Dynamics of phytoplankton, prey and lamprey populations in ecosystem , although the population of lampreys continues to decline in the ecosystem, and the number of plankton also shows a periodic decline, especially with a significant difference between the initial two population peaks, the prey population still shows a relatively stable periodic fluctuation curve, but the rates of growth and decrease tend to be flat, balancing with the population of lampreys.

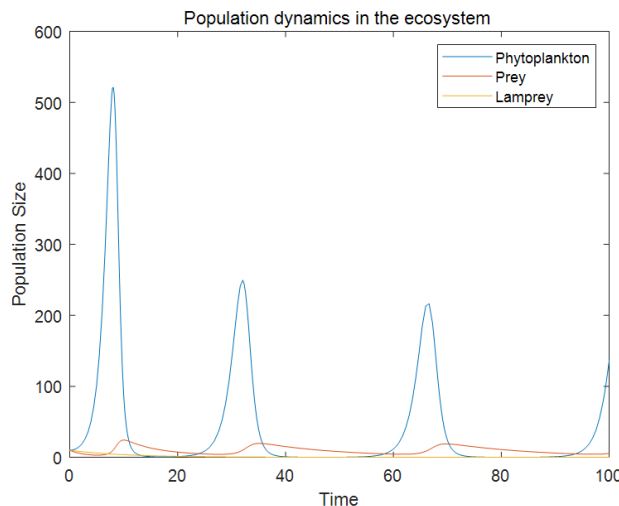


Figure 5. Population dynamics in ecosystem

As shown in Figure 5. Population dynamics in ecosystem, in the continuous fluctuation of the number of plankton and prey, the population of lampreys is still in a shrinking trend, but the gender changes are far apart. Males first experience rapid group expansion under stress, while females first experience a cliff like decline, followed by both tending to flatten out.

3.2. Analysis of experimental results

Through this model, there are valuable insights into the significant impact that shifts in sex ratios within the lamprey population, as influenced by population size, have on their survival, reproduction, and ultimately, the overall stability of the ecosystem. Through theoretical analysis and numerical simulation, the specific impacts of changes in sex ratio on the dynamics of the lamprey population and the stability of ecosystems are presented in detail. The change in sex ratio of lampreys can have a significant impact on the ecosystem, especially for plankton in the environment. Often, a surge in males means a decrease in overall numbers, which maintains the high predatory ability of lampreys and promotes a balance in predator-prey relationships in the ecosystem. However, an increase in the male ratio of lampreys in the absence of nutrients can delay the decline of the population, but it will destroy the stability of the environment and create a vicious circle.

By integrating the sex ratio and reproduction rate using a population dynamics model to simulate changes in the lamprey population over time, a deeper understanding of the impact of management and conservation measures on lamprey populations is provided. This model can help predict the long-term effects of environmental changes on the structure and size of lamprey populations, providing a basis for formulating scientific management strategies.

4. Conclusions

In this paper, the Lotka-Volterra and Predator model is adopted under the background that lampreys have dynamic sex ratios. The dynamic sex ratio of lampreys has a certain role in resisting population decline caused by insufficient nutrients, but the increase of male ratio will lead to the decline of other populations, indirectly destroying population stability, resulting in an irreversible vicious circle. This study underscores the importance of considering sex ratios in population dynamics and conservation efforts. It highlights the need for further research to fully understand the implications of dynamic sex ratios on ecosystem stability and biodiversity.

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