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Multi-objective Integration and Optimization Research on Urban Waste Sorting and Transportation

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Abstract. This paper focuses on the challenges of urban waste sorting and transportation scheduling, establishing a mathematical modelling and optimisation framework that integrates vehicle path planning, multi-vehicle collaborative scheduling, and facility location optimisation. The study first establishes a CVRP model for single-vehicle route optimisation, employing an improved heuristic algorithm (combining PathCheapestArc and the 2-opt operator) to achieve efficient solutions. Next, in multi-vehicle scheduling, the traditional model is expanded to incorporate constraints such as time windows, with a solver used to perform collaborative optimisation. Finally, a two-stage decomposition method is proposed for transfer station site selection and carbon emissions optimisation. using clustering analysis and the P-median model to make the first-stage location decisions, and then embedding carbon emission targets into the second-stage route optimisation. This study innovatively proposes an integrated optimisation framework, designs a hybrid solution method combining precise algorithms and heuristic strategies, and for the first time systematically incorporates carbon emission indicators into transportation scheduling models, providing a scientific decision-support tool for urban waste classification management.

Keywords: Vehicle path optimisation; improved heuristic algorithm; carbon emission; waste sorting.

1. Introduction

With the development of urbanisation, the production of urban solid waste has surged, presenting complex challenges for waste sorting and transportation due to multiple objectives and constraints. Traditional single-optimisation models are insufficient to meet these demands. This paper proposes an integrated framework to address vehicle route planning for waste sorting and transportation, multi-vehicle scheduling, and the selection of transfer stations considering carbon emissions [1-2]. For single-vehicle route optimisation, a CVRP model is established [3], and an improved heuristic algorithm combining PathCheapestArc and the 2-opt operator is used for solution [4]. In multi-vehicle collaborative scheduling, the traditional model is extended to incorporate constraints such as time windows, and a solver is employed to achieve global optimisation. For transfer station site selection and carbon emission control, a two-stage decomposition method is proposed [5]. In the first stage, clustering analysis and the P-median model are used to determine the site selection scheme [6], and in the second stage, carbon emission targets are embedded in path optimisation. This research provides a scientific decision-making method that balances efficiency and environmental protection for urban waste sorting and transportation.

2. Single Vehicle Path Optimisation Model

2.1. Model Establishment

In vehicle path optimisation under single-vehicle capacity constraints, this paper aims to minimise the total driving distance of all transport vehicles and considers the following main constraints: First, let $x_{i,j}^k$ be the decision variable. When the kth vehicle directly passes from point i to point j on its path, $x_{i,j}^k = 1$, otherwise 0. Let y_i^k be the auxiliary variable. If collection point i is served by the kth vehicle, then $y_i^k = 1$, otherwise 0.

1) Objective function (minimum total travel distance):

$$\min Z = \sum_{k=1}^{K}\sum_{i=0}^{n}\sum_{j=0}^{n} d_{i,j} x_{i,j}^{k} \tag{1}$$

2) Unique service constraint: Each collection point must be served by exactly one vehicle.

$$\sum_{k=1}^{\mathrm{K}} y_i^k = 1, \quad i = 1, \ldots, \mathrm{n} \tag{2}$$

3) Vehicle load constraint: The total amount of waste transported by each vehicle in a single trip must not exceed the maximum load capacity Q.

$$\sum_{i=1}^{n} w_i y_i^k, \quad k = 1, \ldots, K \tag{3}$$

4) Vehicle departure and return constraint: Each vehicle must depart from and return to the processing plant for each trip.

$$\sum_{j=1}^{\mathrm{n}} x_{0,j}^{k} = 1, \quad \sum_{i=1}^{\mathrm{n}} x_{i,0}^{k} = 1, \quad k = 1, \ldots, K \tag{4}$$

5) Path connectivity and flow conservation constraint: Except for the processing plant, the number of times a vehicle enters each collection point must equal the number of times it exits, ensuring path continuity.

$$\sum_{j=0}^{\mathrm{n}} x_{i,j}^{k} - \sum_{j=0}^{\mathrm{n}} x_{j,i}^{k} = 0, \quad i = 1, \ldots, n \tag{5}$$

6) Variable value constraints:

$$\mathrm{x}_{\mathrm{i,j}}^{\mathrm{k}} \in \{0,1\}_{\mathrm{i}}^{\mathrm{k}} \in \{0,1\} \tag{6}$$

This model not only ensures that all collection points are efficiently covered, but also minimises transport distances under actual capacity conditions. It is a classic optimisation framework for urban waste collection and transport scheduling and is an NP-hard problem.

2.2. Model Solution

Since the vehicle routing problem is essentially an NP-hard problem, it is difficult to obtain an exact solution through exhaustive search. Therefore, heuristic and meta-heuristic algorithms are used to achieve an approximate optimal solution. This paper uses the RoutingSolver on the GoogleOR-Tools platform for model solution.

OR-Tools' underlying implementation combines multiple local search operators (such as 2-opt, Or-opt, Swap, etc.), adjusting the path structure in each iteration to further reduce the objective function Z. Under the constraint of satisfying all constraints, the solver can output the optimal allocation plan for each vehicle and its specific transportation path within a finite time.

In the model, the PathCheapestArc strategy is used as the initial solution generation method, combined with GuidedLocalSearch for global optimisation. The model automatically determines the optimal transport route for each vehicle and strictly satisfies the constraint that the amount of waste transported by each vehicle in a single trip does not exceed 5 tonnes.

The solution results show that a total of 15 transport vehicles were scheduled to cover all 30 collection points, with all waste effectively collected and transported back to the processing plant. The specific transport paths, total waste volume, and path distances for each vehicle are shown in Table 1.

Table 1. The specific transport routes, total waste volume, and route distances for each vehicle

Vehicle Number	Transportation Route (Collection Point Number)	Total Waste Volume (tonnes)	Path Distance (km)
1	0→10→25→0	5.00	81.89
2	0→3→19→0	4.60	78.48
3	0→2→11→0	4.00	32.24
…	…	…	…
14	0→28→16→0	4.80	73.47
15	0→4→15→0	5.00	56.07

The total transportation distance for all vehicles is 1,108.19 km, with each vehicle transporting no more than 5 tonnes of waste, fully compliant with the model constraints and actual requirements.

From the above results, it can be seen that using the OR-Tools platform for modelling and solving problems can efficiently achieve task allocation and path optimisation for waste transport vehicles, significantly reducing the total transport distance and improving collection and transport efficiency. This method provides a feasible and efficient theoretical tool and numerical support for urban waste transport scheduling optimisation.

3. Multi-vehicle Collaborative Scheduling Model

3.1. Model Establishment

The multi-vehicle collaborative scheduling model needs to be further expanded based on the single-vehicle model, taking into account real-world urban waste sorting, collaborative scheduling of multiple vehicle types, and more complex physical constraints

First, assume that there are N waste collection points in the city, numbered 1, 2, ..., N, and the processing plant is numbered 0. Waste is divided into $K = 4$ categories, each of which must be transported by a dedicated corresponding vehicle type. The maximum payload of the kth vehicle type is Q_k (tonnes), the maximum volume is V_k (m³), the unit transportation cost per distance is C_k (yuan/km), and the actual number of vehicles per category is M_k. The amount of waste category k generated at collection point i in a day is $w_{i,k}$ (tonnes), its volume is $v_{i,k}$ (m³), and the distance between any two points is $d_{i,j}$ (km).

The decision variables are expanded to $x_{i,j}^{m,k}$, i.e., multidimensional decision variables representing different types of vehicles, different types of waste, and different vehicle numbers. This indicates whether the mth vehicle of the kth type travels from i to j ($x_{i,j}^{m,k} = 1$) and $y_i^{m,k}$ indicates whether the mth vehicle of the kth class serves collection point i.

In addition, the multi-vehicle collaborative scheduling model also has significant extensions in terms of constraints compared to the single-vehicle model. The specific changes are as follows:

1) Coverage constraints: Each type of waste must be served by the corresponding type of vehicle, and each type of waste cannot be mixed or split. The mathematical expression is $\sum_{m=1}^{M_k} y_i^{m,k} = \delta_{i,k}$.

2) Multiple capacity and volume constraints: Each vehicle must simultaneously satisfy the dual constraints of maximum load Q_k and maximum volume V_k for each type of waste.

3) Multi-objective and multi-cost: The path objective function expands from the total cost of a single vehicle type to the weighted total cost of multiple vehicle types ($(\sum_{k,mi,i,j} C_k d_{i,j} x_{i,j}^{m,k})$).

4) Coordinated scheduling: The service correspondence between various vehicle types and waste types at collection points must be allocated from a global perspective, increasing the complexity and practical significance of the problem.

Therefore, the mathematical model can be summarized as follows:

$$\min Z = \sum_{k=1}^{K}\sum_{m=1}^{M_k}\sum_{i=0}^{N}\sum_{j=0}^{N} C_k d_{i,j} x_{i,j}^{m,k}$$
$$\text{s.t.} \begin{cases} \sum_{m=1}^{M_k} y_i^{m,k} = \delta_{i,k}, \\ \sum_{i=1}^{N} w_{i,k} y_i^{m,k}{}_k, \\ \sum_{i=1}^{N} v_{i,k} y_i^{m,k}{}_k, \\ \sum_{j=1}^{N} x_{0,j}^{m,k} = 1, \quad \sum_{i=1}^{N} x_{i,0}^{m,k} = 1, \\ \sum_{j=0}^{N} x_{i,j}^{m,k} = y_i^{m,k}, \sum_{j=0}^{N} x_{j,i}^{m,k} = y_i^{m,k}, \\ x_{(i,j)}^{(m,k)}, y_i^{(m,k)} \in \{0,1\} \end{cases} \tag{7}$$

Where $\delta_{i,k} = 1$ if $w_{i,k} > 0$, otherwise 0.

3.2. Model with the Constraint of 'Maximum Daily Driving Time for Vehicles'

This section introduces a time constraint: during the working hours from 6:00 to 18:00, the daily driving time of each vehicle must not exceed the given threshold H_{max} (e.g., 6 hours). By introducing the time dimension into the existing multi-vehicle capacity-cost model, the constraint expansion can be achieved.

1) Parameter supplementation: Assume that all vehicles have a constant speed of $V = 40\ km/h$. Let $\tau_{i,j} = \frac{d_{i,j}}{v}$ be the travel time of the vehicle on arc(i,j). Let $T_{m,k}$ be the cumulative travel time of the mth vehicle in class k on that day, then

$$T_{m,k} = \sum_{(i,j)\in A} \tau_{i,j} x_{i,j}^{m,k} \tag{8}$$

2) Addition of time constraints: Supplementing the model

$$T_{m,k} \le H_{max}, \quad \forall k = 1,\ldots,K,\ m = 1,\ldots,M_k \tag{9}$$

The remaining objective functions remain unchanged with respect to capacity, volume, and path constraints. Since travel time is proportional to distance, introducing this constraint essentially sets a maximum distance limit VH_{max} for each feasible path.

3) Example of the impact on scheduling results: Taking kitchen waste vehicle 3 as an example, the original route of 105.38 km is feasible when $H_{max} = 6\ h$. If H_{max} is tightened to 2h, it is split into two trips, resulting in a slight increase in total distance.

4) Solver implementation: In OR-Tools, the TravelTime dimension can be added through the additional AddDimension, and the arc consumption $\tau_{i,j}$ and vehicle upper limit H_{max} can be set: Where this dimension is added, the solver can synchronously optimise the route under the triple constraints of capacity, volume and travel time.

3.3. Model Solving

For the multi-vehicle type–multi-waste classification vehicle routing model established, this problem continues to select GoogleOR-Tools' RoutingSolver as the core solver. The solver uses a time limit $T_{max} = 60\ s$ as the termination criterion, and the algorithm complexity is approximately $O(M_k N^2)$.

Based on the solver output, the transport distance, transport cost, and path length structure are analysed, and the model performance is evaluated accordingly.

1) Total cost and total distance of vehicle types.

Table 2. Statistical summary of dispatching four types of vehicles

Type of Waste	Number of Vehicles	Total Distance Travelled/km	Total Cost/yuan	Demand/tonne
Kitchen Waste	5	414.74	1036.85	39.39
Recyclables	2	216.84	433.69	8.39
Hazardous Waste	1	175.47	877.37	2.31
Other Waste	3	257.63	463.74	21.32
Total	11	1064.69	2811.64	71.41

As shown in Table 2, different types of waste exhibit significant stratification in terms of transportation distance and cost: food waste accounts for the highest proportion of costs (36.9%), followed by hazardous waste (31.2%), while recyclables and other waste account for 15.4% and 16.5% respectively. This result aligns closely with the unit mileage cost per vehicle and the total mass of each waste type, indicating that the model effectively utilises vehicle capacity and achieves minimised transportation costs on a global scale.

2) Single-path length distribution. Most paths are concentrated in the 70–120 km range, with the extreme shortest and longest paths being approximately 32 km and 175 km, respectively. The relative concentration of path lengths indicates that under the combined influence of capacity constraints and distance costs, the solver tends to generate vehicle tasks that are ‘load-optimised and distance-balanced,’ thereby reducing the overall total cost.

3) Path spatial layout. Fig. 1 shows the optimal collection and delivery routes for each vehicle type. The overall path layout aligns with the distance and cost statistics in Table 2, validating the model's reasonableness.

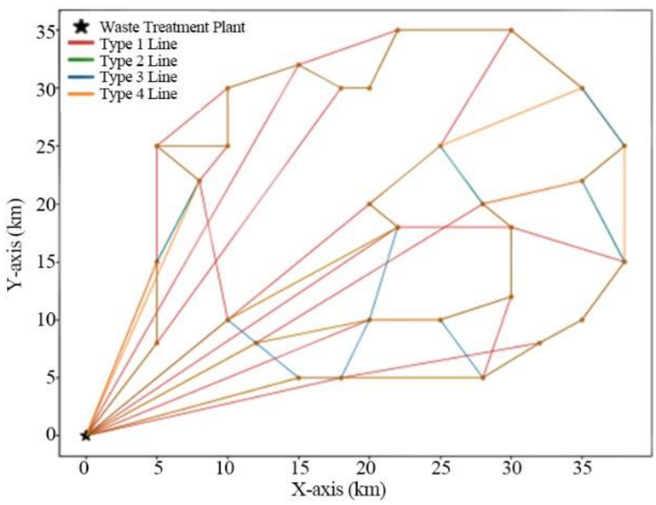

Fig. 1 Optimal collection routes for different types of vehicles

4. Transit Station Site Selection and Carbon Emission Optimisation Model

4.1. Model Establishment

In the base model, in addition to transit station construction costs and transportation costs, vehicle carbon emissions must also be included as an objective, ensuring that both transportation costs and carbon emissions are minimised while satisfying all constraints. Therefore, the objective function in this section combines fixed construction costs, transportation costs, and carbon emissions into a single objective, with constraints including transfer station activation and flow constraints, service coverage and flow conservation, vehicle and transfer station capacity/volume constraints, time window constraints (vehicle arrival time $\mathrm{t}_{\mathrm{p}\to\mathrm{j}}^{\mathrm{m},k}$ must be within its open window), load variable associations (The load variable $L_{p,q}^{m,k}$ is equal to the load of that segment of transport if and only if $x_{p,q}^{m,k}=1$.), and decision variable domains.

The complete model is as follows:

$$\min \sum_{\mathrm{j}} T_j z_j + \sum_{k,m,q,q} C_k d_{p,q} x_{p,q}^{m,k} + \lambda \sum_{k_k,m,d} \left(\alpha_k + \beta_k L_{p,q}^{m,k}\right) d_{p,q} x_{p,q}^{m,k}$$

$$\text{s.t.} \begin{cases} \sum_{m=1}^{M_k} \sum_{p} x_{p,j}^{m,k} \le M_k z_j, \quad \sum_{m=1}^{M_k} \sum_{i} y_{i,j}^{m,k} \le |\mathcal{I}| z_j, \quad \forall j \in \mathcal{J}, k \in \mathrm{IX}. \\ \sum_{m=1}^{M_k} \sum_{j \in \mathcal{J}} y_{i,j}^{m,k} + \sum_{m=1}^{M_k} x_{i,0}^{m,k} = 1, \quad \forall i \in I, k \in \mathrm{IX} \\ \sum_{\mathrm{q}} x_{p,q}^{m,k} = \sum_{\mathrm{r}} x_{r,p}^{m,k}, \quad \forall p \in (I \cup \mathcal{J}), m, k. \\ \sum_{i \in I} \sum_{j \in \mathcal{J}} w_{i,k} y_{i,j}^{m,k} \le Q_k, \quad \sum_{i \in I} \sum_{j \in \mathcal{J}} v_{i,k} y_{i,j}^{m,k} \le V_k, \quad \forall \mathrm{m,k}, \\ \sum_{\mathrm{i \in I}} w_{i,k} y_{i,j}^{m,k} \le S_k, \quad \forall \mathrm{j,k}. \\ \mathrm{a_j z_j \le t_{p\to j}^{m,k} \le b_j z_j}, \quad \forall \mathrm{p,m,k,j}. \\ 0 \le L_{p,q}^{m,k} \le Q_k x_{p,q}^{m,k}, \quad \sum_{\mathrm{p,q}} L_{p,q}^{m,k} = \sum_{\mathrm{j \in I}} \sum_{\mathrm{j} \in \mathfrak{J}} w_{i,k} y_{i,j}^{m,k} \\ \mathrm{z_j, x_{p,q}^{m,k}, y_{i,j}^{m,k} \in \{0,1\}}, \quad \mathrm{L_{p,q}^{mk} \ge 0}. \end{cases} \tag{10}$$

4.2. Two-Stage Solution Model

Considering that the basic integrated model is large in scale and difficult to solve directly, this paper adopts a two-stage decomposition strategy of ‘site selection first, then route selection,’ coupling the selection of transfer stations with vehicle route optimisation and solving them step by step.

Stage 1: Transfer station site selection and point-to-station allocation. In the first stage, only the activation status z_j of the transfer stations and the assignment of each collection point to a transfer station $y_{i,j}^k$, without considering specific vehicle routes. At this point, the total flow of each type of waste k at transfer station j must not exceed its storage capacity $S_k z_j$, and each point must select one transfer station:

$$\begin{aligned} \min \quad & \sum_{j} T_j z_j + \sum_{k=1}^{4} \sum_{i=1}^{N} \sum_{j \in \mathcal{J}} C_k d_{i,j} y_{i,j}^k, \\ \text{s.t.} \quad & \sum_{j \in \mathcal{J}} y_{i,j}^k = 1, \quad \forall i \in I; k = 1, \ldots, 4, \\ & \sum_{i=1}^{N} w_{i,k} y_{i,j}^k \le S_{j,k} z_j, \quad \forall j \in \mathcal{T}; k = 1, \ldots, 4, \\ & \mathrm{z_j \in [0,1], y_{i,j}^k \in [0,1]}. \end{aligned} \tag{11}$$

Where T_j is the construction cost of the transfer station (yuan), d_{ji} is the distance from point i to station j (km), C_k is the unit transport cost per kilometre for vehicle type k (yuan/km), $w_{i,k}$ and $s_{j,k}$ are the maximum storage capacities at point i and transfer station j (tonnes) respectively.

This model is a mixed integer programming model, and commercial MIP solvers can be used to efficiently find the optimal or near-optimal solution.

Stage 2: Vehicle path optimisation within transfer stations. After obtaining the transfer station activation vector $\{z_j^*\}$ and point-to-station assignment $[y_i^k{}_j^*]$ in Stage 1, Stage 2 constructs independent CVRP subproblems for each activated transfer station j and each waste type k.

$$I_{j,k} = \left\{ i \in I \mid y_{ij}^{k*} = 1 \right\} \tag{12}$$

Indicates all collection points of type k assigned to transfer station j. For this set of points, let $M_{j,k}$, be the number of vehicles of type k. Solve for

$$\begin{aligned}
\min \ & \sum_{m} \sum_{p,q \in \{j\} \cup I_{j,k}} C_k d_{p,q} x_{p,q}^{m,k}, \\
\text{s.t.} \ & \sum_{m} \sum_{q} x_{i,q}^{m,k} = 1, \quad \forall \mathrm{i} \in \mathrm{I}_{\mathrm{j,k}}, \\
& \sum_{q} x_{p,q}^{m,k} = \sum_{r} x_{r,p}^{m,k}, \quad \forall p \in \{j\} \cup \mathcal{I}_{j,k}, m, \\
& \sum_{i \in I_{j,k}} w_{i,k} x_{p,i}^{m,k} \le Q_k, \quad \forall m, \\
& \mathrm{x}_{\mathrm{p,q}}^{\mathrm{m,k}} \in \{0,1\}.
\end{aligned} \tag{13}$$

At this stage, CVRP solvers such as OR-Tools can be invoked in parallel to fully utilise the allocation results from the first stage to decompose large-scale problems into multiple medium-scale subproblems, significantly improving solution efficiency.

Two-stage collaboration and iteration. The two stages use an iterative mechanism to ensure feasibility. If the second stage has no solution, it backtracks to adjust the first stage's location or allocation plan. This decomposition-solving-feedback mechanism balances global location decisions with local path optimisation, effectively balancing model complexity and solution quality.

4.3. Path and Results Display

Fig. 2 shows the optimal circuit diagram for each transfer station and each type of waste (where the red star indicates the transfer station):

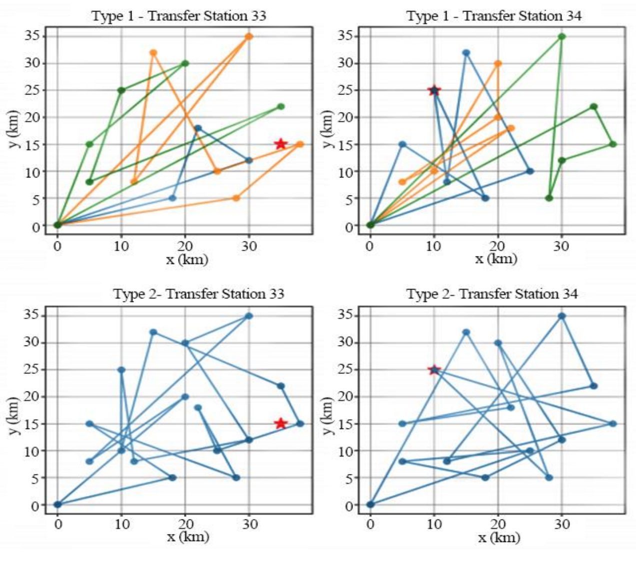

Fig. 2 Optimal delivery routes for waste types 1 and 2 at transfer stations 33 and 34

The numerical solution is as follows:

1) Transfer station 33

Optimal delivery route of $I_{33,1} = \{29,20,13,26,22,9,25,4,14,10,23,17,7,5\}$ is $[33 \to 29 \to 20 \to 13 \to 33], [33 \to 29 \to 20 \to 13 \to 33]$, $[33 \to 10 \to 23 \to 17 \to 7 \to 5 \to 33]$. And Types 2 and 3 are single chains, while other types of waste are divided into two chains.

2) Transfer station 34

Optimal delivery route of $I_{34,1} = \{2,11,15,1,21,6\} \cup \{28,3,30,19,16\} \cup \{8,18,27,24,12\}$ is $[34 \to 2 \to 11 \to 15 \to 1 \to 21 \to 6 \to 34], [34 \to 28 \to 3 \to 30 \to 19 \to 16 \to 34], [34 \to 8 \to 18 \to 27 \to 24 \to 12 \to 34]$. The remaining types are all single-chain loops.

The above results indicate that enabling only two transfer stations is sufficient to cover all 30 collection points; each type of waste route fully utilises vehicle load capacity, dividing the point set into several ‘load-optimised, route-streamlined’ loops; the first-stage point-to-station allocation and second-stage route optimisation efficiently collaborate, achieving an integrated optimal solution for site selection and scheduling.

5. Summary

This study establishes an integrated optimisation framework for urban waste sorting and transportation scheduling, achieving integrated solutions for vehicle route planning, multi-vehicle coordination scheduling, and transfer station site selection. For single-vehicle route optimisation, a CVRP model is established and solved using an improved heuristic algorithm that combines PathCheapestArc and the 2-opt operator. Secondly, the study extends the traditional model and introduces constraints such as time windows, and utilised a solver to achieve global optimisation for multi-vehicle collaborative scheduling. For transfer station site selection and carbon emissions control, a two-stage decomposition method was proposed: the first stage uses clustering analysis and the P-median model to determine transfer station site selection schemes, while the second stage embeds carbon emissions targets into path optimisation, balancing site selection efficiency with environmental protection requirements.

The research findings provide practical decision-making tools for urban waste sorting management and have practical application value in improving transportation system efficiency and reducing environmental impact. Future research could further explore optimisation methods for more complex scenarios, such as dynamic demand response and addressing uncertainty factors, to promote the deep integration of theoretical research and engineering practice.

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