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Modeling the Great Lakes Basin based on dynamic network flows

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Abstract. The Great Lakes are the largest freshwater lakes in the world and involve various stakeholders. the paper develop a Basin model of the Great Lakes based on dynamic network flow and propose an optimization algorithm for dam control to obtain the best water level.First, We regard lakes and rivers in the Great Lakes as edges and nodes of a dynamic network flow to build a Dynamic Hydrological Network Flow Model where the capacity of edges relate to lakes'maximum water levels and flow is transmitted through the rivers. To explore the interaction between lakes and rivers in the basin, the paper first model lakes by establishing a water balance equation that analyzes the change in water level and fitting results to confirm the correctness of our model. Then the paper use collected data to reveal that the flow depends on the water level of the upstream and downstream Lakes. Finally, analyze the mutual relation between lakes and rivers in the St. Lawrence Basin.

Keywords: Dynamic Hydrological Network Flow, Multivariate Multi-objective Planning Model, Bidirectional Process.

1. Introduction

The Great Lakes are the world's largest group of freshwater lakes, comprising 21 percent of the freshwater on the Earth's surface. In addition, the water in the lakes is used for various purposes, water levels are determined by the amount of water entering and leaving the lakes, and changes in water levels can significantly impact stakeholders. Water levels in the Great Lakes result from the interaction of several manmade and natural factors, with major flow control mechanisms including the Compensating Works Compensation Works at Sault Ste. Marie and the Moses Saunders Dam in Cornwall. Dynamic control of lake water inflows and outflows is particularly important as local jurisdictional policies and seasonal and environmental changes in the water basin have different expected impacts, affecting the ecosystems of the Great Lakes region, with significant impacts on stakeholders.(Data sources:https://www.glerl.noaa.gov,https://www.greatlakescc.org)

2. Dynamic Hydrological Network Flow Model of St. Lawrence Basin

2.1. Motivation of Dynamic Network Flow

In order to obtain the spatial and temporal distribution of hydrological characteristics in the St. Lawrence Basin, the paper establish a hydrological model[1] of the Great Lakes based on dynamic network flow. Among them, each side represents a lake, the maximum water level of the lake determines the capacity of the side, and the node represents the river, the flow is transmitted between the lakes through the river, and finally reaches the sink from the source point, which is considered as Upstream tributaries, glaciers and the Atlantic Ocean.

Although Lake Michigan and Lake Huron are often considered to be two separate lakes, the paper consider them to constitute a single lake domain because they are connected to each other by the St.Mary River, and because the relative depth and width of waterways results in the average level of the surfaces of the two lakes converging to the same level over a long period of time. At the same time, the paper consider Lake St.Clair. i.e., five lakes as edges of the dynamic network flow.

The dynamic network flow the paper build satisfies the basic properties of network flows:

Capacity constraints: for each edge,i.e., lake, there is a certain capacity that will not be exceeded.

Flow constancy: the paper have a net river flow because the river constantly flows and the rise in water level is insignificant. The chained dynamic network flow model the paper modeled is shown in the Figure 1 below:

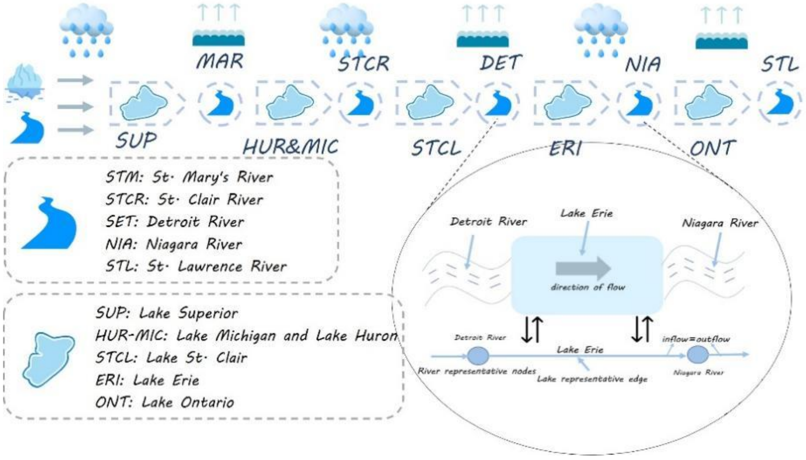


Figure 1: Dynamic network flow model

By modeling the hydrological phenomena in the St.Lawrence Basin through a dynamic network flow model, the paper want to discover the correlation between the lake level and the river flow to regulate the flow of the river, and thus the lake level, by managing the dams on the river.

2.2. From River Model and Lake Model to Basin Model

According to Science Water Loss from the Great Lakes[2], changes in the water levels ofthe Great Lakes are mainly influenced by precipitation, evaporation, river inflow and outflow, and landmark runoff. Accordingly, the paper firstly obtain the water level change through the change in the water volume of the lakes:

$$\Delta L = \frac{\Delta V}{A} \tag{1}$$

A is the area of the lake, ΔL isthe water level increase.

A water balance equation is developed for a single lake to obtain the change of water volume:

$$\Delta V = P - E + Q_{in} - Q_{out} - I \tag{2}$$

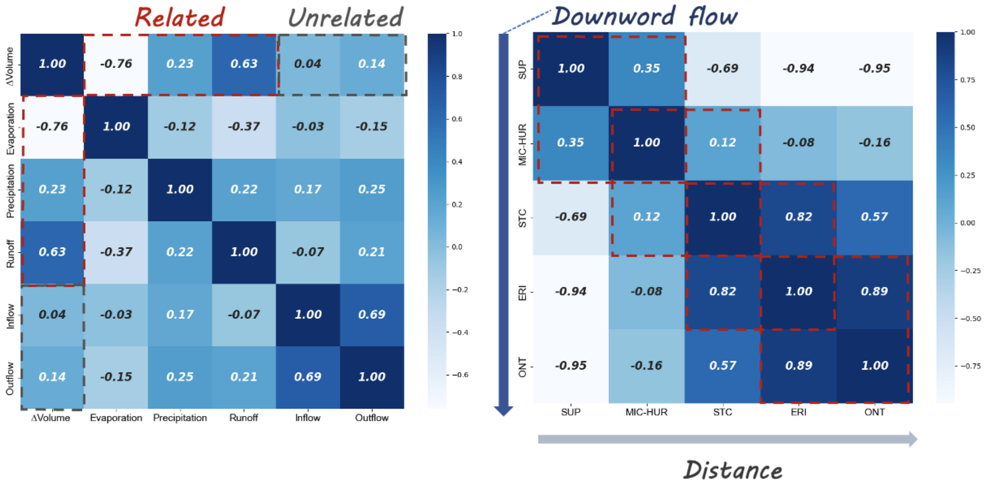
where ΔV represents the change in the water volume over a specific time period, Q_{in} denotes the total flow into the lake and Q_{out} denotes the total flow out of the lake, P represents precipitation, E and I represent evaporation and runoff versus groundwater respectively.

Next, the paper count the monthly volume growth of the five lakes (SUP, MIC-HUR, STC, ERI, ONT), which can be formulated as:

$$\Delta L = \frac{\Delta V}{A} \tag{3}$$

where ΔL isthe water level on the first day of next month minus the water level on the first day of current month.

By analyzing the volume growth for the Great Lakes, the paper found a clear correlation between the volume growth of the five lakes. volume growth between lakes near each other tend to be more correlated, as shown in Figure 2 below by the darker colors of the closer grids.



	ΔVolume	Evaporation	Precipitation	Runoff	Inflow	Outflow
ΔVolume	1.00	-0.76	0.23	0.63	0.04	0.14
Evaporation	-0.76	1.00	-0.12	-0.37	-0.03	-0.15
Precipitation	0.23	-0.12	1.00	0.22	0.17	0.25
Runoff	0.63	-0.37	0.22	1.00	-0.07	0.21
Inflow	0.04	-0.03	0.17	-0.07	1.00	0.69
Outflow	0.14	-0.15	0.25	0.21	0.69	1.00

	SUP	MIC-HUR	STC	ERI	ONT
SUP	1.00	0.35	-0.69	-0.94	-0.95
MIC-HUR	0.35	1.00	0.12	-0.08	-0.16
STC	-0.69	0.12	1.00	0.82	0.57
ERI	-0.94	-0.08	0.82	1.00	0.89
ONT	-0.95	-0.16	0.57	0.89	1.00

Figure 2: Correlation coefficient matrix of different factors and different lake levels

As shown in the Figure 2 above, this is consistent with our model of dynamic network flows:the Great Lakes are connected by natural waterways, forming a continuous system of water bodies. This connection allows changes in water levels in the upstream lakes to affect the downstream lakes directly.

To validate model, the paper fit the data of five lakes separately. evaporation data, precipitation data, and runoff data have been collected from relevant websites. Firstly, the monthly flow volume VF of the river is obtained from the flow of the river

$$VF = F \cdot T \tag{4}$$

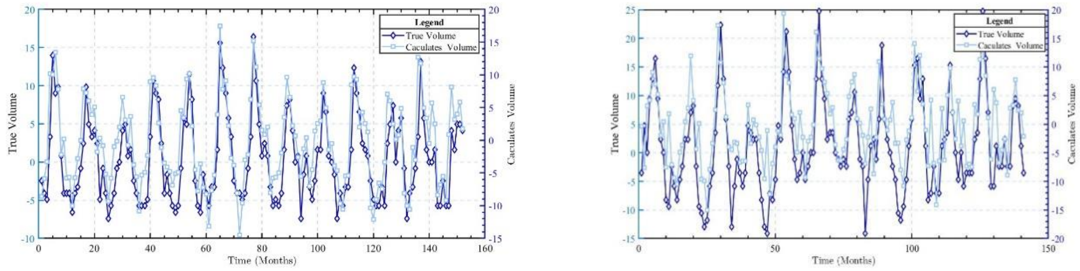
The paper assume that all lakes within the Great Lakes flow out of the corresponding inflow river Rin in the network:

$$Q_{in} = VF_{R_{in}} \tag{5}$$

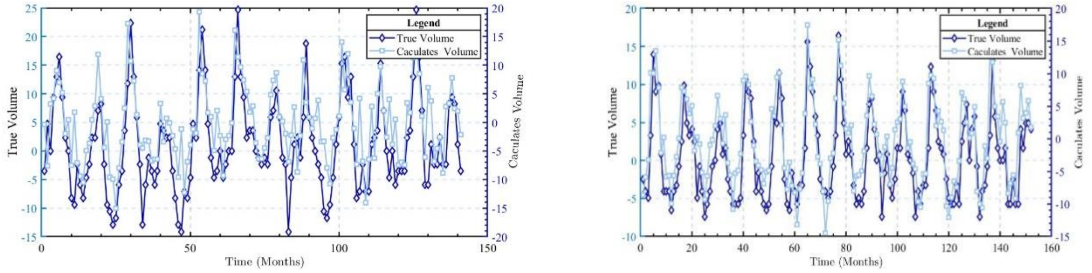
The paper assume that all lakes within the Great Lakes flow out of the corresponding outflow river Rout in the network:

$$Q_{in} = VF_{R_{in}} \tag{6}$$

The paper fitted the data for the Great Lakes and the results of the fit are shown in Figure 3.



(a) SUP (b) MIC-HUR



(c) ERI (d) ONT

Figure 3: Fitting the net increase in water volume

Prof.Andrew Gronewold of the University of Michigan's research[3] shows that for rivers, the flow of the river is less affected by evaporation and precipitation, and its upstream and downstream water levels show a direct correlation, correlation analysis of the data confirms the paper point of view: in the case of SUP, for example, the correlation coefficient between the flow of the river and the level of the upstream lake is 0.953.

Based on Torricelli's law, which states that the volume of water exiting the pipe is:

$$V = A \cdot \sqrt{2gh} \tag{7}$$

From the point of view of conservation of energy, the square of the flow velocity of the river is correlated with the height drop, except for the cross-sectional area and other factors which are constant by the characteristics of the terrain, so the paper find the relationship between the two as follows:

$$VF = k_1 \cdot \sqrt{L_{upstream}} + k_2 \cdot \sqrt{L_{downstream}} + b \tag{8}$$

In this case, the paper set the downstream level to 0 because the St. Lawrence River empties into the Atlantic Ocean.

The paper fit for the St. Mary's River, which flows out of Lake Superior and into Lakes Michigan and Huron, is shown in Figure The paper obtained R-squared=0.8714 which is greater than 0.85, and the fit is considered to be better, and the final fitted equation is:

$$VF_{STM} = 167.7 \cdot \sqrt{L_{SUP}} - 28.93 \cdot \sqrt{L_{MIC-HUR}} - 1881 \tag{9}$$

The fitting surface plot of St. Mary's River's flow are as below Figure 4:

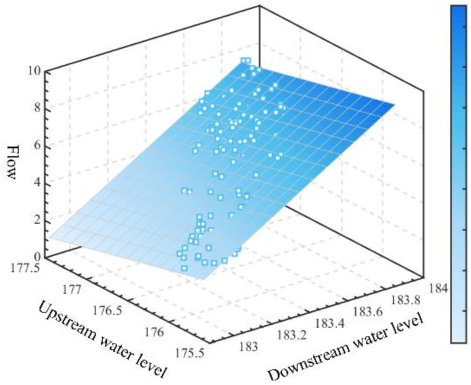


Figure 4: Fitting surface plot of St. Mary's River's flow

The parameters for other lakes were obtained respectively as shown in Table 1:

Table 1: Weights for each indicator

	K1	K2	b	R2
STCR	55.1676	63.8465	-1563.3942	0.867
DET	257.2108	-145.7234	-1465.1651	0.873
NIA	162.7281	-16.2344	-1991.8539	0.938
STL	94.6306	0.5000	-798.2392	0.324

The coefficients for the upstream lakes are usually positive. In contrast, those for the downstream lakes are usually negative, a result consistent with the paper conjectures and practical factors. The absolute value of the former in the results is usually larger than the latter, which indicates that the upstream lakes have a greater influence on the river's flow rate than the downstream lakes. It's worth noting that the poor performance on STL will not matter.

After quantitative analyses of rivers and lakes, the paper have the physical laws of lake levels and river flows in the Great Lakes basin. Therefore, the paper use the flow of water as a clue to provide a specific description of the hydrological characteristics of the basin.

The St. Lawrence River Basin consists of some sub-basins (abbreviated SUB) such as the Lake Superior Sub-basin. Influences within the watershed are mainly represented by changes in the water level of the lake, which are often interconverted with the amount of change in water volume to the water level to facilitate the use of the Lake Model for analyses. The influence between basins is mainly realized by the river that connects them. The flow of this river determines the outflow from the upstream lake and the inflow from the downstream lake, which in turn affects the water levels of the lakes in the two basins. The following equation gives the water levels of the lakes in the watersheds:

$$\Delta V_i = Q_{in}^i - Q_{out}^i + I_i + P_i - E_i = VF_{i-1} - VF_i + P_i - E_i + I_i L_i = L_i + \frac{\Delta V_i}{S_i} \tag{10}$$

Where i represents the i−th lake/riverin a network link.Besides, according to the River Model, the paper get the law that the upstream lakes and the downstream lakes together determine the water level of the river:

$$VF_i = k_{i1} \cdot \sqrt{L_i} + k_{i2} \cdot \sqrt{L_{i-1}} + b \tag{11}$$

The paper dynamic network flow model of river and lake interactions is shown in Figure 5:

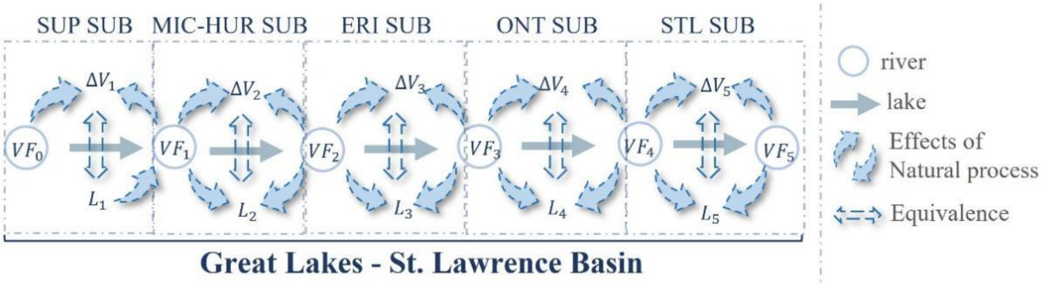


Figure 5: Basin Hydrological Model based on River-Lake Interaction

3. Multi-interest Based Water Level Decision Model

To get the optimal water level that meets the interests of multiple parties, the paper choose to construct a decision-making model for water level based on multiple stakeholders based on the given 22 years of data from a long-term perspective.

3.1. Seasonal Analysis using STL

As the impacts of climate change become increasingly visible, coupled with in- creased human activities, the fluctuations in the water levels of the Great Lakes have far-reaching impacts on local ecosystems, economies, and community life. The paper integrated the relevant data of the Great Lakes to conduct a preliminary visual inspection and statistical examination of them. The paper found that a distinct seasonality characterizes the water level changes of the Great Lakes. To further explore its seasonality, we used STL (Seasonal-Trend decomposition using LOESS) for seasonal decomposition, which has higher flexibility to adapt to irregular and time-varying seasonal patterns in the data than the classical seasonal decomposition methods.

We perform the STLbased on the data given for the Great Lakes for the period 2000 to 2022:

$$L_t = T_t + N_t + E_t \tag{12}$$

Where Lt is the water level, Nt is the seasonal component, and Et is the residual com- ponent. This is shown in the following Figure 6.

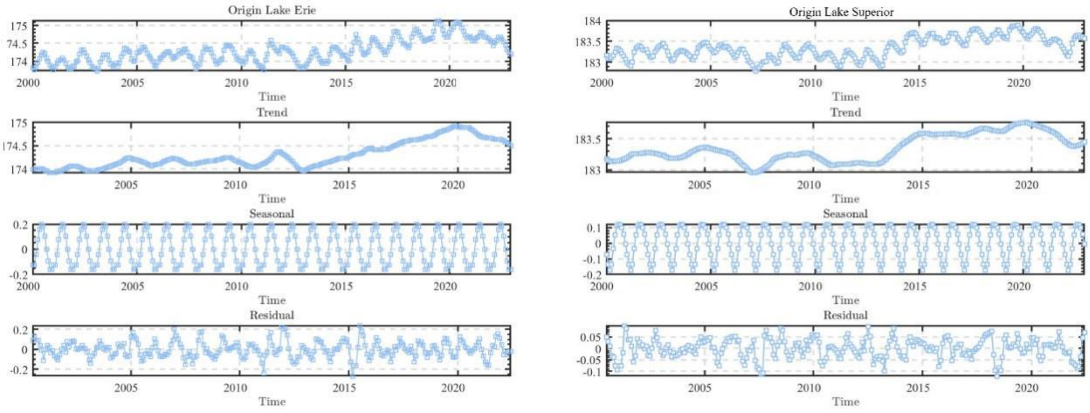


Figure 6: Decomposition results of STL and SUP

The peaks and valleys on the seasonal component charts occur at a fixed frequency, suggesting that changes in the water levels of the Great Lakes exhibit a consistent pattern over the same time period each year. This regularity may be closely related to climate change, snow and ice melt, seasonal variations in rainfall, and other environmental factors. Therefore, based on such significant seasonality, it’ s reasonable to hypothesize that the seasonal components are similar yearly.

After removing the seasonal and residual components, the trend component Tt shows a narrowing of the data range and increased stability. This implies that the long- term trend is smoother, whereas seasonality and stochastic fluctuations are the main causes of data volatility. The smoothness of the trend component provides us with a solid basis for analyzing and planning water level control measures.

For the stability S, The paper can analyze the 20-year trend fluctuations with the magni- tude of seasonal variations.

$$S=\delta(T_t)+\delta(N_t) \tag{13}$$

Where $\delta(T_t)$ represents the variance of trend components, and $\delta(N_t)$ represents the variance of seasonal components. The paper believe that the S obtained by adding the two can fully reflect the water level stability of the lake in the past twenty years.

3.2. Multiple Cost-benefit Indicator Construction

To explore the relationship between the benefits and costs of various stakeholders and the Great Lakes water level based on the Ontario Lake sub-problem mentioned in the Appendix and its related information[4][5], the paper have explored the relationship between the benefits and costs of five stakeholders and the water level. Next, the paper will establish cost and benefit functions for five stakeholders, and further refine the study of each lake by adjusting the corresponding parameters,as in fig 7.

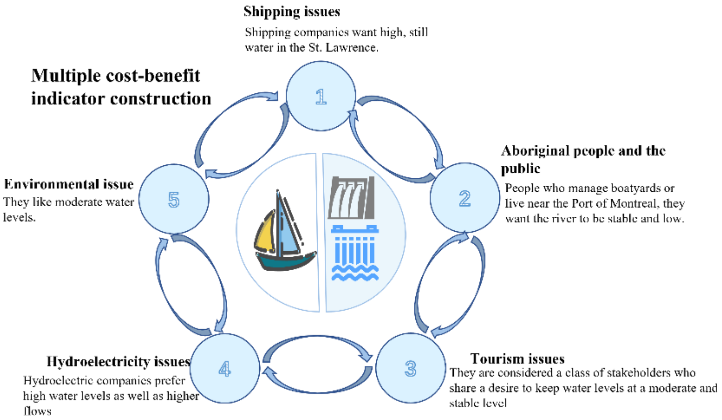


Figure 7: Relevant stakeholders’ desires

Shipping issues[6] and higher water levels are a boon. Shipping companies’ operating costs are mainly fuel-related, which may be reduced in station- ary and high water conditions. The revenue of the shipping company is related to the volume of goods transported, which can increase at high water levels. The cost and revenue functions can be expressed as:

$$\begin{cases} C_1(L,S)=e^{-bL}-S \\ R_1(L,S)=b\cdot L-C_1 \end{cases} \tag{14}$$

Where b denotes the coefficient of return for cargo transportation, which increases linearly as the waterline rises, and e^{-bL} denotes the cost of fuel, which is assumed to decrease exponentially as the water level L rises. The more stable the water level is the lower the cost will be.

As the water level line rises, neighborhoods must spend more money on Aboriginal people and the public to protect themselves from flooding[7]. The paper define $M = k\,(L\text{-}L_{safe})$,especially when the water level line exceeds the safe water level line L_{safe}, the cost starts to rise sharply, where k denotes the rate of increase of the cost with the rise of water level. To simplify the model, the paper define $L_{safe} = 0.8 \cdot L_{\max}$. For this purpose, the cost function can be expressed as:

$$C_2 = k \cdot \max\{0,(L - L_{safe})\} + S \tag{15}$$

The role of the benefit to the residents is negligible.

The paper consider recreational boaters, fishing boats, and tourists as a group of stakeholders who share a common desire to keep the water level at a moderate and stable level[8]. The paper consider the cost of water level changes to the property owners or the fishermen for repairing and controlling the fishing boats, as the water level is stable and not hazardous. Some recreational activities maybe organized around the area to generate revenue. The cost and benefit functions are:

$$\begin{cases} C_3 = c \cdot S \\ R_3 = \alpha - \beta(L + S) - C_3 \end{cases} \tag{16}$$

where c is the average restoration cost per unit of water level deviation induced, where α is the maximum number of activities at the desired level, and β is a positive coefficient indicating the rate of decrease in the number of activities when the water level deviates from the desired level.

Hydroelectric companies and others prefer high water levels as well as higher flows. The costs to such companies include the operating costs of managing reservoir levels and the adjustment costs that may be incurred at non- ideal levels. The adjustment cost decreases as the water level increases, and the paper set$L_{allow} = 1.2 \times L_{\min}$, p is the parameter the paper set. The revenue of these companies is generally positively correlated with the water level, for this reason, the cost function and revenue function are as follows.

$$\begin{cases} C_4 = -\,p \cdot (L - L_{allow}) \\ R_4 = (L - L_{allow}) \cdot (p - S) - C_4 \end{cases} \tag{17}$$

Environmentalists who want the water level to show seasonal fluctuations[9], too low or too high, may affect the survival of certain organisms, then the paper can write their revenue function:

$$R_5 = \gamma\Big(\sum_{i}^{5} |L_i - L_{opt}|\Big) \tag{18}$$

Where L_i is the current water level for a particular environmental protection target and L_{opt} is that ideal water level, and the paper specify that the ideal water level is $\bar{L} = \frac{1}{22}\sum_{i=1}^{22} L_i$.

The functions$L_{\min}$ and$L_{\max}$ are the maximum and minimum water levels of each lake for the last 22-year period of data.

Finally, the paper consider the benefit functions of the different stakeholder parties as different objectives and next, perform multivariate multi-objective planning for the requirements of this task.

3.3. Multi-objective Planning Solution Based on NSGA-II Algorithm

Based on the above analysis, the paper establish the revenue maximization equation for each stakeholder as follows:

$$\begin{cases} 183.0 < L_{SUP} < 183.5 \\ 176 < L_{MIC\&LAK} < 177 \\ 175.0 < L_{STC} < 175.5 \\ 174.0 < L_{ERI} < 174.5 \\ 74.5 < L_{ONT} < 75.5 \end{cases} \tag{19}$$

To ensure that the solved water level is within an acceptable range, the paper set the constraint conditions here as the upper and lower limits of Great Lakes over the past twenty years.

Obviously, the demands for water level and stability among various stakeholders are contradictory, meaning that satisfying one stakeholder requires sacrificing the interests of another, and the paper cannot satisfy all stakeholders simultaneously. In this case, adopting the Pareto efficiency criterion can achieve the optimal balance of fairness and efficiency as much as possible.

NSGA-II[10] (Non-dominated Sorting Genetic Algorithm II) is an algorithm used to solve multi-objective optimization problems. Its algorithm process mainly includes initializing the population, fast non-dominated sorting, and calculating crowding distance. This enables NSGA-II to not only quickly find diverse Pareto frontier solution sets when dealing with complex multi-objective optimization problems but also maintain the diversity of solution sets, making it a widely used algorithm in multi-objective optimization.

NSGA-II will provide us with a set of Pareto optimal solutions in an appropriate interval, which can improve on one objective without worsening the other, even if no other solution can do better simultaneously. To meet the needs of all relevant stake- holders to the greatest extent possible.

For the obtained set of optimal solutions, in order to comprehensively consider the interests of all parties, the paper use theminimax strategy to find the optimal solution. In the end, the paper obtain the following solution as shown in Table 2:

Table 2: The optimal solution obtained based on the strategy of minimax

Lake	SUP	MIC&LAK	STC	ERI	ONT
Level(cm)	18349.93	17699.64	17550.00	17410.92	7525.00

When it comes to determining the optimal water level at any time, the paper can choose to calculate the optimal water level Libest based on calculations and add the average seasonal components of previous years. When calculating the Li for the i-th month, this process can be formulated as:

$$\frac{1}{22}\sum_{j=1}^{22} N_{ij} \tag{20}$$

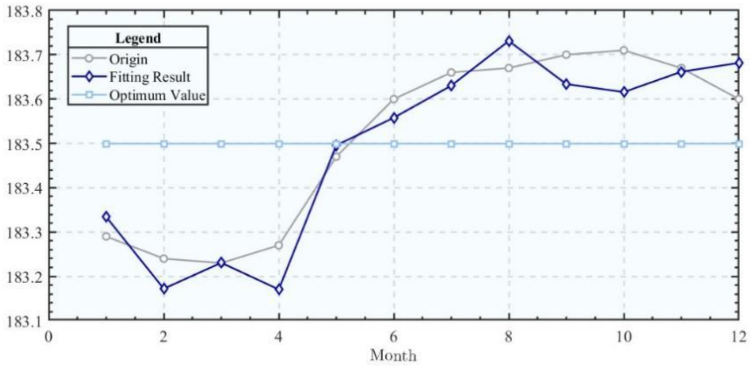


Figure 8: Result comparison

Taking Lake Superior in 2014 as an example, the paper find that in the result Figure 8. The monthly data, after incorporating the seasonal component, matches the previous performance, validating the reasonableness of the paper model.

4. Conclusions

This study developed a Dynamic Hydrological Network Flow Model to analyze the interactions between lakes and rivers in the Great Lakes and St. Lawrence Basin. By modeling lakes as edges with maximum water levels and rivers as nodes transmitting flow, we revealed that river flow is influenced by the water levels of upstream and downstream lakes. The model's accuracy was confirmed through data fitting and water balance equations.

To optimize water levels for various stakeholders, we constructed a multi-objective planning model using NSGA-II to find Pareto optimal solutions and applied a minimax strategy to determine the optimal water level. Seasonal decomposition analysis of lake water level data captured consistent seasonal patterns, allowing us to propose optimal water levels throughout the year. This approach enhances water level management, balancing ecological, economic, and social interests, providing a comprehensive solution for the Great Lakes Basin's water resource management.

References

[1] Wu K, Johnston C A. Hydrologic response to climatic variability in a Great Lakes Watershed: A case study with the SWAT model[J]. Journal of hydrology, 2007, 337(1-2): 187-199.Watershed Council. Influences on Great Lakes Water Levels. Tip of the Mitt Watershed Council, 2021.

[2] Gronewold A D, Stow C A. Water loss from the Great Lakes[J]. Science, 2014, 343(6175): 1084-1085.

[3] Anderson E J, Stow C A, Gronewold A D, et al. Seasonal overturn and stratification changes drive deep-water warming in one of Earth’s largest lakes[J]. Nature communications, 2021, 12(1): 1688.

[4] Hartig J H, Krantzberg G, Alsip P. Thirty-five years of restoring Great Lakes Areas of Concern: Gradual progress, hopeful future[J]. Journal of Great Lakes Research, 2020, 46(3): 429-442.

[5] Kayastha M B, Ye X, Huang C, et al. Future rise of the Great Lakes water levels under climate change[J]. Journal of Hydrology, 2022, 612: 128205.

[6] Byrnes T A, Dunn R J K. Boating-and shipping-related environmental impacts and example management measures: A review[J]. Journal of Marine Science and Engineering, 2020, 8(11): 908.

[7] Woolway R I, Kraemer B M, Lenters J D, et al. Global lake responses to climate change[J]. Nature Reviews Earth & Environment, 2020, 1(8): 388-403.

[8] Leira M, Cantonati M. Effects of water-level fluctuations on lakes: an annotated bibliography[J]. Ecological effects of water-level fluctuations in lakes, 2008: 171-184.

[9] Liu Y, Ren Z, Qu X, et al. Seasonal water level fluctuation and concomitant change of nutrients shift microeukaryotic communities in a shallow lake[J]. Water, 2020, 12(9): 2317.

[10] Verma S, Pant M, Snasel V. A comprehensive review on NSGA-II for multi-objective combinatorial optimization problems[J]. IEEE access, 2021, 9: 57757-57791.