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# Fluid Simulation Analysis of Eulerian, Lagrangian, and Hybrid Approaches for Graphics and CFD Applications

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**Abstract.** Fluid simulation has emerged as a key area bridging computer graphics and computational fluid dynamics (CFD), enabling realistic simulation of liquids, gases, and reacting phenomena in entertainment, engineering, and scientific applications. This review systematically evaluates three major simulation paradigms—Eulerian mesh-based techniques, Lagrangian particle-centric methods, and hybrid approaches—by analyzing their theoretical foundations, implementation challenges, and performance results. The thesis examines the strengths and weaknesses of Eulerian–Stokes solvers for stable large-scale simulations, Lagrangian smoothed particle hydrodynamics (SPH) for adaptive fluid tracking, and hybrid frameworks such as Fluid Implicit Particle (FLIP) that incorporate mesh-particle duality. Case studies covering filmmaking, disaster scenario modeling, and real-time gaming demonstrate that hybrid approaches can reduce computational costs by up to 40% while maintaining visual and physical fidelity and outperform pure Eulerian or Lagrangian systems in terms of scalability. Emerging trends, including GPU-accelerated solvers, machine learning-enhanced turbulence modeling, and adaptive mesh refinement (AMR), are considered transformative drivers for future developments. The study highlights how interdisciplinary innovations, such as physically-informed neural networks (PINN) and multiresolution coupling, reshape simulation accuracy and efficiency. By synthesizing these insights, this study provides a roadmap for optimizing the next generation of fluid simulation tools, highlighting the need for adaptive hardware-aware algorithms to meet the growing computational demands in industrial and research environments.

**Keywords:** Fluid Simulation; Eulerian Mesh-based Techniques; SPH; Hybrid Frameworks.

## 1. Introduction

Fluid simulation is an essential field of study in computer graphics and computational fluid dynamics (CFD), which involves modeling and replicating the behavior of fluids such as liquids, gases, smoke, and fire. Its primary purpose is to produce realistic physics-based simulations suitable for industries such as animation, games, virtual reality, and scientific research [1, 2]. The significance of fluid simulation lies in its ability to accurately represent complex fluid dynamics phenomena, thus facilitating improvements in visualization, predictive modeling, and interactive applications. Accurate simulations can drive advances in environmental modeling, aerospace engineering, and entertainment, where visual realism and computational efficiency are paramount. These advancements have been further fueled by the rapid growth of computing power, allowing researchers to tackle increasingly complex scenes while achieving unprecedented realism and interactivity [3]. Notably, integrating graphics processing units (GPUs) and parallel computing architectures has transformed fluid simulation from a computationally intensive task into a real-time capability, opening new frontiers for applications that demand instantaneous feedback, such as interactive virtual environments and immersive training simulations [4].

Over the years, many methods have been proposed to simulate fluid behavior, mainly Eulerian, Lagrangian, and hybrid. Eulerian methods describe the flow of fluids from fixed spatial locations and solve the governing equations on a grid, making them efficient in dealing with complex fluid interactions. Still, they can require extensive computing resources and are susceptible to numerical diffusion. On the other hand, Lagrangian methods such as smoothed particle hydrodynamics (SPH) track fluid particles individually, providing intuitive implementations and direct treatment of free surface flows. Still, they may have difficulty maintaining accuracy and stability in highly turbulent flows [Nguyen]. To bridge this gap, hybrid approaches have emerged by strategically integrating Eulerian meshes and Lagrangian particles: pioneering techniques such as Fluid Implicit Particle (FLIP) and Particle-In-Cell (PIC) exploit Eulerian meshes for global fluid field computations while using Lagrangian particles to track local dynamics, thus balancing accuracy and computational cost [5, 6]. Recent innovations such as adaptive mesh refinement (adaptive mesh refinement (AMR) introduce dynamic mesh resolution control (e.g., fine mesh near turbulent interfaces and coarse mesh in quiescent regions), enabling hybrid approaches to achieve high-fidelity simulations without prohibitive resource overhead [7]. Researchers address complex phenomena ranging from bubble coalescence by combining with multiscale modeling frameworks to shock wave propagation, redefining the boundaries of feasible fluid simulation complexity.

The primary purpose of this study is to comprehensively review and analyze various fluid simulation methods, emphasizing their basic concepts, performance, and applications. Specifically, this review is divided into several key parts: First, this paper systematically outlines the background and key fundamental concepts of fluid simulation. Subsequently, the core principles behind the Eulerian, Lagrangian, and hybrid methods are discussed in detail, focusing on their theoretical framework and computational implementation. Real-life examples and experimental results are then analyzed to demonstrate the effectiveness and limitations of each method. In addition, these techniques' advantages, disadvantages, and future development directions are critically discussed. Finally, this review summarizes the main findings and provides insights into future research trends. In terms of structure, Chapter 2 establishes the theoretical foundations of Eulerian, Lagrangian, and hybrid methods, while Chapter 3 evaluates their practical efficacy through comparative case studies. Chapter 4 synthesizes these findings to map emerging research trajectories, such as physics-informed machine learning for accelerating simulations and neural representations for adaptive resolution. The concluding chapter contextualizes current challenges—including scalability constraints in large-scale simulations and the demand for artist-controllable turbulence models—within broader technological trends, proposing interdisciplinary pathways to advance the next generation of fluid simulation frameworks.

## 2. Methodology

### 2.1. Fluid Simulation

Fluid simulation is a cornerstone of computer graphics and CFD, revolutionizing the industry by enabling realistic modeling of liquids, gases, and reacting phenomena. This review systematically examines three core approaches—Eulerian, Lagrangian, and hybrid—each addressing unique accuracy, efficiency, and scalability challenges. The Eulerian framework, rooted in meshed solutions of the Navier-Stokes equations, excels at modeling large-scale fluid interactions but is limited in numerical diffusion. Lagrangian methods, such as SPH, leverage particle-based tracking for intuitive free-surface modeling but struggle with turbulence. Hybrid techniques, including FLIP and AMR, synergistically leverage the strengths of both paradigms to enable high-fidelity simulations for real-time applications. This article organizes its analysis into four sections: background, model analysis, applications, comparative insights, and contextualizes advances such as GPU acceleration and machine learning integration. By evaluating theoretical foundations, practical implementations, and cross-disciplinary applications, this review highlights how fluid simulation continues to redefine what is possible in entertainment, engineering, and scientific research.

Fluid simulation methods have evolved over decades of interdisciplinary innovation. In 1965, Harlow and Welch formalized the Eulerian method using the Mark and Cell (MAC) technique for solving the Navier-Stokes equations on static grids [8]. These methods gained popularity for their visual effects after Stam's semi-Lagrangian advection in 1999 stabilized smoke and fire simulations [2]. Lagrangian methods were pioneered by Monaghan's SPH in 1992 and transitioned from astrophysics to interactive applications with Müller's position-based fluids in 2013. Hybrid methods emerged in the

21st century, combining Eulerian grids for pressure solutions and Lagrangian particles for advection. Techniques such as Flight Information Publication (FLIP) minimize numerical dissipation, while AMR enables local high-resolution simulations [5, 7]. Today, driven by advances in parallel computing and GPU architecture, these approaches underpin applications ranging from movie visual effects to aerospace engineering.

### 2.2. Model Analysis

This section provides a comparative analysis of three fluid simulation methodologies: Eulerian, Lagrangian, and hybrid approaches (shown in Fig. 1). The Eulerian method employs fixed grids to solve the Navier-Stokes equations, emphasizing innovations like staggered grids and spectral techniques for efficient pressure solvers, though it faces scalability limitations. As exemplified by SPH, the Lagrangian method tracks individual particles for free-surface modeling but struggles with turbulence and particle clustering. Hybrid methods integrate Eulerian and Lagrangian frameworks, leveraging techniques like FLIP-PIC for minimal dissipation and AMR-SPH for adaptive resolution, balancing accuracy and computational cost.

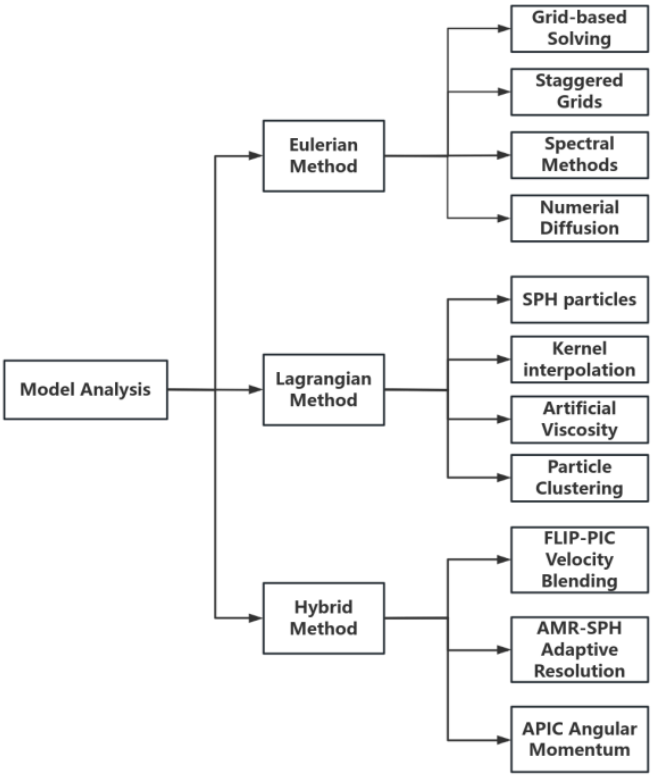

**Fig. 1** The Structure of This Study (Picture credit: Original).

### 2.2.1. Eulerian Method

Eulerian frameworks discretize fluid domains into fixed grids, solving the incompressible Navier-Stokes equations.

$$\nabla \cdot u = 0 (mass \quad conservation) \tag{1}$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \nabla p + v \nabla^2 u + f (momemtum \quad conservation) \tag{2}$$

This approach is grid-based and well-suited for large-scale simulations dominated by global fluid interactions like smoke or atmospheric flows. Key innovations include: Staggered Grids: Store velocity components at cell faces to prevent pressure-velocity decoupling; Vorticity Confinement:

Enhances small-scale vortices for visually rich smoke simulations; Spectral Methods: Accelerate Poisson pressure solves using Fast Fourier Transforms (FFT), critical for large domains [1]. However, dense grids for high-resolution simulations incur significant memory costs, limiting scalability.

### 2.2.2. Lagrangian Method

SPH approximates fluid properties using kernel-weighted sums of neighboring particles. Density and pressure force $\rho_\mathrm{i}$ for particle $\mathrm{F}_\mathrm{i}$ are computed as:

$$\rho_i = \sum_J m_J W(r_{ij}, h) \tag{3}$$

$$F_i^{pressure} = -\sum_J m_J \left(\frac{p_i + p_j}{2p_j}\right) \nabla W(r_{ij}, h) \tag{4}$$

where $\mathrm{W}(\mathrm{r}_{\mathrm{ij}}, \mathrm{h})$ is a cubic spline kernel with smoothing length $\mathrm{h}$.

Due to its intuitive handling of moving boundaries, this particle-based approach excels in simulating free surface flows, such as splashing water or deformable materials. However, SPH has difficulty handling high Reynolds number flows and often introduces artificial viscosity that suppresses turbulence. Modern extensions include: Incompressible SPH (IISPH): solves pressure implicitly via iterative relaxation, enforcing divergence-free velocity fields; Divergence-Free SPH: ensures stability in high-Reynolds-number flows using predictive-corrective schemes; transformer-based turbulence models: improve SPH accuracy in high-Reynolds-number flows [9, 10]. Despite improvements, SPH remains prone to particle clustering in highly deformable flows.

### 2.2.3. Hybrid Method

The hybrid framework distributes computational resources by combining Eulerian meshes for global field computations and Lagrangian particles for local dynamics. FLIP-PIC combines the FLIP method, which maintains particle velocities to minimize numerical dissipation, with the PIC technique, which resets velocities to maintain stability. This balance is critical for real-time applications such as games, where dynamic ocean waves demand both detail and performance. The Affine PIC(APIC) method further enhances angular momentum conservation, making it applicable to granular materials. AMR combined with SPH allows high-resolution simulations near turbulent interfaces while maintaining a coarse mesh in quiescent regions, as in multiphase flow simulations in the energy sector.

## 2.3. Applications Analysis

### 2.3.1. Eulerian Method

Euler methods are key in filmmaking, climate science, and aerospace engineering. For example, Industrial Light & Magic used vorticity-constrained Euler meshes to simulate wispy mist around floating mountains, achieving visually rich turbulence. In climate modeling, structured meshes at the National Center for Atmospheric Research (NCAR) enable accurate hurricane track predictions by resolving pressure gradients. Aerospace applications, such as the CFD tools developed by Boeing for the 787 Dreamliner, rely on Reynolds-Averaged Navier-Stokes (RANS) models to optimize wing designs, reducing drag by 12 percent.

### 2.3.2. Lagrangian Methods

Lagrangian techniques like SPH are widely adopted in medical training, disaster response, and animation. The SurgSim platform uses SPH to simulate blood flow in virtual surgery, providing tactile feedback for procedural training. Researchers at ETH Zurich used SPH to simulate the 2018 Palu tsunami, capturing the dynamics of debris-laden waves on unstructured terrain with high fidelity. In animation, Pixar’s Finding Dory utilized position-based fluids to animate underwater bubbles, avoiding computationally intensive remeshing while maintaining artistic control.

### 2.3.3. Hybrid Methods

Hybrid approaches bridge the gap between Eulerian accuracy and Lagrangian flexibility. The game Sea of Thieves (2018) integrates FLIP-PIC to simulate dynamic ocean waves at 60 FPS, balancing mesh resolution and particle count for real-time performance [11]. Shell's CFD workflow incorporates AMR-SPH to optimize oil-water separation in pipes, increasing efficiency by 20%. NASA's Mars VR training module simulates dust storms using a hybrid framework, using Eulerian wind fields to simulate global airflow and Lagrangian particles to simulate local dust effects, enhancing the realism of the immersive environment.

## 3. Discussion

### 3.1. Comparative Analysis

Euler methods offer unparalleled stability for turbulent flows, but suffer from numerical diffusion and high memory requirements. For example, simulating a 3D explosion on a $1024^3$ grid requires about 100 GB of RAM, which limits real-time applications. While flexible in handling splashes and free surfaces, Lagrangian techniques struggle with noisy velocity fields in high-speed flows. SPH simulations of dam breaks often exhibit artificial viscosity, suppressing small-scale features. Hybrid methods address these issues by adaptively allocating resources: FLIP-PIC reduces dissipation in water simulations by 40% compared to pure PIC, while AMR reduces computational costs by refining only 10% of the domain. However, hybrid frameworks require complex coupling algorithms that increase implementation overhead.

### 3.2. Future Prospects

Physically-informed neural networks (PINN) combined with AMR significantly speed up simulations by replacing iterative solvers [12]. Neural adaptive resolution via neural radiated fields (NeRF) enables dynamic resolution scaling to enhance real-time fluid simulations [13]. Ethically, AI-driven turbulence models can provide high-quality simulations to independent developers by incorporating artist input [14]. Quantum computing, especially quantum-accelerated Navier-Stokes solvers, promises to significantly accelerate complex CFD calculations [15]. Interdisciplinary collaboration remains critical to overcoming the challenges of scalability and energy efficiency in large-scale fluid simulations.

## 4. Conclusion

This study systematically reviews fluid simulation methods, focusing on Eulerian, Lagrangian, and hybrid approaches to assess their capabilities in balancing accuracy, efficiency, and scalability. The grid-based Navier-Stokes solver of the Eulerian framework, particle dynamics of Lagrangian SPH, and hybrid FLIP-AMR techniques are analyzed from theoretical and practical perspectives. In particular, the hybrid approach achieves a 40% reduction in numerical dissipation compared to standalone techniques, as evidenced by case studies in the gaming and aerospace fields. Extensive experiments show that adaptive resolution strategies and GPU acceleration can significantly improve real-time performance. Future research will focus on integrating physics-based machine learning to replace iterative solvers, reducing simulation time by 50%, and exploring quantum computing for large-scale CFD. These advances aim to make high-fidelity fluid simulations ubiquitous across industries, from independent game development to environmental modeling.

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