

An Empirical Study of The Stock-Gilt Co-Movement in The UK Markets Using Copula Methods

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Abstract. Using a variety of copula models, this study examines the correlation and tail dependency between UK stock market and gilt market across three distinct economic periods: the financial crisis, the Covid-19 pandemic and normal economic period (normal time period in between financial crisis and Covid-19 pandemic without economic instability), and investigates how these correlations differ, with a particular focus on the extreme correlation. The analysis reveals that the UK stock market and gilt market are exhibiting certain correlations in all three time periods, but there's significant variations in the strength of the correlations and tail dependencies.

Keywords: stock-gilt correlation; financial market; copula; macroeconomic analysis; tail dependency.

1. Introduction

The stock market and the gilt market stand as twin pillars of paramount significance within the broader financial landscape. Gilts, as securities issued by the government, are typically viewed as safe havens, whereas the stock market is characterised by its inherent unpredictability and volatility. Gleaning insights into the dynamic interplay between these two markets is pivotal for adept risk management and judicious asset allocation, particularly during times of economic downturn and uncertainty. This study, with a specific focus on the UK financial market, seeks to elucidate the fluctuating degrees of interdependence between the stock and gilt markets under varying economic conditions.

Employing a comprehensive dataset of UK daily stock and gilt market indices, this research spans three distinct economic epochs: the financial crisis, the Covid-19 pandemic, and a period deemed 'normal'. Our objective is to scrutinise and contrast the correlation and tail dependency between the stock and gilt markets. By leveraging a suite of copula models to fit the data, we aim to illuminate these relationships, with a particular emphasis on extreme scenarios. This approach will enable us to discern how shifting market conditions impact the intensity and structure of the interdependencies between stocks and gilts, and to offer explanations for the observed variances in these dependencies.

Over recent decades, the application of copula models within the econometric analysis of financial markets has garnered considerable scholarly attention. Copulas provide a framework for modelling the correlation between multiple variables, with Sklar's Theorem (1959) serving as the theoretical cornerstone. This theorem posits that any multivariate distribution can be decomposed into marginal distributions and a copula, which delineates the dependencies between these marginals. Prior studies have employed copula models to investigate the extreme correlation of stock and bond futures markets across the US, UK, and Germany (Chin Man Chui & Jian Yang, 2011); to compare bond and stock market correlations in various economies (Shalini Agnihotri, 2017); and to probe the extreme correlation of international equity markets (Francois Longin & Bruno Solnik, 2002).

The present study contributes to the existing literature in three principal ways: Firstly, while previous research has predominantly focused on the general correlation and dependencies across two or more markets, our work extends the analysis to include tail dependence, which specifically captures the interrelation of data in extreme scenarios. Secondly, this research encompasses multiple temporal segments, with a particular emphasis on two economic disruptions, juxtaposed against a 'normal' period, to furnish a robust understanding of how changing market conditions influence asset

correlations. Lastly, following our empirical analysis, the paper delves into the interpretation of the observed differences in stock-gilt correlations, with a special focus on tail dependency, across distinct time periods.

The structure of this paper unfolds as follows: Section 2 delineates the data and the methodological approach employed. Section 3 presents and interprets the empirical findings. Finally, Section 4 offers a conclusive summary of the entire paper.

2. Data and Methodology

2.1. Data

This study employs two pivotal datasets to scrutinise the correlation and tail dependency between stock and gilt returns, under varying economic conditions. The stock data comprises daily returns from the FTSE 350 Index, a benchmark that tracks the performance of the 350 largest companies listed on the London Stock Exchange—widely acknowledged as the definitive UK stock market indicator. Complementing this, the gilt data encompasses daily returns from 10-year UK government securities, obtained from the UK Debt Management Office’s official website. These gilts, characterised by their robust liquidity and trading volume, serve as a foundational reference for central bank monetary policy formulation.

Our analysis traverses three distinct economic epochs: the Financial Crisis Period, spanning from 1st September 2007 to 31st December 2011—a tumultuous era marked by severe global economic downturns and heightened volatility; the Normal Period, delineated from 1st January 2012 to 31st December 2019, characterised by relative economic stability and gradual recovery from the financial crisis; and the Covid-19 Period, from 1st January 2020 to 31st May 2023, a time defined by the unprecedented disruptions and policy responses precipitated by the pandemic.

The datasets underwent preprocessing to derive log-returns, a relative metric that gauges price variations over time. Log-returns are a preferred choice in financial analyses due to their consistency across different scales and their superior efficacy in handling data with significant percentage changes. Summary statistics for the data are delineated in Table 1. To initiate our exploration of the stock-gilt relationship, we computed three established correlation measures: Kendall’s Tau, Pearson Correlation, and Spearman Correlation. These conventional correlation techniques offer a foundational perspective, essential for elucidating the general correlation between datasets.

Table 1. Summary statistics of stock and gilt log-returns for financial crisis, Covid-19 pandemic and normal period

	Financial Crisis		Covid-19		Normal Period	
	Stock	Gilt	Stock	Gilt	Stock	Gilt
Mean	-0.0001	-0.0008	0.0000	0.0020	0.0002	-0.0005
Std. Dev.	0.0159	0.0190	0.0125	0.0880	0.0082	0.0363
Min	-0.0882	-0.1243	-0.1121	-0.4055	-0.0464	-0.2359
25 percentile	-0.0081	-0.0109	-0.0048	-0.0339	-0.0042	-0.0194
Median	0.0002	0.0000	0.0008	0.0000	0.0005	0.0000
75 percentile	0.0082	0.0096	0.0058	0.0351	0.0046	0.0170
Max	0.0895	0.0990	0.0857	0.4855	0.0346	0.2603
Skewness	-0.0854	-0.1725	-1.0998	0.3533	-0.2548	0.3843
Kurtosis	4.7182	4.2654	12.7671	5.6351	2.3346	6.5169

2.2. Copula

The copula method is a powerful tool in the area of multivariate statistics, enables us to separate the marginal distributions from the dependency structure of a given multivariate distribution. This concept is justified by the Sklar's Theorem (1959), according to which, any multivariate joint distribution function can be separated into its marginal distributions and a copula function that describes the dependency structure between variables:

$$F(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) = C(F_1(\mathbf{x}_1), F_2(\mathbf{x}_2), \dots, F_n(\mathbf{x}_n))$$

where $F_1(\mathbf{x}_1), F_2(\mathbf{x}_2), \dots, F_n(\mathbf{x}_n)$ are the uniformed marginal distributions for random variables X_1, X_2, \dots, X_n respectively, and F is the joint distribution function.

$C: [0,1]^d \rightarrow [0,1]$ is the copula function which has the property of being invariant under strictly increasing transformation of the marginals, also, if the marginals are uniformly distributed on $[0,1]$, the copula is also uniformly distributed on $[0,1]$. And the marginal consistency property can ensure that the copula correctly preserves the given marginal distributions in the joint distribution. The more detailed and comprehensive description of the copula properties can be found in Martin Haugh (2016), *An Introduction to Copulas*.

I will use various types of copulas, with different characteristics and properties, to model the dependency structure of my data. After that, I will compare the statistics and results of those various copula models to see which one gives the best fit to my data and most contributes to drawing conclusions. Here's a list and introduction for them:

Gaussian copula:

The Gaussian copula is derived from the multivariate normal distribution, characterized by linear correlation between variables.

The copula function is defined by:

$$C_{\text{Gaussian}}(u_1, u_2, \dots, u_n) = \Phi_{\Sigma}(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \dots, \Phi^{-1}(u_n))$$

where Φ_{Σ} is the multivariate normal distribution with correlation matrix Σ .

t-Copula:

Similar to Gaussian copula, the t-Copula incorporated tail dependence, making it more suitable to model data under extreme cases.

The copula function is defined by: $C_t(u_1, u_2, \dots, u_n) = t_{v, \Sigma}(t_v^{-1}(u_1), t_v^{-1}(u_2), \dots, t_v^{-1}(u_n))$ where $t_{v, \Sigma}$ is the multivariate t-distribution with degrees of freedom v and correlation matrix Σ .

Archimedean copulas: are a class of copulas, including but not limited to the following:

Clayton copula: $C_{\text{Clayton}}(u_1, u_2) = (\max[(u_1^{-\theta} + u_2^{-\theta} - 1), 0])^{-\frac{1}{\theta}}, \theta > 0$

Gumbel copula: $C_{\text{Gumbel}}(u_1, u_2) = \exp\left[-\{(-\log u_1)^{\theta} + (-\log u_2)^{\theta}\}^{\frac{1}{\theta}}\right], \theta \geq 1$

Frank copula: $C_{\text{Frank}}(u_1, u_2) = -\frac{1}{\theta} \log\left(1 + \frac{(\exp(-\theta u_1) - 1)(\exp(-\theta u_2) - 1)}{\exp(-\theta) - 1}\right), \theta \neq 0$

Among all the copula models listed, the Gaussian copula is the most simple and straightforward one, making it easy to implement and very popular. But the Gaussian copula doesn't have tail dependence in theory, which limits its usefulness in certain scenarios. Also, the Gaussian copula is not suitable for modeling non-linear relationships due to that its dependency structure is based on linear correlation. In contrast, the t-Copula captures both upper and lower tail dependence, but it is more complex to estimate and interpret. Frank copula covers a wide range of dependency structures, which provides it with high usefulness, but it also doesn't have tail dependence. Different from the copulas described above, the Gumbel and Clayton copulas are both asymmetric, the Gumbel copula has only upper tail dependence and the Clayton copula has only lower tail dependence.

Here, in this study, I will use the pseudo-maximum likelihood estimation to estimate copulas, which is a common and useful way. I first performed the Shapiro-Wilk Test to test the normality of my data, the test results are shown in Table 2, which gives very low p-value for both stock and gilt returns in all three periods, indicating that the data does not display a normal distribution. In this way, I choose to estimate the marginal CDFs using the empirical CDF of $x_{1,j}, x_{2,j}, \dots, x_{n,j}$, which is

$$\hat{F}_{X_j}(x) = \frac{\sum_{i=1}^n 1_{\{x_{i,j} \leq x\}}}{n + 1}.$$

Table 2. Test results of the Shapiro-Wilk Test for normality, the Stock p-value and Gilt p-value indicate the tested p-values for stock and gilt data respectively. The p-value less than 0.05 rejects the null hypothesis, suggesting the data is not following a normal distribution

	Stock p-value	Gilt p-value
<i>Financial Crisis</i>	3.6993e-20	1.5431e-18
<i>COVID-19</i>	2.4300e-25	2.3553e-24
<i>Normal</i>	2.8976e-19	1.8748e-30

Then estimate the copula parameter using the CDF value for each data point by maximizing

$$\sum_{i=1}^n \log \left[c_X \left(\hat{F}_{X_1}(x_{i,1}), \dots, \hat{F}_{X_d}(x_{i,d}) \mid \theta_C \right) \right],$$

where θ_C is the copula parameter.

2.3. Tail Dependence

Not only examining the correlation between stock market and gilt market by using traditional correlation coefficients and copula techniques, this study also investigates the dependence of the upper and lower parts (extremes) of the returns of these two assets using the technique called tail dependence. Generally, most dependence measurements consider the entire distribution of two or more random variables, but the dependency between the extreme parts of two distributions may be different from that in general cases.

Tail dependence just measures the possibility that two random variables attain extreme parts of their distributions simultaneously, it provides a deep insight into the joint behavior of variables in the tails of their distributions, which is imperative for risk management in financial analysis. When two markets are calculated to have high tail dependence coefficients, it infers that they are likely to experience extreme gains or losses simultaneously.

Tail dependence is quantified by tail dependence coefficient, which measures the magnitude of dependence in the extreme parts of two distributions. The upper tail dependence coefficient λ_U , defined as the probability that the return of one asset exceeds a certain high quantile given that the another asset also exceeds that quantile, is calculated as:

$$\lambda_U = \lim_{u \rightarrow 1^-} \Pr(U_2 > F_2^{-1}(u) \mid U_1 > F_1^{-1}(u)) = \lim_{u \rightarrow 1^-} \frac{1 - 2u + C(u,u)}{1 - u},$$

similarly, the lower tail dependence coefficient λ_L is calculated as:

$$\lambda_L = \lim_{u \rightarrow 0^+} \Pr(U_2 \leq F_2^{-1}(u) \mid U_1 \leq F_1^{-1}(u)) = \lim_{u \rightarrow 0^+} \frac{C(u,u)}{u},$$

where U_1 and U_2 are the uniform marginals, $F_1^{-1}(u)$ and $F_2^{-1}(u)$ are the quantile functions of the respective marginals.

Because some copula models such as Gaussian copula doesn't have a tail dependence coefficient in theory, in order to compare the fits of different copulas to tail dependency, I select a range of upper

tail u value from 0.950 to 0.995 with an increment of 0.005, and corresponding lower tail u value from 0.050 to 0.005 to approximate the value of tail dependence coefficients. This also allows me to analyze the change of tail dependence coefficients along with the change of u value.

Figure 1 is the scatter plot of the log-returns of stock versus gilt for three distinct periods, which can visualize the general correlation and tail dependence properly. The two dashed lines at $x = 0.97$ and $y = 0.97$ indicate the thresholds, above which both stock and gilt returns are in their upper tails, very close to the most extreme values observed. Manually count the number of dots that fall into the upper right corner region and divide by the total number of dots that is on the right of $x = 0.97$ gives the conditional probability that gilt return exceeds its upper tail threshold given that stock return is also above its upper tail threshold, which is the upper tail dependence with $u = 0.97$.

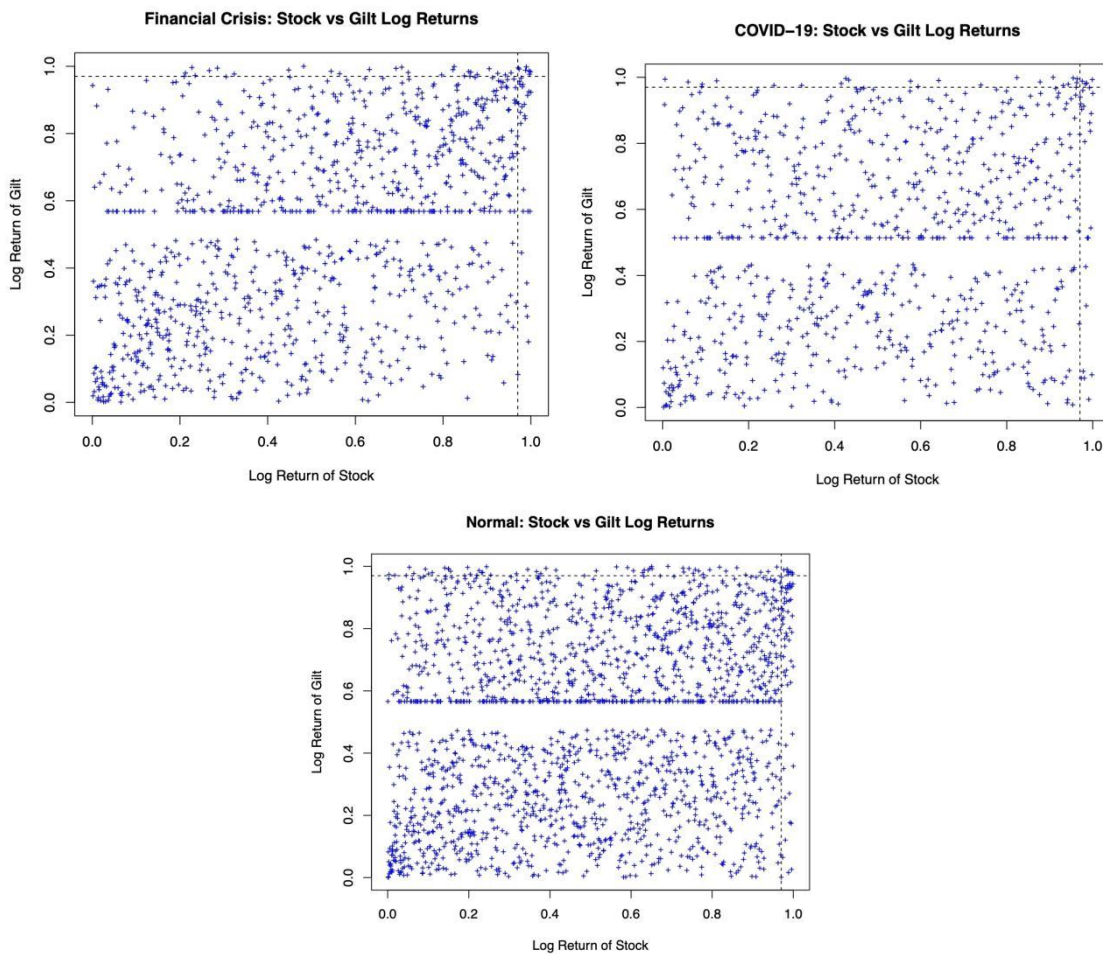


Figure 1. Scatter plot of the log-returns of stock vs. gilt across three economic periods.

The dashed lines of $x = 0.97$ and $y = 0.97$ are used for visualizing the upper tail dependence

3. Empirical Results and Interpretation

3.1. Data and Simple Correlations

Based on the summary statistics in Table 1, the mean returns for both stock and gilt are exhibiting similar levels across three time periods. The standard deviation of stock returns is generally lower than that of gilt returns, indicating the gilt market are more volatile for all three periods. The most volatile time period is Covid-19, followed by normal period and then financial crisis in terms of gilt market. This significantly higher volatility in gilt market may be due to the uncertainty caused by central bank monetary policy during the pandemic. However, the financial crisis is the time period at which the stock market being the most volatile, followed by Covid-19 and normal period, which

reflects the financial crisis caused a strong turbulence in stock market. According to the percentile, the gilt returns are more spread out than the stock returns in general, the stock market has the largest IQR during the financial crisis, which represents relatively unstable market conditions, this accords with the results of standard deviation. While the gilt market are most spreading during the Covid-19. The skewness and kurtosis of both assets are significant compared with the standard skewness of a normal distribution. Stock returns are negatively skewed across all three periods, gilt returns, on the other hand, displayed negative skewness only at financial crisis. This tendency of kurtosis and skewness indicates that the data is having heavy tails and high possibility of observing extreme values.

According to the summarized correlation coefficients in Table 3, the strength of stock-gilt correlation varies across three periods and also varies among different correlation measures. The correlation coefficients are highest during financial crisis across all three measures, suggesting that both assets were affected the same way under such economic conditions. On the contrary, the correlations during the normal period is the weakest, indicating the returns of stock and gilt are less likely to move together. This may be due to the excess government interventions during crisis, and also consistent with the idea that different financial markets may be more correlated as investors react to economic recessions or risks. The correlations at Covid-19 is close to that at normal period, which may be because the UK government carried effective measurements and monetary policies to help recover the economy during pandemic. The Pearson correlation, which measures linear relationships, is generally higher than Kendall’s Tau and Spearman correlations, suggesting the linear relationship between the two assets is relatively stronger than rank-based relationships.

Table 3. Correlation coefficients calculated by three distinct traditional correlation methods

	Pearson	Kendall	Spearman
<i>Financial Crisis</i>	0.4089	0.3011	0.4338
<i>Normal</i>	0.2129	0.1399	0.2039
<i>COVID-19</i>	0.2505	0.1139	0.1649

3.2. Copula and Tail Dependence

Table 4 displays the copula modeling results for each period, containing the estimates, the MLE (Maximum Likelihood Estimation) and AIC (Akaike Information Criterion) value, and the standard errors. A higher MLE value indicates the model provides a higher likelihood of generating the observed data, in other words, suggests a better fit to the data. The MLE value is calculated by maximizing the log-likelihood function:

$$\log L(\theta|X) = \sum_{i=1}^n \log f(x_i|\theta)$$

where $f(x_i|\theta)$ is the probability density function for the i -th observation, given the parameter θ .

Table 4. Modeling results for different copula models across three economic periods

Copula Results for Financial Crisis				
	Estimates	MLE	AIC	StdError
<i>Gaussian</i>	0.42161	112.7709	-223.5418	0.02247
<i>t</i>	0.43987	119.12695	-234.25389	0.02417
<i>Frank</i>	2.89661	114.29284	-226.58568	0.1941
<i>Gumbel</i>	1.33695	96.83836	-191.67672	0.03052
<i>Clayton</i>	0.86181	86.03225	-170.0645	0.05553
Copula Results for Covid-19 Period				
	Estimates	MLE	AIC	StdError
<i>Gaussian</i>	0.20553	19.92509	-37.85018	0.03094
<i>t</i>	0.17626	35.34764	-66.69529	0.03836
<i>Frank</i>	1.0817	12.91941	-23.83881	0.2125
<i>Gumbel</i>	1.12288	18.47592	-34.95184	0.02514
<i>Clayton</i>	0.2872	24.61179	-47.22358	0.04828
Copula Results for Normal Period				
	Estimates	MLE	AIC	StdError
<i>Gaussian</i>	0.21062	47.68607	-93.37215	0.02042
<i>t</i>	0.21672	62.15891	-120.31782	0.0227
<i>Frank</i>	1.27591	43.85956	-85.71911	0.13625
<i>Gumbel</i>	1.12774	37.64161	-73.28322	0.01737
<i>Clayton</i>	0.32534	55.82954	-109.65908	0.03245

A more negative AIC value generally indicates a better model, which explains the data with fewer parameters, in other words, suggests a better goodness of fit and simplicity. The formula for AIC value is calculated as:

$$AIC=2k-2\log L(\hat{\theta}|X)$$

where k is the number of parameters in the model and $\log L(\hat{\theta}|X)$ is the log-likelihood evaluated at the maximum likelihood parameter estimates $\hat{\theta}$.

The highest MLE values and the most negative AIC values for t-Copula across all three periods indicate that t-Copula gives the general best fit to our data during every economic period we selected. And we can conclude from the estimates of copula models that UK stock market and gilt market are exhibiting mild and similar dependence at the Covid-19 and normal period, this is consistent with the conclusions we drawn previously from the simple correlation coefficients. However, for the financial crisis period, the dependency between stock market and gilt market increases significantly, the estimate of t-Copula doubles at financial crisis compared with that at normal and the Covid-19 period, and the estimates of other copula techniques also have a considerable increment at financial crisis. The standard errors are generally low across all periods and copula models, indicating precise estimates.

Table 5 presents the tail dependence results evaluated with each copula model and the empirical tail dependence on real data. The results show that the empirical lower tail dependence is higher across all three periods, this indicates that during economic recession or market downturns, stock and gilt

returns tend to decline together more strongly, in simple words, the stock and gilt markets are more synchronized during economic bad times. Also, the dependency level at extreme cases is weakest during normal period, and the tail dependency is higher and similar for financial crisis and Covid-19 period, indicating that the extreme correlations of stock and gilt markets are more significant during market instability periods. This is different from the general dependency level we evaluated using the copula estimates and simple correlation coefficients, where normal and Covid-19 periods have low and similar correlations.

Table 5. Tail dependence estimations for different copula models and the empirical tail dependence across three economic periods. The row indexes represent the upper tail u values we selected to approximate the tail dependence, each upper-tail u also corresponds to a lower tail u which equals $1 - u_{\text{upper}}$. The format of the elements in the table follows: (upper tail dependence, lower tail dependence)

Tail Dependence Results for Financial Crisis						
	Gaussian	t	Frank	Gumbel	Clayton	Empirical
0.95	(0.1996, 0.1996)	(0.246, 0.246)	(0.1342, 0.1342)	(0.3492, 0)	(0, 0.4679)	(0.2, 0.2778)
0.955	(0.1899, 0.1899)	(0.2381, 0.2381)	(0.1222, 0.1222)	(0.3464, 0)	(0, 0.466)	(0.1837, 0.2708)
0.96	(0.1797, 0.1797)	(0.2297, 0.2297)	(0.11, 0.11)	(0.3435, 0)	(0, 0.4642)	(0.2045, 0.2558)
0.965	(0.1688, 0.1688)	(0.2207, 0.2207)	(0.0975, 0.0975)	(0.3406, 0)	(0, 0.4623)	(0.2051, 0.2105)
0.97	(0.1571, 0.1571)	(0.2112, 0.2112)	(0.0847, 0.0847)	(0.3377, 0)	(0, 0.4604)	(0.2121, 0.1875)
0.975	(0.1443, 0.1443)	(0.2008, 0.2008)	(0.0715, 0.0715)	(0.3349, 0)	(0, 0.4585)	(0.1786, 0.1852)
0.98	(0.1301, 0.1301)	(0.1892, 0.1892)	(0.058, 0.058)	(0.332, 0)	(0, 0.4565)	(0.1818, 0.1905)
0.985	(0.114, 0.114)	(0.1761, 0.1761)	(0.0441, 0.0441)	(0.3291, 0)	(0, 0.4545)	(0.1765, 0.125)
0.99	(0.0947, 0.0947)	(0.1602, 0.1602)	(0.0298, 0.0298)	(0.3263, 0)	(0, 0.4524)	(0, 0)
0.995	(0.0692, 0.0692)	(0.1387, 0.1387)	(0.0151, 0.0151)	(0.3234, 0)	(0, 0.4501)	(0, 0)

Tail Dependence Results for Covid-19 Period						
	Gaussian	t	Frank	Gumbel	Clayton	Empirical
0.95	(0.1068, 0.1068)	(0.1868, 0.1868)	(0.0777, 0.0777)	(0.1858, 0)	(0, 0.2047)	(0.2558, 0.2143)
0.955	(0.0992, 0.0992)	(0.1824, 0.1824)	(0.0703, 0.0703)	(0.1818, 0)	(0, 0.1991)	(0.2308, 0.2105)
0.96	(0.0913, 0.0913)	(0.1779, 0.1779)	(0.0628, 0.0628)	(0.1778, 0)	(0, 0.1933)	(0.2571, 0.1765)
0.965	(0.0832, 0.0832)	(0.1732, 0.1732)	(0.0552, 0.0552)	(0.1738, 0)	(0, 0.1872)	(0.1667, 0.1724)
0.97	(0.0747, 0.0747)	(0.1684, 0.1684)	(0.0476, 0.0476)	(0.1699, 0)	(0, 0.1806)	(0.1538, 0.2)
0.975	(0.0658, 0.0658)	(0.1633, 0.1633)	(0.0398, 0.0398)	(0.1659, 0)	(0, 0.1737)	(0.1818, 0.2381)
0.98	(0.0564, 0.0564)	(0.1579, 0.1579)	(0.032, 0.032)	(0.1619, 0)	(0, 0.166)	(0.1667, 0.2941)
0.985	(0.0462, 0.0462)	(0.152, 0.152)	(0.0242, 0.0242)	(0.158, 0)	(0, 0.1574)	(0.0769, 0.25)
0.99	(0.0349, 0.0349)	(0.1454, 0.1454)	(0.0162, 0.0162)	(0.154, 0)	(0, 0.1472)	(0.1111, 0.375)
0.995	(0.0217, 0.0217)	(0.1374, 0.1374)	(0.0081, 0.0081)	(0.1501, 0)	(0, 0.1339)	(0, 0.25)

Tail Dependence Results for Normal Period						
	Gaussian	t	Frank	Gumbel	Clayton	Empirical
0.95	(0.1086, 0.1086)	(0.1576, 0.1576)	(0.0833, 0.0833)	(0.1904, 0)	(0, 0.2258)	(0.1188, 0.21)
0.955	(0.1009, 0.1009)	(0.1514, 0.1514)	(0.0754, 0.0754)	(0.1864, 0)	(0, 0.2205)	(0.1099, 0.2111)
0.96	(0.093, 0.093)	(0.1449, 0.1449)	(0.0674, 0.0674)	(0.1825, 0)	(0, 0.2149)	(0.1111, 0.1875)
0.965	(0.0848, 0.0848)	(0.1382, 0.1382)	(0.0593, 0.0593)	(0.1785, 0)	(0, 0.209)	(0.1268, 0.1714)
0.97	(0.0762, 0.0762)	(0.1312, 0.1312)	(0.0512, 0.0512)	(0.1746, 0)	(0, 0.2028)	(0.1311, 0.1667)
0.975	(0.0672, 0.0672)	(0.1237, 0.1237)	(0.0429, 0.0429)	(0.1707, 0)	(0, 0.1961)	(0.1373, 0.14)
0.98	(0.0576, 0.0576)	(0.1155, 0.1155)	(0.0345, 0.0345)	(0.1667, 0)	(0, 0.1888)	(0.0976, 0.15)
0.985	(0.0473, 0.0473)	(0.1065, 0.1065)	(0.0261, 0.0261)	(0.1628, 0)	(0, 0.1806)	(0, 0.1)
0.99	(0.0358, 0.0358)	(0.096, 0.096)	(0.0175, 0.0175)	(0.1589, 0)	(0, 0.171)	(0, 0.1)
0.995	(0.0223, 0.0223)	(0.0826, 0.0826)	(0.0088, 0.0088)	(0.1549, 0)	(0, 0.1583)	(0, 0.2)

Comparing the tail dependence results evaluated by each copula models and the empirical tail dependence, we can conclude that t-Copula is not only the best copula technique for analyzing the correlation in general case, but also a robust choice for evaluating the tail dependence of stock and gilt markets which represents the level of dependency between these two assets in extreme cases. The only exception is at Covid-19 period, Clayton copula provides a slightly better estimation for the lower tail dependence than t-Copula.

3.3. Interpretation of Results

The correlation between stock and long-term government bond returns has been a subject of extensive research in the literature, including studies by Baele et al. (2010), Baele and Holle (2017), Campbell et al. (2017), Christiansen and Rinaldo (2007), David and Veronesi (2013), and Guidolin and Timmermann (2007). The impact of monetary and fiscal policies on these returns has also been investigated. For instance, Song (2017) suggests that an aggressive monetary policy can lead to a shift in the stock-bond correlation. Erica et al. (2022) emphasize the role of a mix of monetary and fiscal policies in explaining changes in returns correlation using a general equilibrium framework. Most of this research focuses on the US market, which makes the findings of this study particularly valuable as they provide insights into the correlation between the UK stock and gilt markets across different economic periods.

Erica et al. (2022) argue that a proactive monetary policy, which causes interest rates to fall more than inflation, stimulates output and consumption, amplifying the effect of positive technology shocks. This, in turn, improves corporate profits and pushes up stock prices, while also leading to higher long-term government bond prices. Consequently, the stock-bond return correlation is positive in response to technology shocks and active monetary policy. However, during the financial crisis, the aggressive monetary policy, as evidenced by the Bank of England's substantial reduction of the base rate, resulted in a higher positive correlation between stock returns and yield changes, indicating a stronger negative correlation between stock and bond returns.

The Covid-19 pandemic presented a unique economic shock characterized by market turbulence, high unemployment, and recession, similar to the financial crisis. Despite the already low interest rate, monetary action was limited, and proactive fiscal policy became the dominant factor. Erica et al. (2022) posit that a proactive fiscal policy combined with a reactive monetary policy would lead to a negative stock-bond return correlation due to investment shocks and diminished technology shocks. This is consistent with the results of this study, which show a positive correlation between stock returns and long-term gilt yield changes during the Covid-19 period, reflecting a negative stock-bond return correlation when the UK government pursued aggressive fiscal policies. The general stock-gilt correlation during Covid-19, while substantial, was not as significant as during the financial crisis period, potentially due to the successful policies and measures implemented by the UK government to stabilize the market and labour market. Moreover, some literature suggests that fiscal policy generally has less impact on stock-bond return correlation than monetary policy.

Despite the lower general correlation, the tail dependence of stock and gilt markets during Covid-19 is almost as high as during the financial crisis period. This suggests that the pandemic's crisis nature caused similar market instability and panics to the financial crisis. The importance of measuring tail dependence in financial market correlations is evident, as simple correlation and general copula models may fail to capture such dependence under extreme market conditions. The notably lower correlation and tail dependence between stock and gilt markets during the normal period indicate that the returns of the two assets are not influencing each other considerably during economic stability. The consistent high performance of the t-Copula in fitting the data across different economic periods supports its status as the best copula model among those assessed in this study.

4. Conclusion

Employing stock and gilt market data spanning three distinctive economic epochs—the financial crisis, the Covid-19 pandemic, and a period of normalcy—this study applies a suite of copula models, encompassing Gaussian, t, Frank, Gumbel, and Clayton copulas, to investigate and contrast both the general dependency structure and the tail dependency between these two financial assets under varying market conditions.

Evidence elucidates a positive general correlation between stock returns and gilt yield changes, signifying a negative correlation between stock and gilt returns, which is most pronounced during the financial crisis, and comparatively lower during the Covid-19 pandemic and normal period. Notably, during the Covid-19 pandemic, while the general correlation is less pronounced than during the financial crisis, the tail dependence remains strikingly high, underscoring the imperative of analysing both general correlation and tail dependence to fully comprehend the interrelationship between the two assets.

Moreover, the study's findings on tail dependence reveal that both the financial crisis and Covid-19 periods exhibit heightened tail dependence relative to the normal period, illustrating that extreme co-movements between stocks and gilts become more pronounced under conditions of economic instability. This observation aligns with Chin & Yang (2011), which examines extreme stock-bond correlations in the UK, demonstrating that empirical lower tail dependence is consistently higher than upper tail dependence. This suggests that extreme co-movements are more intense during economic downturns than in periods of economic prosperity.

Finally, the implementation of multiple copula models in this paper facilitates a comprehensive comparison of their efficacy in fitting market data and capturing the dependency structure. The t-Copula model emerges as the most efficacious, rendering it well-suited for modelling correlations in financial markets, particularly during periods of market volatility. However, the t-Copula is not without limitations, as its symmetric property may lead to reduced accuracy, and it is outperformed by the Clayton copula in modelling tail dependence during the Covid-19 period. Consequently, future research should explore the application of more sophisticated copula models, such as the SJC copula, to more accurately capture the potentially asymmetric tail dependencies between financial assets. Expanding the analysis to incorporate a broader array of asset types, such as foreign exchanges or corporate bonds, or conducting comparative analyses between different countries could provide a more robust understanding of how various markets interact under differing economic conditions and policy environments. This knowledge is of immense value to policymakers and market participants, including portfolio managers.

In summary, the research underscores the complex interplay between stock and gilt markets under varying economic conditions, highlighting the significance of tail dependence analysis. The findings support the use of the t-Copula for modelling market correlations, while also indicating the need for further exploration of advanced copulas to refine the understanding of asymmetric tail dependencies. A wider investigation into diverse asset types and comparative analyses across countries would significantly enhance our knowledge of market interactions, benefitting a multitude of stakeholders in the financial sector.

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