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Development and Application of a Python Program Simulating the Orbit of a Lunar Probe

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Abstract. This paper presents the development and application of a Python program designed to simulate the orbital trajectory of a lunar probe using the fourth-order Runge-Kutta (RK4) method, a method known for its accuracy and computational efficiency. The program models the entire journey of a lunar probe, from its initial launch on the Earth to its stable circular orbit around the Moon. By allowing users to input specific parameters for acceleration and deceleration phases, the program generates orbital paths relative to both the Earth and the Moon, offering valuable insights into mission planning and trajectory optimization. One significant application demonstrated in this paper is the simulation of the Chang'e 5 mission, China's first lunar sample return mission. By incorporating Doppler data from Chang'e 5, the program effectively simulates the probe's orbital path, highlighting its potential for future real-world lunar exploration missions. Despite some simplifying assumptions, such as treating the Earth as a perfect sphere and assuming instantaneous acceleration, the program provides a robust framework for simulating lunar probe trajectories with high accuracy and computational efficiency.

Keywords: Fourth-order Runge-Kutta (RK4) method, Python Program, lunar probe, orbital simulation.

1. Introduction

Lunar probes are unmanned spacecraft used to investigate the Moon. They provide detailed information about the Moon's surface, composition, and environment by taking photographs or collecting lunar soils. The Soviet Luna 2 was the first lunar probe to make contact with the Moon, achieving this on September 13, 1959; and the United States' first successful lunar probe, Ranger 7, captured close-up images before crashing into the Moon on July 31, 1964.[1] Lunar probes have significantly evolved in these decades. The Chang'e 5 mission, one of the most recent lunar probes and China's first lunar sample return mission, successfully landed on the Moon on December 1, 2020 and was expected to return 2 kilograms of lunar samples.[2]

Despite the evolution of lunar probes, the trajectory of them have not changed. Upon launch, a lunar probe first achieves an initial velocity to establish a stable circular orbit around the Earth. At a specific moment, the probe accelerates to transition from this circular orbit to an elliptical trajectory. Once the probe reaches the point of closest approach to the Moon, it decelerates in order to establish a stable circular orbit around the Moon. This sequence of maneuvers ensures the probe can ultimately establish a stable circular orbit around the moon, crucial for the success of lunar missions.

Among the series of maneuvers, certain orbital parameters can be obtained mathematically, such as the initial orbital velocity around Earth; while other parameters, including the precise timing and magnitude of acceleration to es- cape from the Earth's gravitational field, are extremely challenging to obtain using analytical methods. Therefore, computational methods are preferable to determine these parameters.

Nevertheless, many computational methods used to determine orbital parameters are not efficient and have significant error. Many people use Euler's method to approximate a set of differential equations iteratively and use the result to determine the orbit.[3] This method is inefficient and has relatively lower accuracy. However, accuracy is one of the most important aspects in mission planning and spacecraft launch. The trajectory of a spacecraft is the basis of a spacecraft's all functions.[4]

This paper focuses on developing a Python program that models the complete journey of a lunar probe—from its initial launch on Earth to establishing a stable circular orbit around the Moon. The program allows users to input parameters for acceleration and deceleration phases and generates the resulting orbital paths with respect to both the Earth and the Moon.The program employs the fourth-order Runge-Kutta (RK4) method, a numerical method with high accuracy and computational efficiency in solving differential equations that significantly outperforms traditional methods like Euler's method.

The program is designed to assist users in determining the optimal orbital parameters for lunar probes through a trial-and-error process. One significant application is in simulating the Chang'e 5 mission, China's first lunar sample return mission, which aims to bring back 2 kilograms of lunar samples.[2] By incorporating Doppler data from Chang'e 5, which provides insights into the probe's orbital period around the Moon, the program effectively simulates the probe's journey from its launch on Earth to its stable orbit around the Moon. This capability demonstrates the program's potential in supporting real-world lunar exploration missions.

2. Method

This section introduces some fundamental physics formulas and the fourth-order Runge-Kutta Method, a numerical method for solving ordinary differential equations.

2.1. Fundamental Physics Formulas

Newton's Second Law states that the net force exerted on an object with constant mass is equal to the product of its mass and acceleration.

$$F = ma \tag{1}$$

where F denotes the net force, m denotes the mass of the object, and a denotes the acceleration of the object.

Centripetal force is the net force exerted on an object to keep it moving along a circular path. In uniform circular motions, the magnitude of the centripetal force can be calculated as below:

$$F_c = \frac{mv^2}{r} \tag{2}$$

where Fc denotes the centripetal force, m denotes the mass of the object, v denotes the velocity of the object, and r denotes the radius of the circular orbit.

Gravitational force is a force that exists between any two objects with mass in the universe. This force pulls the objects toward each other. The magnitude of the gravitational force can be calculated as below:

$$F_g = \frac{GMm}{r^2} \tag{3}$$

where Fg denotes the gravitational force, G denotes the gravitational constant, M denotes the mass of one object, m denotes the mass of another object, and r denotes the distance between the objects.

2.2. Derivation of Orbital Velocity Based on Previous Formulas

Orbital velocity is the velocity needed for a spacecraft to maintain uniform circular motion at a specific distance from a planet with a specific mass.

First, equate the net force to the centripetal force and the gravitational force:

$$Fnet = Fc = F_g$$

Next, substitute the expressions for centripetal force and gravitational force according to equations 2 and 3:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

where m denotes the mass of the spacecraft, M denotes the mass of the planet, v denotes the velocity of the spacecraft, r denotes the distance between the spacecraft and the planet, and G denotes the gravitational constant.

Solving for the orbital velocity v, yields:

$$v = \sqrt{\frac{GM}{r}} \tag{4}$$

2.3. Numerical Method for Differential Equation

Accurately simulating the spacecraft's journey requires solving a set of differential equations millions of times. Although modern computers are incredibly powerful, selecting an efficient numerical method is crucial. The fourth-order Runge-Kutta method (RK4) is an excellent choice for this scenario and is usedin the program.[4]

Runge-Kutta methods are a family of iterative techniques for approximating solutions to differential equations. Unlike Euler's method, which takes a single point per step, Runge-Kutta methods evaluate multiple points within each step. The RK4 method uses four intermediate points within each step. The RK4 method is designed to minimize the error by considering the Taylor expansionof the exact solution around the current point. Therefore, the slopes at the intermediate points are weighted more heavily than the slopes at initial andfinal points.

Consider the following initial value problem:

$$\frac{dx}{dt} = f(t, x)$$

with initial condition $x(t_0) = x_0$.

The RK4 method approximates the solution over small time steps h $(h > 0)$ by defining:

$$x_{n+1} = x_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$t_{n+1} = t_n + h$$

where,

$$k_1 = f(t_n, x_n)$$
$$k_2 = f\left(t_n + \frac{h}{2}, x_n + \frac{h}{2}k_1\right)$$
$$k_3 = f\left(t_n + \frac{h}{2}, x_n + \frac{h}{2}k_2\right)$$

$$k_4 = f(t_n + h, x_n + hk_3)$$

Here x_{n+1} is the approximation of xt_{n+1}.Note that the middle two slopes are weighted more, as previously explained.

The RK4 method is significantly more efficient than Euler's method. It requires much less computation time to achieve the same accuracy.

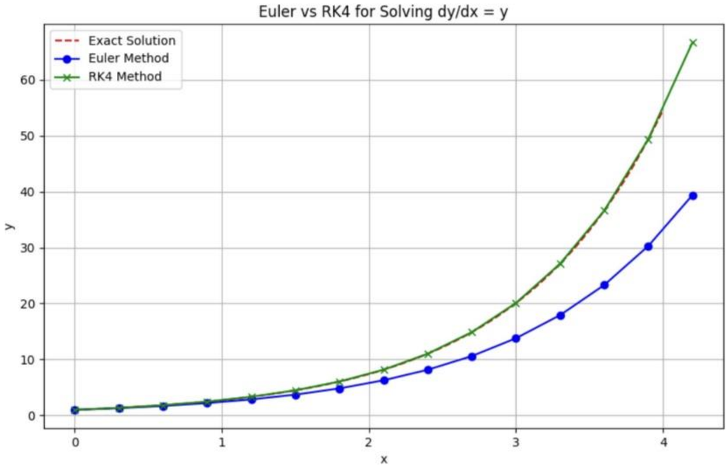

Fig.1 The predicted solution to the differential equation $\frac{d_y}{d_x} = y$ with initial condition $x = 0,\ y = 1$ using three methods: analytical method (shown as the orange dotted line), RK4 method with step size of 0.3 (shown as the green line), and Euler's method with step size of 0.3(shown as the blue line).

In Fig.1, the step sizes for RK4 method and Euler's method are the same (0.3), but the accuracy of the prediction varies significantly. The RK4 method's prediction almost overlaps with the exact solution, while Euler's method's prediction is far off.

3. Discussion

To simulate the journey of a spacecraft using a Python program, recording the spacecraft's position and velocity at each "moment" is necessary. In mathematical language, x, y, v_x, v_y are recorded at each infinitesimal time period dt. In the program, $, x, y, v_x, v_y$ at each time period is called the "state" of the spacecraft at that time.

In order to achieve this, four differential equations $dx/dt, dy/dt, dvx/dt, dvy/dt$ and their initial conditions are needed. Once having these, we can determine the position of the spacecraft at each moment, enabling us to graph its journey.

The Python program also enables users to enter some conditions, namely the timing and magnitude of the spacecraft's acceleration to escape the gravitational field of the Earth. The program achieves this by simply multiplying the appropriate percentage of v_x and v_y at the right time.

spacecrafts decelerate to an appropriate speed to orbit circularly around the moon when it is closest to the moon. The program has a mechanism to determine the right moment for deceleration, so users do not need to enter conditions.

3.1. Differential Equation

Before deriving the relevant differential equations, some assumptions are made for convenience while not affecting the accuracy of the result: 1) The Earth is fixed at the origin. 2) The initial position of the spacecraft is at $(r_{earth}, 0)$. 3)The initial velocity of the spacecraft is pointing upward (so that it will start orbiting around the Earth counterclockwise). 4) The initial position of the Moon is at $(x_{moon}, 0)$. 4) The Moon is orbiting around the Earth counterclockwise. Fig.2 illustrates the assumptions.

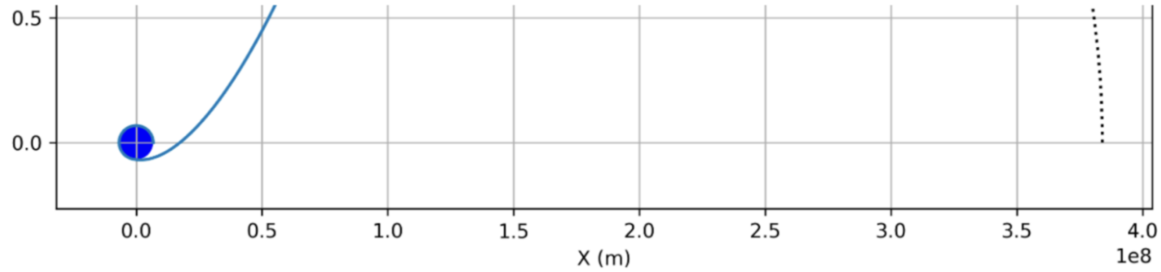

Fig. 2 The blue circle represents the earth, the blue line represents the spacecraft's journey, and the grey dotted line represents the Moon's journey

3.1.1. Derivation of Differential Equations

The first pair of differential equations is:

$$\frac{dx}{dt} = v_x \tag{5}$$

$$\frac{dy}{dt} = v_y \tag{6}$$

The other pair of equations is harder to derive. From Newton's Second Law, one can obtain:

$$\frac{dv_x}{dt} = a_x = \frac{F_x}{m} \tag{7}$$

where a_x denotes the acceleration in the x direction, m denotes the mass of the spacecraft, and F_x denotes the net force in the x direction exerted on the spacecraft.

$$Fx = Fx, earth + Fx, moon \tag{8}$$

where $Fx, earth$ denotes the x-component of the Earth's gravitational force and $Fx, moon$ denotes the x-component of the Moon's gravitational force.

To calculate the x-component of the Earth's gravitational force, orthogonal decomposition is required. According to equation 3,

$$F_{x,earth} = Fg \cdot \frac{x}{r} = \frac{-GMm}{r^2} \cdot \frac{x}{r} = \frac{-GMmx}{r^3} = \frac{-GMmx}{(x^2+y^2)^{3/2}} \tag{9}$$

where x and y denote the x –and y –coordinates of the spacecraft respectively, r denotes the distance between the spacecraft and the Earth, M denotes the mass of the earth, and m denotes the mass of the moon.

$F_{x,moon}$ can also be obtained. However, since the Moon's position is changing, we need to introduce two variables, x_{moon} and y_{moon} to obtain $F_{x,moon}$. Note that we assume the moon's initial position at $(r_{earth}, 0)$ and that it orbits around the earth counterclockwise. Therefore,

$$x_{moon}(t) = rcos\left(\frac{2\pi t}{T}\right) \tag{10}$$

$$y_{moon}(t) = rsin\left(\frac{2\pi t}{T}\right) \tag{11}$$

Where r denotes the distance between the moon and the earth, T denotes the period of moon's rotation around the earth.

According to equations 10 and 11, it can be obtained that

$$x' = x - x_{moon} = x - rcos(\frac{2\pi t}{T}) \tag{12}$$

$$y' = y - y_{moon} = y - rsin(\frac{2\pi t}{T}) \tag{13}$$

Where x' denotes the horizontal distance between the Moon and the spacecraft and y' denotes the vertical distance between the Moon and the spacecraft.

Using a similar logic that we deduced equation 9, it can be obtained:

$$F_{x,moon} = \frac{-GM'mx'}{(x'^2+y'^2)^{3/2}} \tag{14}$$

Where M' denotes the mass of the moon

Plugging in the values of x', y' from equations 12 and 13, we obtain equation 14:

$$F_{x,moon} = \frac{-GM'm(x-rcos(\frac{2\pi t}{T}))}{\left(((x-rcos\left(\frac{2\pi t}{T}\right))^2+(y-rsin(\frac{2\pi t}{T}))\ ^2\right)^{3/2}} \tag{15}$$

According to equations 8, 9, and 15, one can obtain:

$$F_x = \frac{-GMmx}{(x^2+y^2)^{3/2}} + \frac{-GM'm(x-rcos(\frac{2\pi t}{T}))}{\left((x-rcos\left(\frac{2\pi t}{T}\right))^2+(y-rsin(\frac{2\pi t}{T}))\ ^2\right)^{3/2}} \tag{16}$$

According to equations 7 and 16, $\frac{dv_x}{dt}$ can be obtained.

$$\frac{dv_x}{dt} = \frac{F_x}{m} = \frac{-GMmx}{(x^2+y^2)^{3/2}} + \frac{-GM'(x-rcos(\frac{2\pi t}{T}))}{\left((x-rcos\left(\frac{2\pi t}{T}\right))^2+(y-rsin(\frac{2\pi t}{T}))\ ^2\right)^{3/2}} \tag{17}$$

The exact same method can be used to derive $\frac{dv_y}{dt}$.

$$\frac{dv_y}{dt} = \frac{-GMy}{(x^2+y^2)^{3/2}} + \frac{-GM'(y-rsin\left(\frac{2\pi t}{T}\right))}{\left((x-rcos\left(\frac{2\pi t}{T}\right))^2+(y-rsin(\frac{2\pi t}{T}))^{\ 2}\right)^{3/2}} \tag{18}$$

For clarity, all four differential equations are listed below:

$$\frac{dx}{dt} = v_x \tag{19}$$

$$\frac{dy}{dt} = v_y \tag{20}$$

$$\frac{dv_x}{dt} = \frac{F_x}{m} = \frac{-GMmx}{(x^2+y^2)^{3/2}} + \frac{-GM'(x-rcos(\frac{2\pi t}{T}))}{\left((x-rcos\left(\frac{2\pi t}{T}\right))^2+(y-rsin(\frac{2\pi t}{T}))^{\ 2}\right)^{3/2}} \tag{21}$$

$$\frac{dv_y}{dt} = \frac{-GMy}{(x^2+y^2)^{3/2}} + \frac{-GM'(y-rsin\left(\frac{2\pi t}{T}\right))}{\left((x-rcos\left(\frac{2\pi t}{T}\right))^2+(y-rsin(\frac{2\pi t}{T}))^{\ 2}\right)^{3/2}} \tag{22}$$

3.1.2. Initial Conditions for Differential Equations

Since we assume the initial position of the spacecraft to be at $(r_{earth}, 0)$, the following can be obtained:

$$x = r_{earth}$$

$$y = 0$$

According to equation 4 and the assumption that the initial orbital velocity of the spacecraft is pointing upward, it can be obtained:

$$v_x = 0$$

$$v_y = \sqrt{\frac{GM}{r_{earth}}}$$

where r_{earth} denotes the radius of the earth.

3.2. Acceleration and Deceleration

The gravitational force is not the only factor that affects the spacecraft's journey. Acceleration and deceleration from spacecraft's rockets also play a significant role, and these can be implemented in the program.

3.2.1. Acceleration

Time and magnitude for the acceleration are both user-entered variables. The program magnifies the v_x and v_y at the designated time and magnitude indicated by the user.

3.2.2. Deceleration

Unlike acceleration, the timing and magnitude of deceleration is not a user- entered variable. The program automatically determines the time when the spacecraft is closest to the Moon and decelerates it to the appropriate magnitude so that it can achieve a circular orbit around the Moon.

The spacecraft is closest to the Moon when its velocity relative to the Moon is perpendicular to the displacement vector between it and the moon. This condition is equivalent to the following equation involving the dot product of the vectors:

$$\vec{v}' \cdot \vec{r} = 0 \tag{23}$$

where $\vec{v}'$ is the spacecraft's velocity with respect to the Moon and $\vec{r}$ is the displacement vector pointing from the Moon to the spacecraft. The program decelerates the spacecraft at the moment when equation 23 is satisfied.

Figure 3 visually demonstrates that the spacecraft is at its closest approach to the Moon when its velocity with respect to the Moon is perpendicular to the displacement vector between it and the Moon.

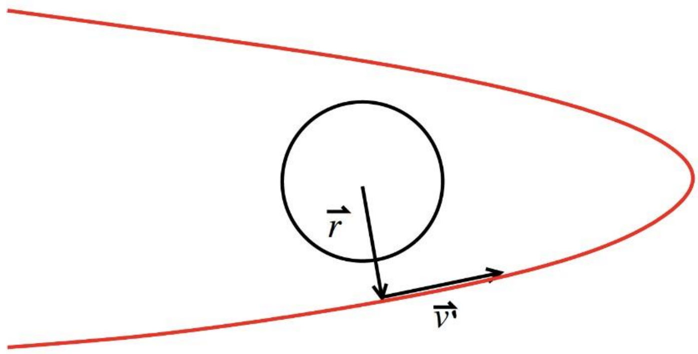

Fig.3 The spacecraft's journey (as red line) around the moon (as black circle), and labels the velocity vector of the spacecraft with respect to the moon as well as the displacement vector between the spacecraft and the moon.

Besides knowing the time of the deceleration, finding its magnitude is crucial. To do so, we need to know the spacecraft's current and target velocity with respect to the Moon.

To find the spacecraft's velocity current with respect to the Moon ($\vec{v}'$), we subtract the moon's velocity from the spacecraft's velocity with respect to the Earth.

$$\vec{v'} = \vec{v} - \overrightarrow{v_{moon}} \tag{24}$$

Equations 10 and 11 indicate the moon's position with respect to time.

Differentiate these equations yield the $x-$ and $y-$ components of the moon's velocity with respect to time:

$$v_{x,moon}(t) = -rsin\left(\frac{2\pi t}{T}\right) \cdot \frac{2\pi}{T} \tag{25}$$

$$v_{y,moon}(t) = rcos\left(\frac{2\pi t}{T}\right) \cdot \frac{2\pi}{T} \tag{26}$$

Therefore, according to equations 24, 25, and 26, the current velocity of the spacecraft with respect to the Moon can be obtained.

$$\vec{v'} = \left[v_x + rsin\left(\frac{2\pi t}{T}\right) \cdot \frac{2\pi}{T}, v_y - rcos\left(\frac{2\pi t}{T}\right) \cdot \frac{2\pi}{T}\right] \tag{27}$$

The magnitude of the velocity vector $\vec{v}'$ is given by:

$$|(v')^{\rightarrow}| = \sqrt{\left(v_x + r\sin\left(\frac{2\pi t}{T}\right)\cdot\frac{2\pi}{T}\right)^2 + \left(v_y - r\cos\left(\frac{2\pi t}{T}\right)\cdot\frac{2\pi}{T}\right)^2}$$

The target velocity is the orbital velocity of the spacecraft around the moon with respect to the moon. According to equation 4, it can be obtained by

$$|\overrightarrow{v_{target}}| = \sqrt{\frac{GM'}{r}} \tag{28}$$

Where v_{target} denotes the target velocity, M' denotes the mass of the moon, r denotes the distance between the moon and the spacecraft.

Equations 27 and 28 give the target velocity and current velocity, respectively. Therefore, the magnitude of the deceleration can be obtained. The last step is to convert the target velocity of the spacecraft with respect to the moon back to its velocity with respect to the Earth. This step is necessary because the variables (v_x and v_y) in the differential equations represent the spacecraft's velocity with respect to the Earth. Without this conversion, the differential equations cannot iterate correctly. This can be done using the following formula:

$$\vec{v} = \overrightarrow{v'} + \overrightarrow{v_{moon}}$$

3.3. Example Program Output

After trial and error, the following pair of parameters allow the spacecraft to orbit around the moon extremely near its surface: time to acceleration $= 3939$ seconds after launch, acceleration rate= 40.19%

This pair of parameters is entered in the program, and the program outputis as shown in figure 4 and figure 5.

Figure 4 appears corkscrew-shaped after orbiting around the Moon, which is normal since the Moon is not static with respect to the Earth. Figure 5 appears to be a perfect circle that has a radius barely bigger than that of the Moon's, as expected.

4. Application

The program can be used to simulate the orbit of Chang'e 5, a spacecraft for which we can access its doppler data on GitHub.

4.1. Introduction to Doppler Effect

The Doppler effect is a phenomenon that relates the frequency of the harmonic waves generated by a moving source with the frequency measured by an observer moving with a different velocity from that of the source.[6] In simpler words, the frequency received by the observer is higher when the source and the observer are moving towards each other, and is lower when they are moving away from each other. The formula below calculates the received electromagnetic wave frequency from the source frequency.

$$f_r = f_s\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \tag{29}$$

Where f_r denotes received frequency, f denotes source frequency, v denotes relative speed (between the source and the receiver), and c denotes the speed of light.

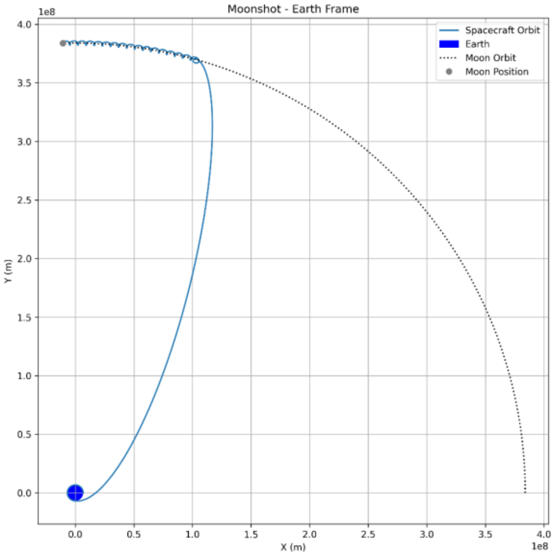

Fig.4 The spacecraft's journey with respect to the Earth. The Earth is denoted by the blue circle, the spacecraft's journey is denoted by the blue line, the Moon's journey is denoted by the grey dotted line, and the Moon is denoted by the small grey circle.

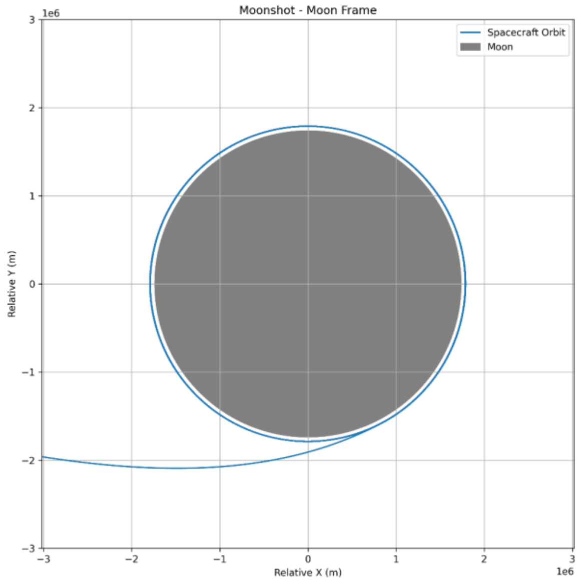

Fig.5 The spacecraft's journey with respect to the Moon. The spacecraft's journey is denoted by the blue line, and the Moon is denoted by the big grey circle.

4.2. Determine the Period of Chang'e 5's orbit

Chang'e 5's period can be determined by its Doppler Data, which can be accessed on GitHub. On selected dates that Chang'e 5 is orbiting around the moon in a circular orbit (from November 30, 2020 to December 5, 2020), frequency vs time graphs are made, as shown in Fig.6.

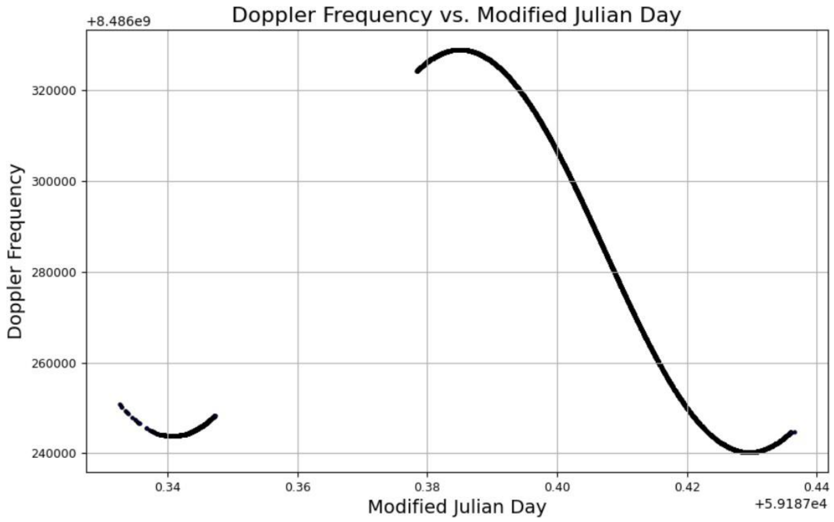

Fig.6 The Doppler Frequency of Chang'e 5 on December4, 2020. The x-axis is time, measured by Modified Julian Day, and the y-axisis Doppler Frequency,measured by Hertz.

The gap on Fig.6 is possibly caused by the electromagnetic wave emitted by Chang'e 5 being blocked by the moon and thus unable to be received by the receiver on the Earth. According to equation 29, the peak on the graph suggests Chang'e 5 ismoving away from the earth at its fastest speed while the trough on the graph suggests Chang'e 5 is moving toward the earth at its fastest speed.

Therefore, the time interval between the peak and the trough is approxi- mately half of the period. The period of Chang'e 5 can then be calculated. Itis approximately 7724 seconds.

4.3. Infer Chang'e 5's Journey

To infer Chang'e 5's journey, knowing its radius of orbit around the moon is crucial. Equation 30 can be used to infer the radius based on the period andthe mass of the Moon, which are both known.

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \tag{30}$$

where T denotes the period of the orbit, r denotes the radius of the orbit, G denotes the gravitational constant, and M denotes the mass of the central object.

Plugging in the values for the gravitational constant, mass of the Moon, and the radius of the orbit, yields $T = 1.949e^6$ meters.

Using trial and error method, the following pair of parameters allows the spacecraft to have an orbit around $1.949e^6$ meters from the center of the moon: time to acceleration $= 3941.5$ seconds after launch, acceleration rate $= 40.1901\%$. It is important to note that other combination of parameters may possibly yieldsthe same result. Fig. 7 and Fig.8 below shows the program output when the above parameters are given.

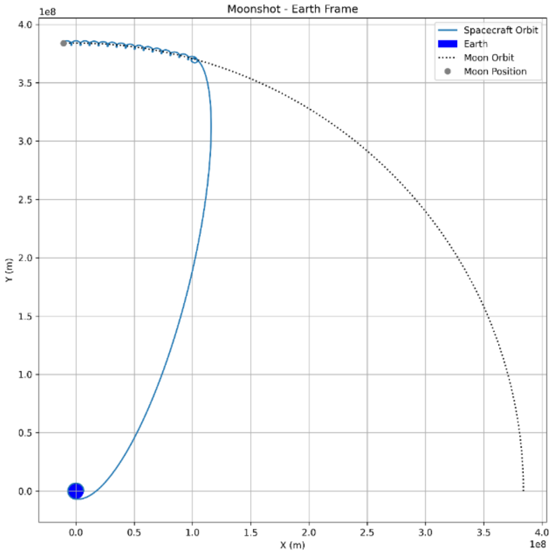

Fig.7 One possibility of Chang'e 5's journey with respect to the Earth. The Earth is denoted by the blue circle, the spacecraft's journey is denoted by the blue line, the Moon's journey is denoted by the grey dotted line, and the Moon is denoted by the small grey circle.

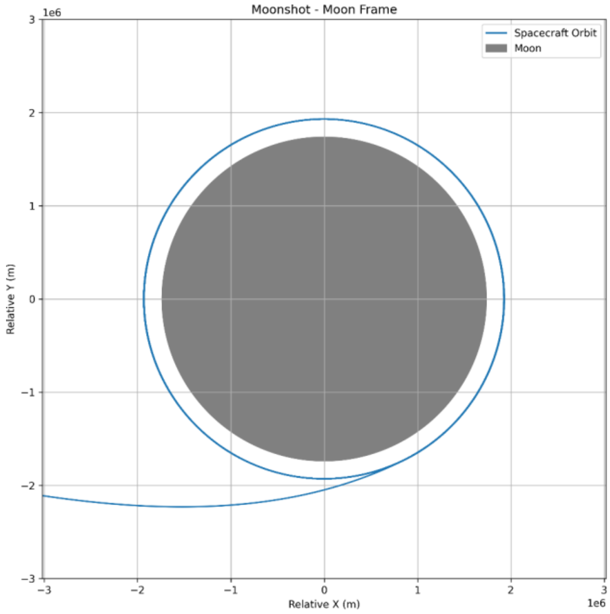

Fig.8 One possibility of Chang'e 5's journey with respect to the Moon. The Earth is denoted by the blue circle, the spacecraft's journey is denoted by the blue line, the Moon's journey is denoted by the grey dotted line, and the Moon is denoted by the small grey circle.

5. Conclusion

In conclusion, this project utilized the accurate fourth-order Runge-Kutta method, derived differential equations, and integrated them into a Python program to simulate the orbit of a lunar probe.

On the other hand, this project has some drawbacks. There are some simplifying assumptions, such as treating the Earth as a perfect sphere, assuming a perfectly circular lunar orbit, and assuming the spacecraft can instantaneously accelerate and decelerate.

Despite the drawbacks, the project overall provides a robust framework for lunar exploration and future mission planning.

Acknowledgements

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