

# The impact of gender ratio variation in lamprey populations based on population dynamics and food chain interactions on ecosystems

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**Abstract.** This study examines the influence of gender ratio variation in lamprey populations on ecosystems. Lampreys have a complex life cycle, and their role in the ecosystem is explored by analyzing their growth stages and interactions within the food chain. The research indicates that changes in lamprey gender ratios significantly affect population dynamics and interactions within the food chain. Using forward differencing methods to solve nonlinear differential equations, simulations were conducted on lamprey population size, gender ratios, and their impact on other components of the food chain. The results demonstrate that fluctuations in lamprey gender ratios not only affect their own survival capabilities but also may lead to drastic fluctuations in other biological populations within the food chain, thereby influencing the overall stability of the ecosystem. By modeling the impacts of sex ratio variations, the research highlights how alterations in the proportion of male and female lampreys can disrupt ecosystem stability, influencing not only lamprey populations but also the broader community of organisms they interact with. Understanding these dynamics is essential for effective conservation and management strategies aimed at preserving biodiversity and ecosystem resilience in freshwater environments.

**Keywords:** Lamprey, Sex ratio, Ecosystem.

## 1. Introduction

Changes in the sex ratio of lamprey populations can have profound ecological impacts on freshwater ecosystems. Lampreys, known for their complex life cycle and significant roles as both predators and prey, interact extensively within food webs, affecting the dynamics of multiple species. Understanding how variations in lamprey sex ratios influence ecosystem stability is crucial for conservation and management efforts [1-2].

Lampreys undergo a life cycle encompassing various stages from larvae to adults, with each stage playing a distinct ecological role. Initially hatching as eyeless larvae that filter-feed on microorganisms, they later metamorphose into parasitic adults primarily preying on fishlike salmonids. This lifecycle complexity underscores their intricate interactions within food chains, where they serve as both predators and prey.

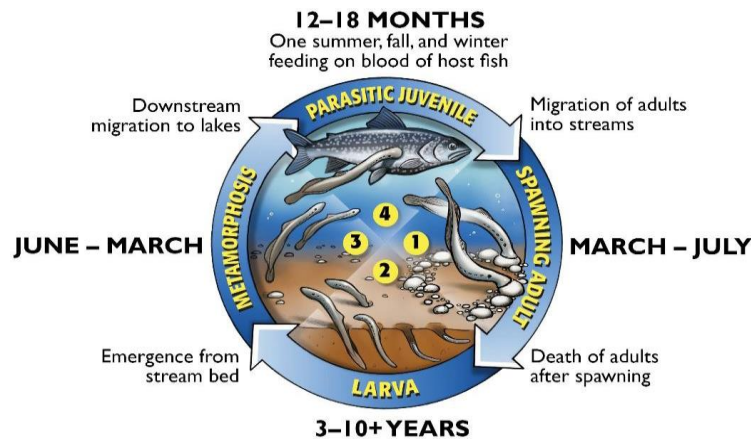
The focus of this study is to investigate how changes in lamprey sex ratios impact population dynamics and food chain interactions in freshwater ecosystems. Specifically, this paper aims to elucidate how alterations in the proportion of male and female lampreys affect not only their own survival but also the stability of the broader ecosystem. Such changes can lead to cascading effects on prey populations and subsequently affect the entire food web structure. To address this issue, this paper adopts a modeling approach that integrates population dynamics and food chain interactions. Utilizing the Lotka-Volterra model adapted for lamprey-prey systems, this paper simulates how varying sex ratios influence population sizes over time. This paper hypothesizes that fluctuations in sex ratios, influenced by factors such as food availability and environmental conditions, will result in nonlinear responses in both lamprey populations and their prey [3-4].

By employing the forward difference method to solve these complex nonlinear differential equations, this paper aims to provide insights into the mechanisms underlying ecosystem responses to sex ratio variability in lamprey populations. Through this approach, this paper explores scenarios where sex ratio adjustments by lampreys may lead to destabilization or resilience within freshwater ecosystems. The data for this paper comes from: <https://www.comap.com/contests/mcm-icm>.

## 2. Ecological impacts of changes in the sex ratio of lamprey

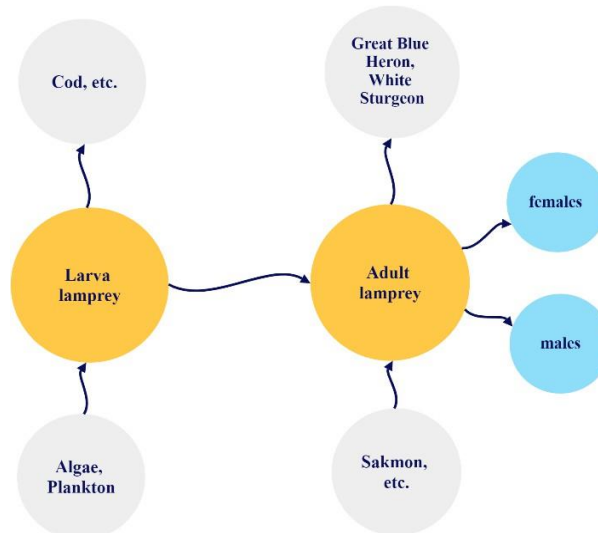
### 2.1. Introduce the Lamprey Growth Cycle and Food Chain

The lamprey has a complex life cycle, and the researchers involved suggest the seven stages of Embryo, Pro-larva, Larva, Transformer, Juvenile, Adult, and Senescent to describe it[5-6]. After hatching from their eggs, lampreys will first go through an eyeless larval stage. This stage has no teeth and feeds on microorganisms that filter the water. When the eyeless larvae grow to a certain size they enter the metamorphosis stage. Next, the lamprey enters the larval stage where feeding eyes are present, and the adult stage where it migrates upstream to spawn. Finally, the lamprey dies after spawning. In this question, in order to unify and simplify the life course of the lamprey, we divide it from a macroscopic perspective, focusing on the larval as well as the adult stages. The figure 1 below shows the growth stages of the lamprey:



**Figure 1.** Stages of growth of the lamprey (credit: Great Lakes Fishery Commission).

Lampreys are an important part of the food web of many freshwater fish, birds and mammals. Lamprey larvae feed primarily on algae and plankton, and are eaten by cod. Adult lampreys feed primarily on salmonids and are consumed by great blue herons and white sturgeon. The food chain associated with the lamprey can be represented in the figure 2:



**Figure 2.** The lamprey food chain.

## 2.2. Lamprey - Prey Model

The lamprey lives in lakes or marine habitats and migrates upstream to spawn, which makes the problem extremely complex if its spawning grounds and its hosting grounds are considered separately. In order to simplify the problem, we expect the lamprey to be in a larger area, and although it will migrate to spawn, it will still not leave this area.

According to the information[4], it is known that the invasive sea lampreys from the Laurentian Great Lakes remain in freshwater lakes or rivers during their parasitic stage. This indicates that our hypothesis or expectation is reasonable. In the following, we mainly use the Great Lakes population of the lamprey as an example to examine the effect of the sex-variable characteristics of the lamprey on the ecosystem.

We consider the population of lampreys in the smallest unit of years. Let the number of larvae of lampreys in the  $t$ -th year be  $l(t)$ , the number of female adults of lampreys be  $f(t)$ , and the number of male adults of lampreys be  $m(t)$ .

Since the reproduction of lampreys is mainly related to the number of female adults, the birth of larvae depends on factors such as the number of eggs laid by female adults, the hatching rate of lamprey eggs, and the survival rate of larvae. Lamprey larvae are also limited by the environmental carrying capacity  $N_1$  of the larvae. If the average effective egg production of female adult lamprey is  $r_1$ , then according to the Logistic Model, the effective births of lampreys are:

$$r_1 f \left(1 - \frac{l}{N_1}\right) \quad (1)$$

Let the effective larval-to-adult transformation ratio for each year be  $T$ , and according to Hypothesis 2, this larval-adult transformation process is the final point in time for sex differentiation in individual lampreys. In the  $t$ -th year, a portion of lamprey larvae undergo the transformation from larvae to adult form. It is assumed that within this subset of larvae,  $s(t)/(1+s(t))$  transform into male adults, while  $1/(1+s(t))$  transform into female adults. Then the effective conversion rates of larvae to females and males to adults in the year of completion of the larval-adult transformation process are:

$$r_f = \frac{1}{1+s} T, r_m = \frac{s}{1+s} T \quad (2)$$

Then the number of larvae of the lamprey that are reduced each year by the larval-adult transformation process is:

$$l \cdot T \quad (3)$$

From this we can get the pattern of change in the number of larvae of the lamprey as:

$$\frac{dl}{dt} = r_1 f \left(1 - \frac{l}{N_1}\right) - l \cdot T \quad (4)$$

In ecosystems, adult lampreys generally play the role of predators or parasites. Organisms that prey on lampreys exist but are relatively rare. Especially in the Laurentian Great Lakes, the sea lamprey is generally at the top of the food web [7]. Therefore, when studying sea lamprey populations in the Laurentian Great Lakes, we should primarily consider the interactions between the sea lamprey and its prey.

Let us set the number of objects on which the adult lamprey feeds or parasitizes to be  $x_2(t)$ , and the environmental carrying capacity of this object on which the adult lamprey feeds or parasitizes to be  $K_2$

The adult lamprey cannot survive without the object of predation or parasitism, let its annual mortality rate when it exists alone and is subjected to various factors such as predation by humans, blue herons, and other organisms be  $d_1$ , while the presence of food provides prey for the lamprey, which is equivalent to making the mortality rate of the lamprey lower, or, in other words, facilitating the lamprey's growth, and it can be assumed that this effect is directly proportional to the amount of food bait.

Similarly, adult lampreys are affected by the carrying capacity of their environment. Since females will consume more resources, we should treat the males of lampreys differently when considering the carrying capacity. Considering the above factors and based on the Lotka- Volterra model, we arrive at the following equation:

$$\frac{df}{dt} = r_f l \left(1 - \frac{\lambda f + m}{N_2}\right) + f d_1 \left(-1 + \sigma_1 \frac{x_2}{K_2}\right) \quad (5)$$

$$\frac{dm}{dt} = r_m l \left(1 - \frac{\lambda f + m}{N_2}\right) + m d_1 \left(-1 + \sigma_1 \frac{x_1}{K_2}\right) \quad (6)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{K_2} - \sigma_2 \frac{\lambda f + m}{N_2}\right) \quad (7)$$

Where  $\sigma_i$  reflects the predatory capacity of lampreys for their prey,  $\lambda$  reflects the ratio of resources consumed by adult females to those consumed by adult males.

Because lampreys adjust their sex ratio based on food availability, which is correlated with the number of lampreys and the amount of prey available. Unproductive environments make lamprey increasingly male-biased, whereas productive environments are increasingly less male-biased, and high-density populations are more likely to favor males than low-density populations [8-9].

We modeled the control of sex ratio by lampreys based on the above information. The number of males compared to females in the lamprey individuals that completed the larval- adult transformation process in  $t$  year has been given above as:

$$s(t) = 1 + k \frac{\lambda f(t) + m(t)}{x_2(t)} \quad (8)$$

Where  $k$  reflects the sensitivity of the lamprey to sex change.

To summarize, we get the Lamprey - Prey Model in the ecosystem when lampreys can change sex:

$$\left\{ \begin{array}{l} \frac{dl}{dt} = r_l f \left(1 - \frac{l}{N_1}\right) - l \cdot T, \\ \frac{df}{dt} = \frac{1}{1 + 1 + k \frac{\lambda f + m}{x_2}} T l \left(1 - \frac{\lambda f + m}{N_2}\right) + f d_1 \left(-1 + \sigma_1 \frac{x_2}{K_2}\right) \\ \frac{dm}{dt} = \left(1 - \frac{1}{1 + 1 + k \frac{\lambda f + m}{x_2}}\right) T l \left(1 - \frac{\lambda f + m}{N_2}\right) + m d_1 \left(-1 + \sigma_1 \frac{x_2}{K_2}\right) \\ \frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{K_2} - \sigma_2 \frac{\lambda f + m}{N_2}\right) \end{array} \right. \quad (9)$$

### 2.3. Solving Using the Forward Difference Method

The above equation is a system of nonlinear differential equations of more complex form, for which it is generally difficult to find an analytical solution. Therefore, we use the forward difference method to solve the above system of nonlinear differential equations.

For simplicity, we have used discretized time in terms of years to study changes in the numbers of the two major populations. Combined with the fact that lampreys and most fish generally reproduce at a fixed time each year[10], it is clear that this is reasonable.

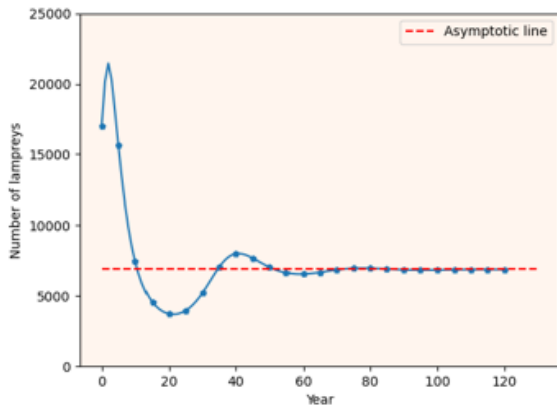
Expressing the differential in the above nonlinear differential equation in difference form, one gets:

$$\left\{ \begin{array}{l} l_{t+1} - l_t = r_1 f_t \left(1 - \frac{l_t}{N_1}\right) - l_t \cdot T, \\ f_{t+1} - f_t = \frac{1}{1 + 1 + k \frac{\lambda f_t + m_t}{x_{2,t}}} T l_t \left(1 - \frac{\lambda f_t + m_t}{N_2}\right) + f_t d_1 \left(-1 + \sigma_1 \frac{x_{2,t}}{K_2}\right) \\ m_{t+1} - m_t = \left(1 - \frac{1}{1 + 1 + k \frac{\lambda f_t + m_t}{x_{2,t}}}\right) T l_t \left(1 - \frac{\lambda f_t + m_t}{N_2}\right) + m_t d_1 \left(-1 + \sigma_1 \frac{x_{2,t}}{K_2}\right) \\ x_{2,t+1} - x_{2,t} = r_2 x_{2,t} \left(1 - \frac{x_{2,t}}{K_2} - \sigma_2 \frac{\lambda f_t + m_t}{N_2}\right) \end{array} \right. \quad (10)$$

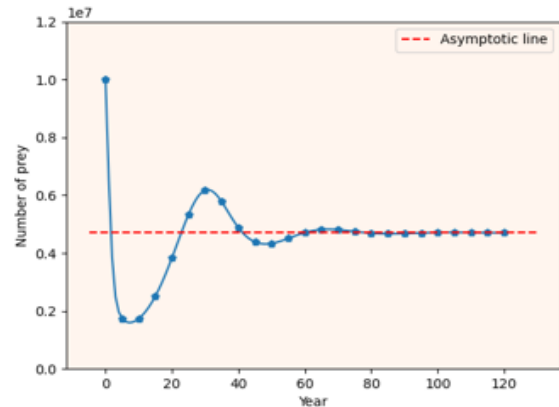
Where  $l_t, f_t, m_t, x_{2,t}$  represents or reflects the size of the two major populations in year  $t$ .

After reviewing information on the lamprey, we learned information on the number of eggs laid by adult females and the survival rate of juveniles. Subsequently, constants were determined by reviewing information such as the natural growth rate of the lamprey's main prey or parasites, The result is obtained, as shown below.

Figures 3 and 4 show the results of solving for the total number of lampreys, and the total number of preys over time when the population of lampreys can change its sex ratio.

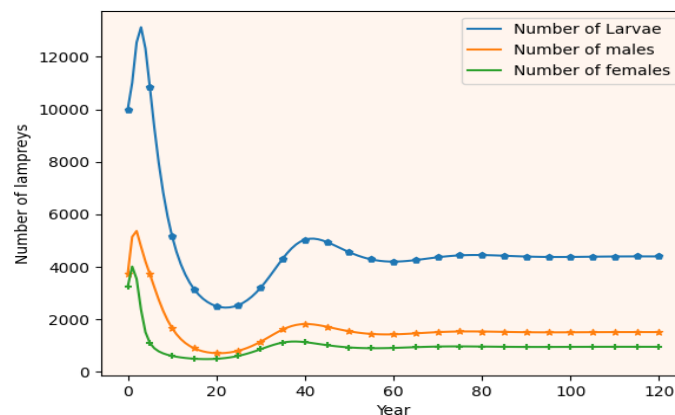


**Figure 3.** Total number of lampreys.

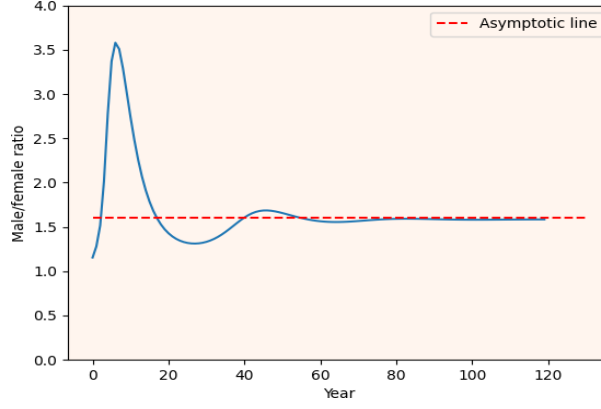


**Figure 4.** T Total number of prey.

Figures 5 and 6 are the results of solving for the number of larvae, females and adult males of lampreys over time, and the ratio of male to female population of lampreys over time, when the population of lampreys can change its sex ratio.



**Figure 5.** Number of lamprey larvae and adults.



**Figure 6.** Ratio of male to female abundance of lamprey.

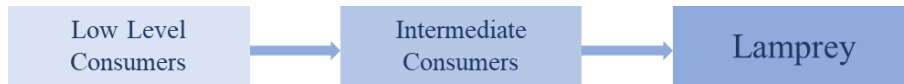
We can see that when the population of lampreys can change its sex ratio, the initial state has sufficient prey and the population of lampreys increases dramatically in a short period of time. In this case, there are far more larvae than adults, and there is a significant increase in the number of male adults over female adults.

However, it was the dramatic short-term increase in the number of lampreys that led to massive predation on prey and a rapid decrease in the number of prey. The decrease in the number of prey led to a decrease in the availability of food for the lampreys, which showed a gradual decrease in the number of lampreys and a consequent gradual increase in the number of prey. As a result, the sex ratio of lampreys declined and stabilized.

#### 2.4. Food Chain-based Population Dynamics Model

Previously, we only considered the binary relationship between the lamprey and the prey of the lamprey, while in real ecosystems, they will also interact with other levels of consumers, and it is logical that we should take them into account when considering the role of the lamprey on the food chain.

However, the mathematical complexity of the solution increases dramatically as the number of populations considered increases, which can affect the effectiveness of the solution. For simplicity, we examine the following ternary food chain. Ternary food chain is shown in figure 7.



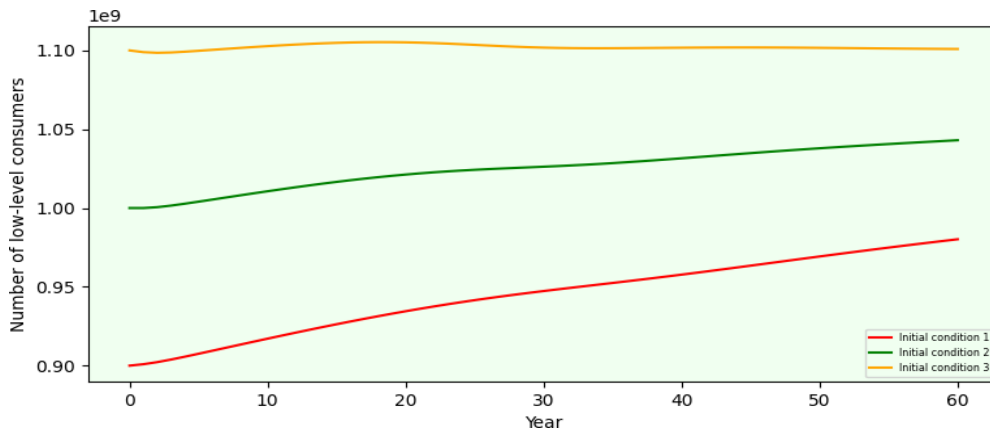
**Figure 7.** Ternary food chain.

Here we set  $x_1(t)$  as the number of low-level consumers such as herbivorous fish and shrimp, and  $x_2(t)$  as the number of intermediate-level consumers such as carnivorous and omnivorous fish, and we can build the following Food Chain-based Population Dynamics Model following the previous Lamprey - Prey Model.

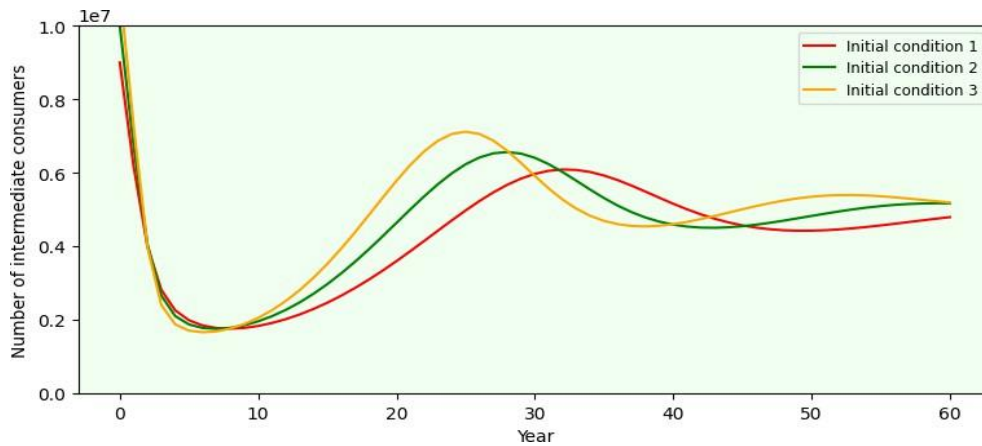
$$\left\{ \begin{array}{l} \frac{dl}{dt} = r_l f \left(1 - \frac{l}{N_1}\right) - l \cdot T, \\ \frac{df}{dt} = r_f T l \left(1 - \frac{\lambda f + m}{N_2}\right) + f d_1 \left(-1 + \sigma_1 \frac{x_2}{K_2}\right) \\ \frac{dm}{dt} = r_m T l \left(1 - \frac{\lambda f + m}{N_2}\right) + m d_1 \left(-1 + \sigma_1 \frac{x_2}{K_2}\right) \\ \frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1}{K_2} - \delta_1 \frac{x_2}{K_2}\right) \\ \frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{K_2} - \sigma_2 \frac{\lambda f + m}{N_2} + \delta_2 \frac{x_1}{K_1}\right) \end{array} \right. \quad (11)$$

Where  $\delta_1, \delta_2$  reflect the strength of interactions between herbivorous fish, shrimp, and carnivorous and omnivorous fish, while  $l, f, m, x_1, x_2$  representing or reflecting the abundance of the three major populations in the year  $t$ .

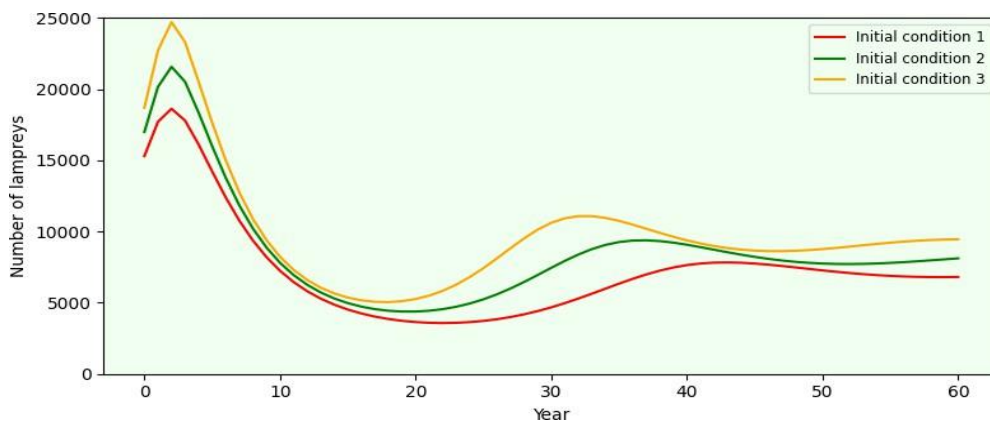
Consulting the relevant information, we determined the relevant parameters, Subsequently, the following three major population size changes were plotted according to different initial value conditions. The initial value conditions have been expressed one by one in the figure and are not listed here.



**Figure 8.** Number of low-level consumers.



**Figure 9.** Number of intermediate consumers.



**Figure 10.** Number of lampreys.

From the three figures 8,9,10, it can be seen that the numbers of the three major populations of low-level consumers, mid-level consumers and lampreys are interacting with each other and influencing each other.

Sex variability in the lamprey, while improving its own survivability, can cause drastic fluctuations in the populations of other organisms. As you can see from the graph, lamprey sex variability can cause drastic fluctuations in the populations of other organisms, making the entire ecosystem less stable.

### 3. Conclusions

In conclusion, this study has highlighted the significant ecological implications of varying sex ratios in lamprey populations within freshwater ecosystems. Through modeling efforts and analysis, this paper has demonstrated that changes in the proportion of male and female lampreys can lead to complex dynamics within food webs, impacting not only their own population dynamics but also cascading effects on prey species and overall ecosystem stability.

The findings underscore the importance of considering sex ratio variability in conservation and management strategies for lampreys and their associated ecosystems. Effective management should take into account the intricate interactions between lampreys and their prey, as well as the broader implications for biodiversity and ecosystem resilience.

Moving forward, further research is needed to refine models and incorporate additional ecological factors that may influence lamprey populations and their interactions within freshwater food webs. By deepening our understanding of these dynamics, this paper can better protect and sustainably manage lamprey populations and the ecosystems they inhabit. This will ultimately contribute to the conservation of biodiversity and the long-term health of freshwater ecosystems worldwide.

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