

CSU's Predictive Power for Stock Expected Returns

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Abstract. This paper aims to investigate the predictive power of cross-sectional uncertainty (CSU) on expected stock returns. Building upon the research of Deshui Yu and Difang Huang, this study extends the sample period to December 2022 and empirically finds that CSU has a strong predictive ability for expected stock returns, outperforming the 13 macroeconomic variables and 4 measures of economic uncertainty proposed by Goyal and Welch (GW). The results show that CSU's predictive power strengthens as the forecast horizon lengthens and passes tests both in-sample and out-of-sample. Moreover, the combined forecast of CSU with macroeconomic variables and economic uncertainty variables can produce better out-of-sample forecast results, especially over longer periods. Economic tests indicate that using CSU to predict stock returns can bring higher economic utility to risk-averse investors. Therefore, this paper concludes that CSU is an effective forecasting tool that can provide investors with additional information for predicting market excess returns.

Keywords: CSU; Stock Expected Returns; Forecasting.

1. Introduction

Economic uncertainty is closely related to asset prices in the financial market. There is already a substantial body of literature examining the impact of aggregate economic uncertainty on economic activity or financial markets. However, Yu and Huang^[1] focus on the impact of cross-sectional uncertainty on expected stock returns. Inspired by Goyal and Santa-Clara's research findings on heterogeneity in risk, which suggest that the predictive power of stock variance for expected returns mainly comes from the idiosyncratic component of average stock variance, Yu and Huang tested the predictive power of cross-sectional uncertainty for stock variance.

Unlike measuring overall financial market uncertainty, cross-sectional uncertainty tracks heterogeneity at the firm level. The measure of cross-sectional uncertainty used in this paper is the CSU variable proposed by Dew-Becker and Giglio^[2], which is the value-weighted average of firm-level implied variance minus market-level implied variance. Using CSU and S&P 500 data from January 1996 to August 2019, the paper empirically tests whether CSU has predictive power for expected stock returns. It also compares CSU with 18 alternative variables, including 13 macroeconomic variables proposed in the GW2008 paper and five other variables measuring economic uncertainty, to determine if there is a significant difference in the predictive power for expected stock returns between CSU and the alternative variables. The paper concludes that CSU has superior predictive power and contains additional information compared to alternative variables. In addition, the paper also tests whether the combined forecast of CSU and alternative variables has better out-of-sample predictive power and further explores the internal mechanism by which CSU predicts excess stock returns.

Based on the research ideas and methods of Yu and Huang, this paper extends the original sample period to December 2022 to study the relationship between cross-sectional uncertainty and stock returns.

The remaining parts of the article are as follows: the second part introduces data and variables, the third part is the model and empirical results, the fourth part is the economic significance test, and the fifth part is the conclusion of the article.

2. Data and Variables

2.1. Variable Definitions and Data Sources

The measure of cross-sectional uncertainty used in this paper is the CSU variable proposed by Dew-Becker and Giglio. CSU is the value-weighted average of firm-level implied variance minus market-level implied variance. In addition, the data for S&P 500 monthly returns, risk-free returns, and 14 macroeconomic indicators all come from the paper by Goyal and Welch^[3]. The measures for overall economic uncertainty come from different scholars' papers and corresponding websites. As shown in Table 1, the variables used in this paper and their specific sources are listed.

Table 1. Variable Definitions and Data Sources

Variable	Variable	Definition	Data Source
Independent Variable	CSU	cross-sectional uncertainty	Dew-Becker and Giglio
Dependent Variable	r	excess market return of S&P500	Goyal and Welch\
Alternative Variables	DP	Log dividend-price ratio	
	EP	Log earning-price ratio	
	DE	Log dividend-payout ratio	
	RVOL	Excess return variance	
	BM	Book-to-market ratio	
	NTIS	Net equity expansion	
	TBL	T-bill rate	
	LTY	Long-term yield	
	LTR	Long-term return	
	TMS	Term spread	
	DFY	Default yield spread	
	DFR	Default return spread	
	INFL	Inflation	
	MRI	Macroeconomic risk index	
CATFI N	Aggregate systematic risk	Allen et al. ^[5]	
EPU	Economic policy uncertainty	Baker et al. ^[6]	
AMU	Aggregate macroeconomic uncertainty	Jurado et al. ^[7]	

Unlike the original text, this paper adopts a sample period of 199601-202212 and omits the alternative variable LIQ because the data for this variable only updates up to 2018, which does not match the overall sample period of this paper.

2.2. Descriptive Statistics

As shown in Table 2, the descriptive statistics for the mean, median, and standard deviation of CSU and alternative variables are listed. The table shows that the mean of CSU is 0.24 and the standard

deviation is 0.08, which is smaller than most of the alternative variables, indicating that CSU has less volatility compared to alternative variables. As shown in Tables 3 and 4, there are more positive correlations between CSU and GW macro variables, among which the correlation between CSU and EP is the highest at -0.55; CSU shows a positive correlation with all economic uncertainty variables, and the correlation between CSU and CATFIN is the highest at 0.54. Overall, the correlation between CSU and alternative variables is not high, preliminarily indicating that CSU and alternative variables may contain different information.

Table 2. Descriptive Statistics

Variable	Mean	Median	Std
Panel A: Cross-sectional uncertainty			
CSU	0.24	0.21	0.08
Panel B: GW predictors			
DP	-4.02	-3.99	0.20
EP	-3.17	-3.11	0.36
DE	-0.85	-0.92	0.41
RVOL	0.15	0.15	0.06
BM	0.27	0.27	0.07
NTIS	0.00	0.00	0.02
TBL	2.02	1.33	2.02
LTY	4.10	4.34	1.65
LTR	0.57	0.54	3.02
TMS	2.08	1.95	1.31
DFY	0.98	0.90	0.40
DFR	0.04	0.06	1.84
INFL	0.19	0.19	0.35
Panel C: Uncertainty predictors			
MRI	0.04	-0.73	2.14
CATFIN	0.24	0.21	0.12
EPU	100.61	84.20	57.51
AMU	0.66	0.62	0.12

Table 3. CSU Correlation Matrix with GW Macro Variables

Variab le	CS U	DP	EP	DE	RVO L	BM	NTI S	TB L	LT Y	LT R	TM S	DF Y	DF R	INF L
CSU	1.00	-0.41	-0.55	0.28	0.49	-0.49	0.08	0.40	0.38	0.05	-0.14	0.24	-0.08	0.00
DP	-0.41	1.00	0.02	0.47	-0.10	0.78	-0.51	-0.49	-0.35	0.05	0.31	0.59	-0.02	-0.20
EP	-0.55	0.02	1.00	-0.87	-0.65	0.22	0.07	0.03	-0.09	0.06	-0.15	-0.47	-0.19	0.05
DE	0.28	0.47	-0.87	1.00	0.53	0.20	-0.31	-0.27	-0.10	-0.03	0.29	0.71	0.16	-0.14
RVOL	0.49	-0.10	-0.65	0.53	1.00	-0.11	0.17	-0.15	0.01	0.00	0.24	0.40	0.13	-0.07
BM	-0.49	0.78	0.22	0.20	-0.11	1.00	-0.28	-0.63	-0.41	0.04	0.45	0.46	0.01	-0.10
NTIS	0.08	-0.51	0.07	-0.31	0.17	-0.28	1.00	0.28	0.47	0.03	0.17	-0.48	0.01	0.08
TBL	0.40	-0.49	0.03	-0.27	-0.15	-0.63	0.28	1.00	0.76	0.03	-0.58	-0.36	-0.08	0.09
LTY	0.38	-0.35	-0.09	-0.10	0.01	-0.41	0.47	0.76	1.00	-0.04	0.09	-0.18	-0.04	0.07
LTR	0.05	0.05	0.06	-0.03	0.00	0.04	0.03	0.03	-0.04	1.00	-0.09	0.04	-0.47	-0.08
TMS	-0.14	0.31	-0.15	0.29	0.24	0.45	0.17	-0.58	0.09	-0.09	1.00	0.34	0.08	-0.05
DFY	0.24	0.59	-0.47	0.71	0.40	0.46	-0.48	-0.36	-0.18	0.04	0.34	1.00	0.10	-0.28
DFR	-0.08	-0.02	-0.19	0.16	0.13	0.01	0.01	-0.08	-0.04	-0.47	0.08	0.10	1.00	-0.08
INFL	0.00	-0.20	0.05	-0.14	-0.07	-0.10	0.08	0.09	0.07	-0.08	-0.05	-0.28	-0.08	1.00

Table 4. CSU Correlation Matrix with Uncertainty Variables

Variable	CSU	MRI	CATFIN	EPU	AMU
CSU	1.00	0.34	0.54	0.17	0.39
MRI	0.34	1.00	0.40	0.47	0.66
CATFIN	0.54	0.40	1.00	0.28	0.45
EPU	0.17	0.47	0.28	1.00	0.63
AMU	0.39	0.66	0.45	0.63	1.0

3. Model and Empirical Results

3.1. In-sample Regression

First, a simple univariate regression model is used to test the predictive power of CSU for stock expected returns:

$$r_{t+1,t+J} = \alpha_J + \beta_J x_t + \varepsilon_{t+1,t+J} \quad (1)$$

$$r_{t+1,t+J} = (r_{t+1} + \dots + r_{t+J})/J \quad (2)$$

where J is the forecast horizon, representing the average excess return of the S&P 500 over J months, and the excess return is the logarithm of the S&P 500 return minus the risk-free return; x is the independent variable, which is CSU and alternative variables after standardization.

CSU and 17 alternative variables are introduced into the model one by one, and in-sample regression is performed over the entire sample period from January 1996 to December 2021. The results are shown in Table 5.

From the in-sample regression results, it can be seen that within a one-month forecast horizon, the regression coefficient of CSU is significant at the 10% level, indicating that for each unit increase in CSU, the expected stock return will decrease by 0.53%. Moreover, CSU's regression obtained a good R2 of 1.47%. Most of the regression coefficients of the 13 macroeconomic variables of GW are not significant, and many variables' R2 do not exceed the standard of 0.5%. Only BM and LTY have significant regression coefficients and produced a good R2. In the regression results of economic uncertainty-related variables, unlike the original paper's conclusion that all economic uncertainty variables' regression results are not significant, in this paper's in-sample regression, EPU produced a significant and good result. Its coefficient is significant at the 1% level, indicating that for each unit increase in EPU, the expected stock return will increase by 0.71%, and the R2 of EPU's regression is 2.63%. Among all alternative variables, EPU's regression result is significantly better than CSU.

In a longer forecast horizon, as shown in the table, CSU's predictive power for expected excess stock returns strengthens as the forecast horizon extends, with its R2 increasing from 1.47% to 10.66%. In the regression results of alternative variables, macroeconomic variables BM and LTY produced results comparable to CSU, and even BM's predictive power was better than CSU in the long term, producing an R2 of 16.11% in a one-year forecast horizon. Among the economic uncertainty variables, EPU consistently shows strong predictive power, with the forecast coefficient significant at the 1% level in each forecast horizon and a larger R2.

In general, in the results of in-sample regression, CSU has a strong predictive power for expected stock returns, and the longer the forecast horizon, the better the predictive effect. However, compared to the original text's conclusion that no alternative variable can defeat CSU, this paper finds that BM, LTY, and EPU all produced regression results that are not worse than CSU.

Table 5. In-sample Regression Results

Predictors	in-sample predictive regression results											
	J=1			J=3			J=6			J=12		
	β	t _{NW}	R ²	β	t _{NW}	R ²	β	t _{NW}	R ²	β	t _{NW}	R ²
Panel A: GW predictors												
DP	0.50	1.25	1.26	0.53	1.57	4.10	0.58	2.32	9.08	0.66	4.07	20.71
EP	0.17	0.43	0.15	0.05	0.14	0.04	0.00	0.01	0.00	0.07	0.33	0.25
DE	0.10	0.25	0.05	0.21	0.73	0.68	0.28	1.35	2.12	0.25	2.00	3.05
RVOL	0.34	1.59	0.60	0.32	1.64	1.51	0.26	1.45	1.85	0.19	1.21	1.72
BM	0.46	1.92	1.10	0.51	2.43	3.81	0.57	3.17	8.81	0.58	3.85	16.11
NTIS	0.38	1.01	0.74	0.43	1.18	2.79	0.42	1.26	4.77	0.28	1.12	3.97
TBL	-0.33	-1.38	0.54	-0.32	-1.48	1.54	-0.35	-1.75	3.43	-0.42	-2.24	8.63
LTY	-0.50	-2.18	1.26	-0.48	-2.19	3.34	-0.47	-2.18	5.81	-0.44	-2.17	8.68
LTR	0.11	0.52	0.06	-0.02	-0.10	0.01	0.09	0.83	0.21	0.05	0.76	0.13
TMS	-0.12	-0.43	0.08	-0.10	-0.36	0.14	-0.03	-0.11	0.02	0.13	0.73	0.85
DFY	-0.22	-0.49	0.25	-0.12	-0.30	0.21	0.05	0.17	0.07	0.18	1.16	1.57
DFR	0.30	0.84	0.48	0.04	0.18	0.03	0.09	0.50	0.23	0.07	0.75	0.26
INFL	0.34	0.94	0.60	-0.12	-0.49	0.20	-0.26	-1.57	1.87	-0.24	-2.31	2.81
Panel B: Uncertainty predictors												
MRI	0.20	0.70	0.21	0.24	1.10	0.90	0.27	1.46	2.06	0.22	1.77	2.41
CATFIN	-0.10	-0.22	0.05	-0.14	-0.40	0.30	-0.10	-0.38	0.29	-0.09	-0.48	0.36
EPU	0.71	3.39	2.63	0.65	4.79	6.41	0.51	5.09	7.27	0.45	4.62	9.96
AMU	-0.25	-0.50	0.33	-0.14	-0.32	0.30	-0.04	-0.12	0.04	0.06	0.25	0.15
Panel A: Cross-sectional uncertainty												
CSU	-0.53	-1.91	1.47	-0.51	-2.05	3.98	-0.48	-2.43	6.43	-0.46	-2.87	10.66

3.2. Out-of-sample Forecast

Using the period from January 1996 to December 1999 as the in-sample period and January 2000 to December 2022 as the out-of-sample forecast period, the historical mean model is used as the benchmark model for out-of-sample forecasting. The results are shown in Table 6.

As can be seen from the table, CSU also shows strong predictive power in out-of-sample forecasting, achieving an R2OS greater than 0.5% in different forecast horizons; however, unlike the in-sample regression, the R2OS for the 6-month forecast horizon is lower than that for the 3-month forecast horizon, indicating that the conclusion that CSU's predictive power strengthens with the extension of the forecast horizon does not hold in out-of-sample forecasting. At the same time, the R2OS for 17 alternative variables is generally less than 0, except for AMU, which achieved an R2OS greater than 0.5%, the rest of the variables could not defeat the historical mean model. Especially BM, LTY, and EPU, which achieved good results in the in-sample regression, have no predictive power out-of-sample.

In general, in the out-of-sample forecasting test, no variable could achieve better forecasting results than CSU. CSU's predictive power for expected stock returns has passed the test in both in-sample and out-of-sample, and CSU has good predictive power. However, the conclusion that CSU's predictive power strengthens with the increase of the time range is not fully proven in the out-of-sample forecasting results. Compared with commonly used macroeconomic variables and economic uncertainty indicators, CSU obviously shows better out-of-sample forecasting effects.

Table 6. Out-of-sample Forecasting Results

Predictors	out-of-sample predictive regression results							
	J=1		J=3		J=6		J=12	
	R_{Os}^2	p	R_{Os}^2	p	R_{Os}^2	p	R_{Os}^2	p
Panel A: GW predictors								
DP	0.24	0.18	-2.47	0.24	5.92	0.01	3.34	0.00
EP	-2.12	0.19	-18.51	0.76	-23.78	0.71	-58.61	1.00
DE	-6.09	0.50	-42.71	0.99	-83.22	1.00	-55.11	0.02
RVOL	-0.37	0.66	-3.15	1.00	-5.68	1.00	-14.26	1.00
BM	-0.86	0.22	2.57	0.01	10.84	0.00	-40.08	0.00
NTIS	-1.51	0.21	-2.15	0.03	-8.72	0.01	-49.44	0.37
TBL	-3.17	0.67	-16.08	0.98	-48.59	1.00	-50.64	0.01
LTY	-1.22	0.50	-5.79	0.85	-10.15	0.88	-21.61	1.00
LTR	-1.06	0.79	-1.66	0.96	-3.58	0.98	-8.97	1.00
TMS	-2.02	0.67	-13.33	1.00	-25.49	0.99	-9.59	0.00
DFY	-3.29	0.21	-47.04	0.85	-90.21	0.99	-85.19	1.00
DFR	-4.48	0.87	-3.36	0.67	-6.73	0.84	-9.59	1.00
INFL	-1.06	0.49	2.32	0.07	-2.96	0.67	-8.11	0.99
Panel B: Uncertainty predictors								
MRI	-4.48	0.17	-46.14	0.66	-72.83	0.73	-104.45	1.00
CATFIN	-2.46	0.56	-13.63	0.99	-18.29	1.00	-29.80	1.00
EPU	-3.41	0.75	-7.62	0.78	-4.00	0.72	-18.23	0.97
AMU	0.78	0.04	-22.98	0.19	-60.95	0.41	-142.51	0.99
Panel C: Cross-sectional uncertainty								
CSU	1.47	0.03	3.05	0.00	1.54	0.00	6.63	0.00

3.3. Forecast Encompassing Tests

Next, Forecast Encompassing Tests are used to test whether the regression prediction of expected returns by CSU and the regression prediction of expected returns by alternative variables contain different information. The specific test model is as follows:

$$\hat{r}_{t+1,t+J}^C = \lambda \hat{r}_{t+1,t+J}^{(Z)} + \phi \hat{r}_{t+1,t+J}^{(CSU)} \quad (3)$$

where, $\hat{r}_{t+1,t+J}^C$ is the combined forecast of excess returns predicted by CSU and one of the alternative variables, Z is the predicted return rate by the alternative variable, which is the yield predicted by the 13 GW variables and the 5 uncertainty variables, and $\hat{r}_{t+1,t+J}^{(CSU)}$ is the predicted yield by CSU, and $\hat{r}_{t+1,t+J}^{(Z)}$ is the predicted yield by the 17 alternative variables. If the combined forecast regression result is significantly greater than 0, it indicates that CSU contains information not contained in the alternative variables. Regression on this model yields the results shown in Table 7.

As can be seen from the table, CSU and the combined forecasts of each alternative variable basically obtained a regression coefficient greater than 0, and most coefficients are close to 1 or even exceeded 1. The HLN statistical test value is basically greater than 1.96, indicating that it is significantly greater than 0, and CSU indeed contains information not contained in the alternative variables.

Table 7. Forecast encompassing tests

Predictors	Encompassing Tests							
	J=1		J=3		J=6		J=12	
	β	HLN	β	HLN	β	HLN	β	HLN
Panel A:GW predictors								
DP	0.66	1.04	0.60	1.47	0.45	1.78	0.32	2.81
EP	1.09	1.99	1.09	3.75	1.07	4.63	1.09	4.80
DE	0.98	2.09	0.90	3.45	0.82	4.47	0.81	5.66
RVOL	0.87	2.15	0.86	3.72	0.87	4.67	0.90	5.01
BM	0.73	1.24	0.63	1.88	0.41	2.07	0.33	2.68
NTIS	0.73	2.09	0.66	3.80	0.61	4.06	0.68	4.06
TBL	1.06	1.95	1.05	3.28	1.00	4.06	0.81	4.46
LTY	1.02	1.78	1.02	3.05	0.98	3.88	0.99	4.45
LTR	0.87	2.03	1.00	3.73	0.97	4.63	0.99	4.93
TMS	0.98	2.12	1.00	3.73	1.00	4.60	0.82	5.26
DFY	0.92	2.02	1.00	4.08	0.99	4.56	0.88	5.39
DFR	0.84	1.88	0.98	3.83	0.96	4.67	0.97	4.90
INFL	0.81	2.06	0.96	3.47	0.78	4.28	0.85	5.14
Panel B:Uncertainty predictors								
MRI	1.02	2.13	0.98	3.69	0.94	4.54	0.89	5.13
CATFIN	1.07	1.93	1.09	3.98	1.10	4.72	1.09	4.88
EPU	0.77	1.78	0.69	2.79	0.74	4.07	0.79	4.75
AMU	0.32	0.72	0.49	1.73	0.66	2.70	0.93	4.34

3.4. Binary Portfolio Forecast

From the above Forecast Encompassing Tests, it can be seen that CSU has information that various macroeconomic variables and economic uncertainty variables do not have. And the correlation between CSU and alternative variables is not high on the whole, the combination of the two forecasts should have smaller volatility and should obtain better out-of-sample forecasting results. Therefore, this paper uses CSU and alternative variables for combined forecasting to explore whether its joint binary portfolio forecasting has better forecasting effects.

Therefore, a binary portfolio forecasting model is used as follows:

$$\hat{r}_{t+1,t+J}^{(C)} = w_1 \hat{r}_{t+1,t+J}^{(1)} + w_2 \hat{r}_{t+1,t+J}^{(2)} \quad (4)$$

where, $\hat{r}_{t+1,t+J}^{(1)}$ and $\hat{r}_{t+1,t+J}^{(2)}$ are the yields obtained by the following models:

$$r_{t+1,t+J}^{(1)} = \alpha_j^{(1)} + \beta_j^{(1)} CSU_t + \varepsilon_{t+1,t+J}^{(1)} \quad (5)$$

That $\hat{r}_{t+1,t+J}^{(1)}$ is the yield predicted by CSU regression, and $\hat{r}_{t+1,t+J}^{(2)}$ is the yield predicted by the respective variable regression.

The following two combination methods are considered: First, the equal-weighted combination forecast, which assigns the same weight to the forecasted yields of CSU and alternative variables, Its advantage is that it does not require estimating the weights of the combined forecast; Second, the DMSFE combination forecast, which calculates the weights of individual models using the mean square error, introduces a discount factor, and controls the proportion of each point's information in the weights. The discount factor used in this paper is 0.75.

As shown in Table 8, the results of the equal-weighted combination forecast regression show that the combination forecast of CSU and alternative variables indeed produced results with stronger out-of-sample forecasting power than using CSU alone, such as the equal-weighted group of CSUS and EPU. Overall, the regression results with a longer forecast period are better than those with a short-term forecast. In addition, although some alternative variables combined with CSU's forecast did not produce better results than CSU's individual forecast, they are basically better than the original individual forecast results of the alternative variables.

Table 9 shows the results of the DMSFE combination forecast regression. Similar to the equal-weighted forecast combination results, most of the combinations of CSU and alternative variables produced a positive R2OS, and the combination forecast of CSU and EPU showed better predictive power than CSU's individual regression, and the out-of-sample R2 increased with the extension of the forecast period.

In general, using CSU and various alternative variables for binary combination forecasting produced better out-of-sample forecasting results than each variable's original individual forecast, especially for macroeconomic variables and economic uncertainty variables. In addition, the combination forecast of CSU and EPU provided more reliable forecast results than CSU's individual forecast.

Table 8. Equal-weighted combination forecast

Predictors	out-of-sample R^2 statistics(%)							
	J=1		J=3		J=6		J=12	
	R_{OS}^2	p	R_{OS}^2	p	R_{OS}^2	p	R_{OS}^2	p
Panel A: Combined with the GW predictors								
DP	0.62	0.08	0.56	0.03	6.06	0.00	14.82	0.00
EP	0.23	0.12	-5.02	0.32	-6.19	0.12	-14.81	0.37
DE	-1.19	0.34	-13.52	0.88	-27.15	0.99	2.13	0.00
RVOL	0.97	0.07	1.44	0.02	0.99	0.02	3.67	0.00
BM	0.18	0.12	2.98	0.00	9.31	0.00	-1.89	0.00
NTIS	0.94	0.13	4.75	0.01	7.77	0.00	2.16	0.00
TBL	-0.12	0.24	-3.13	0.49	-14.15	0.96	4.13	0.00
LTY	0.75	0.11	1.15	0.03	1.71	0.01	6.49	0.00
LTR	0.21	0.21	1.62	0.01	1.85	0.01	7.57	0.00
TMS	0.01	0.23	-2.99	0.55	-6.83	0.62	10.63	0.00
DFY	0.58	0.15	-12.60	0.64	-25.56	0.88	-17.75	0.93
DFR	-1.04	0.52	1.14	0.04	1.11	0.03	7.63	0.00
INFL	0.62	0.12	3.47	0.00	2.83	0.00	8.04	0.00
Panel B: Combined with the uncertainty predictors								
MRI	0.80	0.12	-10.64	0.41	-16.90	0.32	-22.83	0.60
CATFIN	-0.30	0.33	-3.15	0.58	-4.61	0.60	-2.48	0.06
EPU	1.51	0.03	3.28	0.00	5.44	0.00	6.30	0.00
AMU	1.10	0.09	-7.03	0.20	-19.06	0.37	-31.29	0.26

Table 9. DMSFE combination forecast

Predictors	out-of-sample R2 statistics (%)							
	J=1		J=3		J=6		J=12	
	R_{OS}^2	p	R_{OS}^2	p	R_{OS}^2	p	R_{OS}^2	p
Panel A: Combined with the GW predictors								
DP	0.88	0.06	1.14	0.01	5.42	0.00	16.41	0.00
EP	0.36	0.12	-3.95	0.24	-6.69	0.10	-9.34	0.07
DE	-1.35	0.34	-10.12	0.76	-18.95	0.92	16.28	0.00
RVOL	0.94	0.07	2.28	0.00	2.10	0.01	7.47	0.00
BM	0.42	0.10	2.01	0.01	7.25	0.00	6.71	0.00
NTIS	0.89	0.13	5.79	0.00	9.39	0.00	-5.84	0.00
TBL	-0.07	0.23	-2.65	0.41	-4.14	0.39	15.31	0.00
LTY	0.66	0.12	1.56	0.02	2.14	0.00	14.17	0.00
LTR	0.11	0.23	2.07	0.01	3.49	0.00	16.00	0.00
TMS	-0.02	0.23	-1.56	0.29	-2.52	0.22	19.56	0.00
DFY	0.38	0.15	-11.22	0.50	-24.51	0.74	-4.45	0.01
DFR	-1.06	0.51	1.60	0.02	2.28	0.01	16.16	0.00
INFL	0.56	0.13	3.63	0.00	4.74	0.00	16.07	0.00
Panel B: Combined with the uncertainty predictors								
MRI	0.51	0.13	-10.58	0.37	-19.31	0.34	-11.55	0.11
CATFIN	-0.48	0.35	-1.82	0.32	-0.91	0.10	5.78	0.00
EPU	1.52	0.03	4.23	0.00	6.07	0.00	14.68	0.00
AMU	0.87	0.09	-8.87	0.22	-19.60	0.38	-31.30	0.15

4. Economic Test

From the above in-sample and out-of-sample tests, we know that CSU's predictive power for excess returns has passed the statistical test, and the R2 is considerable. After the statistical test is over, it is necessary to conduct an economic test on CSU's predictive power to measure whether CSU's ability to predict returns can be identified by investors and adopt investment strategies to achieve better investment returns.

The mean-variance investor will allocate wealth between the stock market portfolio and the risk-free interest rate bond, and the weight of the investment in risky assets is:

$$w_t = \frac{1}{\gamma} \frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}^2} \quad (6)$$

where, γ is the risk aversion coefficient, \hat{r}_{t+1} is the predicted excess return of stocks using CSU or alternative variables, and $\hat{\sigma}_{t+1}^2$ is the predicted value of the monthly market excess return volatility

estimated using a 10-year rolling window, and r is the excess return predicted by the one-factor model, the risk aversion coefficient used is 3.

Based on the allocation weight, the average utility level that investors can achieve is:

$$U(\mu, \sigma) = \hat{\mu}_p - 0.5\gamma\hat{\sigma}_p^2 \quad (7)$$

where, $\hat{\mu}_p$ is the sample mean, and $\hat{\sigma}_p^2$ is the sample variance. This utility is the difference in utility obtained by investors using CSU or alternative variables to predict excess stock returns and the utility obtained by using the historical mean model. With the risk aversion coefficient γ set to 3, the CER returns and Sharpe ratios that investors can obtain are calculated.

As shown in Table 10, the CER returns that mean-variance investors can achieve by using CSU and various alternative variables to predict expected stock returns are presented. As the table shows, except for a few alternative variables that produce slightly higher CER returns than the returns produced by CSU in a certain forecast period, CSU generally produces better CER returns than alternative variables in each forecast period, bringing better investment utility to investors.

In addition to CER returns, this paper also tests whether each variable can produce a Sharpe ratio higher than that produced by the historical mean model. As shown in Table 11, the Sharpe ratios produced by using various variables for prediction are presented. Similar to the test results of CER returns, using CSU to predict future stock returns can bring higher Sharpe ratios in different forecast periods and are basically better than the results produced by alternative variables.

In general, investors using CSU to predict stock returns can achieve better economic utility, indicating that using CSU to predict excess returns has economic significance. Among the 17 alternative variables, although some variables show different good results in the economic test compared to the in-sample and out-of-sample regression, many variables produce positive economic utility in different periods, but overall, CSU's ability to bring higher returns in different sample periods is basically unmatched by any alternative variable. Using CSU to predict stock returns can indeed bring better economic benefits to risk-averse investors than traditional variable predictions.

Table 10. CER Returns

Predictors	CER gains			
	J=1	J=3	J=6	J=12
Panel A:GW predictors				
DP	3.18	3.58	2.49	0.47
EP	5.31	1.38	-0.06	-3.33
DE	3.04	1.87	-2.57	-2.36
RVOL	-1.12	-1.96	-0.62	-0.65
BM	1.92	4.53	4.44	0.24
NTIS	0.28	1.28	1.84	-3.34
TBL	0.92	-1.31	-1.72	-0.85
LTY	2.74	-0.82	-1.15	-3.35
LTR	-0.47	0.18	0.17	-0.34
TMS	1.18	-1.50	0.02	0.14
DFY	2.96	4.28	2.09	-4.34
DFR	0.17	1.02	0.64	-0.55
INFL	-1.93	1.33	-0.03	-0.94
Panel B: Uncertainty predictors				
MRI	5.12	5.80	1.34	-3.21
CATFIN	1.65	0.01	-1.21	-2.48
EPU	-2.32	-0.08	0.38	-0.88
AMU	6.35	5.72	3.91	-1.81
Panel C: Cross-sectional uncertainty				
CSU	4.94	4.11	2.18	1.99

Table 11. Sharpe ratio

Predictors	Sharp ratio			
	J=1	J=3	J=6	J=12
Panel A:GW predictors				
DP	0.23	0.30	0.24	0.06
EP	0.37	0.12	0.01	-0.43
DE	0.19	0.18	-0.29	-0.30
RVOL	-0.10	-0.17	-0.07	-0.08
BM	0.16	0.38	0.46	0.04
NTIS	-0.09	0.14	0.50	-0.34
TBL	0.04	-0.13	-0.19	-0.11
LTY	0.18	-0.09	-0.13	-0.44
LTR	-0.03	0.02	0.02	-0.04
TMS	0.07	-0.14	0.00	0.01
DFY	0.19	0.50	0.40	-0.57
DFR	-0.01	0.09	0.07	-0.07
INFL	-0.12	0.12	-0.01	-0.12
Panel B: Uncertainty predictors				
MRI	0.36	0.74	0.19	-0.41
CATFIN	0.10	-0.01	-0.13	-0.32
EPU	-0.13	0.00	0.04	-0.11
AMU	0.47	0.72	0.78	-0.15
Panel C: Cross-sectional uncertainty				
CSU	0.34	0.36	0.24	0.35

5. Conclusion

Based on the research ideas and methods of Deshui Yu and Difang Huang, this paper extends the original sample period to December 2022 on the basis of the original paper, testing whether cross-sectional uncertainty has stable predictive power for stock expected returns. The study found that CSU has a strong predictive power for stock expected returns, which is superior to the 13 macroeconomic variables proposed in the GW paper and the 4 variables measuring economic uncertainty. Moreover, the predictive power of CSU for expected returns has obvious economic significance; risk-averse investors can obtain economic utility by using CSU to predict stock expected returns.

In addition, the combination forecast of CSU and popular forecasting factors, that is, macroeconomic variables, and uncertainty variables, can produce good out-of-sample forecasting results. CSU provides additional information for predicting market excess returns, especially in a longer time range.

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