

AI Pricing: Adoption of Artificial Intelligences and Collusive Price

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Abstract. With the growing integration of artificial intelligence (AI) in determining pricing strategies, there is an increasing concern about its potential to foster collusive behavior. Harrington (2012, 2018) underscores the challenge: if AI proves more adept at tacit collusion than humans or if AI-driven collusion is inherently tacit, then it presents a significant hurdle for prosecution under the prevailing interpretation of US antitrust laws. Validating these concerns, Assad et al. (2020) observed collusive price surges linked to the adoption of pricing algorithms among German gas stations. Drawing from game theory—specifically the repeated game paradigm—this paper crafts a foundational mathematical model to analyze competition versus collusion dynamics. It also evaluates the resultant welfare implications of both scenarios. The paper further delves into the broader challenges posed by AI-powered pricing and advocates for potential policy countermeasures, including algorithmic regulation and collusion detection mechanisms.

Keywords: Game Theory, Collusion, Regulation, Algorithm Optimization.

1. Introduction

The dynamic of oligopoly competition has intriguing implications: when one firm reduces its prices, rivals often follow suit. Firms retain roughly their initial market shares but at diminished profit margins. Rather than competing solely on price, these firms sometimes collude, setting prices near monopolistic levels to enhance profitability.

Such collusive practices, however, undermine market competition and resort to unethical methodologies. The direct consequence is a detriment to consumer welfare: elevated prices and potentially diminished product quality and consumers have no choice but to purchase at a price determined by the formation of companies.

Two primary methodologies to detect collusion exist in the Rule of Reason [1] and Per Se Violation [2]. While the former evaluates an agreement's holistic impact on the market, encompassing both its pros and cons, the latter presumes certain agreements, like price-fixing or territorial divisions, as inherently violating antitrust laws. However, it is hard to define collusion behavior. Governments and pertinent bodies often grapple with these intricacies on a case-by-case basis, finding no overarching framework to definitively categorize a firm's actions as collusive. Regulators frequently miss prosecuting potential breaches due to this ambiguity. The complexity intensifies with AI's increasing role in pricing. Collusive behavior, especially with AI involvement, remains more elusive and harder to delineate.

This paper ventures to present mathematical models elucidating competition and collusion dynamics, shed light on potential collusion challenges amidst the rapid evolution of AI, and advocate actionable strategies aiding governmental oversight in identifying and penalizing collusive undertakings.

2. Case Study

The bedrock of fair competition in the U.S. is underpinned by a suite of Federal antitrust laws, bolstered by additional state-specific regulations. At their core, these statutes aim to protect consumers from business practices that unjustly curtail competition, often manifesting in inflated product and service prices [3]

Collusion includes three types—formal collusion, tacit collusion, and price leadership. Formal collusion occurs when firms make a formal agreement to stick to high prices, involving the creation of a cartel that we will mention in part 4, with the most famous example of OPEC. Tacit collusion and price leadership are harder to be proved whether they are unfair competition or just the natural operation of markets, since the firms make informal agreements without actually speaking to their rivals or try to collude by following the prices set by a market leader. [4]

A noteworthy study from ACL 2019 centered on the "S&P 500 Earnings Conference Calls" dataset, which amalgamated both textual and audio traits from CEO dialogues in 2017's earnings calls. This research spawned the Multimodal Deep Regression Model (MDRM) and underscored the gravity of auditory cues. For instance, post-AMD's May 1, 2017, earnings call, the company's stock plummeted by 16.1%. The CEO's statement, while ostensibly positive in text, was tinged with a mean pitch 20% above his average—a potential indicator of wavering confidence.

Airbnb, the global leader in vacation rentals and home-sharing, uses sophisticated AI-driven recommendation systems to optimize user experience. These systems, originally designed to match guests with ideal listings, can also be used to suggest pricing strategies to homeowners based on real-time data. While this can help hosts remain competitive, there's a latent risk: the inadvertent standardization of rental prices. If numerous hosts in a particular region turn to Airbnb's AI for pricing, it's plausible they'd all receive congruent pricing recommendations. Consequently, listings in similar locations, offering comparable amenities, could all converge towards a uniform price. With an AI continually adjusting rates, hosts may feel they're already at the market's cutting edge, diminishing the incentive to undercut competitors or offer unique promotions. While no direct communication occurs between hosts, the AI system, by being the dominant pricing strategy tool, can unintentionally shepherd hosts towards a tacit understanding on pricing benchmarks.

3. Literature Review

Research done by Laffont and Martimort (1997, 2000) unveiled a pivotal notion: transaction costs emerge primarily from agents' asymmetric information, which not only paves the way for collusive agreements but is also constrained by specific modeling architectures. The proposition by Laffont and Martimort, suggesting that collusion can be deterred without any associated costs given uncorrelated types, has been further examined. Quesada (2004) offers a distinct perspective on coalition formation, whereas Jeon and Menicucci (2005) employ a nonlinear model showcasing the advantages accruing to colluding consumers via their purchasing activities. However, there are inherent challenges with these specialized models:

First, there's ambiguity regarding the broader applicability of the outcomes if conditions deviate from the assumptions in the models. Second, Despite the comprehensive nature of the results, the solutions for deterring collusion seem to be limited to particular settings, casting doubt on their efficacy elsewhere. Third, the uniqueness constraint of the models makes it difficult to discern when the hypothesis of collusion is tenable and why it's avoidable in specified contexts.

The study authored by Yeon-Koo Che and Jinwoo Kim at Columbia University serves as a cohesive narrative, amalgamating fragmented insights from prior research and offering a holistic understanding of how transaction costs, arising from agents' private information, can be harnessed to thwart collusion. Their mechanism, though innovative, is underpinned by several pivotal prerequisites which are instrumental in identifying genuine catalysts for collusion.

Chassang and Ortner (2022) delve into the intricacies of collusive behavior in markets and the regulatory mechanisms that can be employed to counteract it. The paper emphasizes the challenges posed by tacit and explicit collusion in different market structures, exploring the economic and legal implications of both. Through a comprehensive examination of contemporary market scenarios, Chassang and Ortner highlight the limitations of traditional antitrust measures and propose innovative

regulatory strategies based on game-theoretical models. The authors underscore the importance of proactive regulation and the adaptability of policy measures to evolving market dynamics.

4. Competitive Modeling Vs. Collusive Modeling

In this section, we are going to analyze classic marketing models, including competitive ones and collusive ones, in order to illustrate the essence of market operation.

4.1. Cournot's Model of Oligopoly

Economists refer to models of "oligopoly" as competition between a small number of sellers, though they involve no restriction on the number of firms; the label reflects the strategic interaction they capture. Both models were studied first in the nineteenth century before the notion of Nash equilibrium was formalized for a general strategic game. The first is due to the economist Cournot (1838). Suppose there are two firms (the industry is a "duopoly"), each firm's cost function is the same, given by $C_i(q_i) = cq_i$ for all q_i ("unit cost" is constant, equal to c), and the inverse demand function is linear where it is positive, given by

$$P(Q) = \max\{\alpha - bQ, 0\} \quad (1)$$

where $\alpha, b > 0$ and $c \geq 0$ are constants. Assume that $c < \alpha$, so that there is some value of total output Q for which the market price $P(Q)$ is greater than the firms' common unit cost c . (If c were to exceed α , there would be no output for the firms at which they could make any profit because the market price never exceeds α .)

The profit maximization problem for two oligopolistic manufacturers can be expressed as follows:

$$\pi_1(Q_1, Q_2) = PQ - cQ_1 = [\alpha - b(Q_1 + Q_2)]Q_1 - cQ_1 = -bQ_1^2 - bQ_1Q_2 + (\alpha - c)Q_1 \quad (2)$$

$$\pi_2(Q_1, Q_2) = PQ - cQ_2 = [\alpha - b(Q_1 + Q_2)]Q_2 - cQ_2 = -bQ_2^2 - bQ_1Q_2 + (\alpha - c)Q_2 \quad (3)$$

One can only control their own production; thus, when aiming to maximize profit, one should consider the production level of the other firm as given:

$$\frac{\partial \pi_1}{\partial Q_1} = -2bQ_1 - bQ_2 + \alpha - c = 0 \quad (4)$$

$$\frac{\partial \pi_2}{\partial Q_2} = -2bQ_2 - bQ_1 + \alpha - c = 0 \quad (5)$$

Organizing equation (4) gives the output decision condition for vendor 1:

$$Q_1 = \frac{\alpha - c - bQ_2}{2b} \quad (6)$$

Organizing equation (5) gives the output decision condition for vendor 2:

$$Q_2 = \frac{\alpha - c - bQ_1}{2b} \quad (7)$$

By associating the reaction functions of the two firms, we can solve for a Nash Equilibrium:

$$Q_1^* = Q_2^* = \frac{\alpha - c}{3b} \quad (8)$$

At this point, the market equilibrium price is:

$$P^* = \alpha - b(Q_1^* + Q_2^*) = \frac{\alpha + 2c}{3} \quad (9)$$

The equilibrium production level is the intersection of two best response curves:

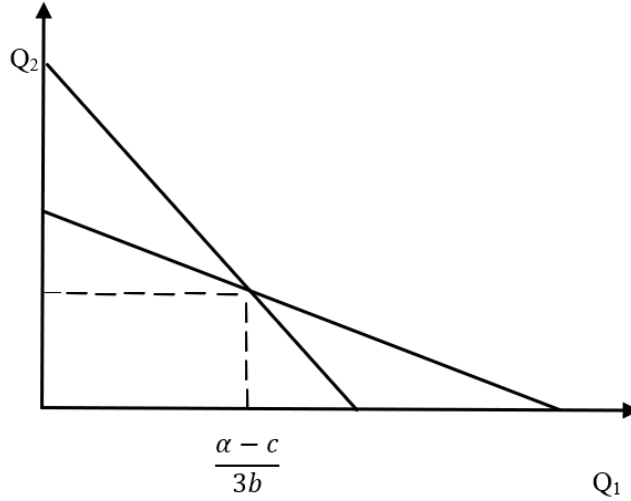


Fig. 1 Graphic expression of production functions

Each firm's profit is:

$$\frac{\alpha + 2c}{3} \times \frac{\alpha - c}{3b} - c \times \frac{\alpha - c}{3b} = \frac{\alpha - c}{3} \times \frac{\alpha - c}{3b} = \frac{(\alpha - c)^2}{9b} \quad (10)$$

And consumers need to pay $\frac{\alpha + 2c}{3}$ to buy the product.

However, if two firms try to collude, they could earn a higher profit and hurt consumer welfare.

To simplify the algebraic calculations, I've assigned specific values to the parameters in our model: $\alpha = 1, b = 1, c = 0$

Consider the following Cournot duopoly problem, market demand in period t is:

$$P_t = \max\{1 - Q_t, 0\} \quad (11)$$

where Q_t is the sum of the output of the two firms in period t .

Each firm $t = 1, 2$ chooses q_t^i every period. Production is costless ($c=0$). So, if the two firms produce q_t^1, q_t^2 in period t , firm i earns profit:

$$\pi_t^i = \max\{(1 - q_t^1 - q_t^2) \times q_t^i, 0\} \quad (12)$$

in period t .

Firm maximizes the discounted value of their profit stream: that is,

$$U_i = \sum_{t \geq 1} \pi_t^i \delta^{t-1} \quad (13)$$

Their discount factor is $\delta = \frac{1}{2}$. I would like to compute a subgame perfect Nash Equilibrium (SPE) where $q_t^1 = q_t^2 = \frac{1}{4}$ for $\forall t \geq 1$.

4.2. Analysis of the Collusive Game

Let's call states $m = \{0, P, NE\}$. Then, I will describe player's strategies and the transition of the state:

$m = 0$:

1. $q_t^1 = q_t^2 = \frac{1}{4}$
2. If either q_t^1 or $q_t^2 \neq \frac{1}{4}$, then we go to state P.
3. Otherwise, the game stays at state 0.

$m = P$:

1. $q_t^1 = \frac{3}{4}$ and $q_t^2 = \frac{3}{4}$
2. If either q_t^1 or $q_t^2 \neq \frac{3}{4}$, then we stay at state P .
3. Otherwise, the game goes to the state NE .

$m = NE$:

1. $q_t^1 = \frac{1}{3}$ and $q_t^2 = \frac{1}{3}$.
2. The game stays at NE forever.

Let's firstly check whether this is a SPE (Subgame Perfect Equilibrium). At $m = 0$, without loss of generality say firm 1 deviates to $q_t^1 = \frac{3}{8}$, which is the best response given $q_t^2 = \frac{1}{4}$. Then the gain in profit from deviating is $\frac{1}{64}$. However, if firm 1 deviates, the game will go to state $m = P$ in next period. Firm 1's loss from deviation is at least $\delta \left(\frac{1}{8} - 0 \right) = \frac{1}{16} > \frac{1}{64}$. Therefore, firm 1 will not have incentive to deviate.

At $m = P$, without loss of generality say firm 1 deviates to $q_t^1 = \frac{1}{8}$, which is the best response given $q_t^2 = \frac{3}{4}$. Then the gain from deviating is $\frac{1}{64}$. However, if firm 1 deviates, then the game will stay at state P instead of going to state NE next period. Therefore, the loss from deviation is $\delta \left(\frac{1}{9} - 0 \right) = \frac{1}{18} > \frac{1}{64}$. This shows that the strategy defined at $m = P$ is SPE.

The fact that strategy defined at $m = NE$ is SPE is trivial. Thus, on the equilibrium path of the SPE. Each firm produces at $\frac{1}{4}$ forever.

4.3. Compare with the Competitive Model

In the competitive model with $\alpha = 1; b = 1; c = 0$, both firms produce at $q_t = \frac{1}{3}$ for every period and this gives them a profit of

$$\sum_{t \geq 1} \left(1 - \frac{1}{3} - \frac{1}{3}\right) \times \frac{1}{3} \times \left(\frac{1}{2}\right)^{t-1} = \frac{2}{9} \quad (14)$$

However, in the collusive model, each firm gets a profit of

$$\sum_{t \geq 1} \left(1 - \frac{1}{4} - \frac{1}{4}\right) \times \frac{1}{4} \times \left(\frac{1}{2}\right)^{t-1} = \frac{1}{4} \quad (15)$$

In the other words, the firm gains a profit by colluding and communicating with each other, and this is going to hurt consumers because previously, in the competitive model, consumers pay a price of $\frac{1}{3}$ to buy the commodity but they need to pay a price of $\frac{1}{2}$ in the collusive model.

5. Issues Raised By the Development of AI

Pricing algorithms play a pivotal role in facilitating potential collusion between firms by harnessing vast online data to instantly price a broad array of products. There are two primary ways in which the ubiquity of these algorithms could jeopardize consumer welfare. Firstly, the algorithm's underlying machine learning can precisely predict demand by meticulously analyzing consumer data to gauge each individual's purchasing intent. Secondly, there's the phenomenon of algorithmic collusion, which we delve into in section five.[7]

Ezrachi, Ariel, and Maurice E. Stucke contend that the widespread use of pricing algorithms can: 1) ease the path to collusion by automating adherence monitoring, 2) facilitate rapid retaliation against firms undercutting prices, and 3) foster "tacit" implicit collusive agreements, which evade detection as they don't rely on explicit communication. Contrary to mere price discrimination, pricing algorithms not only redistribute but can also drive colluding firms to intentionally reduce supply in a market to maintain elevated prices. In addition, the development of AI has brought new problems to researchers, such that AI pricing is capable to help firms collude but not be punished since they could defend themselves by stating they did not collude intentionally.

Furthermore, AI's evolution has posed fresh challenges. Firms can use AI pricing to potentially collude without repercussions, asserting they didn't collude with intent. For instance, Airbnb, an online platform for home and experience sharing, uses "Smart Pricing" to suggest "Price Tips" for hosts. Essentially, this system offers competitors (hosts, in this case) a unified pricing recommendation through its dynamic pricing system, a practice that treads on thin ice legally in the U.S.

However, some believe that concerns about AI's role in collusion might be premature. Research from DeepMind, an Alphabet subsidiary, underscores current AI algorithms' limitations in fostering collusive outcomes. The study points to numerous challenges AI faces in achieving cooperation in games that are neither zero-sum nor wholly cooperative.

Still, while AI-driven collusion may not be an immediate pressing concern, the future potential cannot be ignored. Given AI's rapid learning and adaptability, its capacity for sophisticated cooperation could emerge. As a result, preemptive measures to identify AI's role in pricing and potential collusion remain essential to safeguard consumer interests.

6. Potential Policy Solution

In this section, I will outline two potential solutions to the collusion challenges presented by AI.

The first approach involves regulating algorithms that aim to bolster profit margins for companies, where collusion might be perceived as an optimal route to that end. Broadly speaking, this method would necessitate regulatory measures from federal authorities to embed a restrictive function within the algorithm, denoted as $\lambda \in (0,1)$ which adheres to the formula

$$\lambda \times \text{Firm Profit} + (1 - \lambda) \times \text{Consumer Welfare},$$

where *Consumer Welfare* encapsulating consumer preferences for a variety of goods and services. By manipulating this parameter, we can ascertain a firm's involvement in collusion by maintaining the resulting value within a specified boundary.

The second approach suggests offering leniency to firms employing AI pricing, especially if they assert that any collusive behavior was unintentional. For instance, let's represent the anticipated consumer welfare in a competitive setting V_{com} and in a collusive scenario as V_{col} . In the absence of AI pricing, if the observed consumer welfare dips below V_{col} , regulatory authorities might initiate a dialogue with the firm to scrutinize potential collusive actions. However, recognizing that these actions may be inadvertent due to AI intervention, firms could be granted a certain degree of leeway. A potential policy could stipulate a threshold $\epsilon > 0$. If the actual consumer welfare V falls below $V_{col} - \epsilon$, then the firm would face regulatory consequences.

However, if the welfare lies in the range $V_{col} \geq V \geq V_{col} - \epsilon$, firms would not be penalized. Such a policy acknowledges the unintentional ramifications of AI pricing while also prompting firms to regularly evaluate their algorithms to ensure consumer interests aren't inadvertently compromised.

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