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Introduction and Analysis of Exoplanet Detection Techniques

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Abstract. The exoplanet study is important for exploring life's origins in the universe. With the development of astronomical observation technology, more and more exoplanets have been discovered, and the exploration of habitable planets and the study of planetary atmospheres have become focal areas of study. In this paper, the transit and radial velocity methods for finding exoplanets are introduced and analysed in detail, by introducing the principles of these two methods and showing the calculation progress with specific planetary data. As a result, the parameters of exoplanet can be derived from the principle of doppler effect and binary system. Each method has its advantages and limitations. For radial velocity method, its limitation is that low-mass orbiting planets (planets with mass less than $10M_{Earth}$) cannot be detected because those planets cannot cause detectable and measurable transit signals. Also M_p cannot be directly measured through the radial velocity method unless the value of inclination i is obtained. For the transit method, to get full information and confirm the existence of the planet, complementary measurement is needed, such as radial velocity. Another limitation is the instability of observations. There is a maximum range of the inclination that can observe the transit on the star and nebulae in the universe may block the observing sight, both lead to a limit probability of finding the planet's transit. So, the complementarity between the methods can make the results more comprehensive and precise. From these, it is possible to find more habitable terrestrial planets.

Keywords: Exoplanet; transit; radial velocity.

1. Introduction

As the Earth's resources gradually deplete, the human perspective has turned to the universe, trying to search for habitable planets. In 1995, Mayor first discovered the planet 51 Peg b by observing the Doppler shift of star 51 Pegasi [1]. This discovery showed the presence of exoplanets. Later in 2014, the Earth-size planet Kepler-186f was discovered by NASA's Kepler Telescope, this planet was located in a habitable zone meaning that there may be liquid water on the surface of the planet [2]. The observation indicated that the radius of Kepler-186f is 1.1 times the Earth's radius. This discovery showed the possibility of finding Earth-size exoplanets. Currently, over 4000 exoplanets are confirmed. All these prove the importance of exoplanet exploration.

There are five measurement methods for detecting exoplanets: radial velocity measurement, transit observation, direct imagining, microlensing, and astrometry [3]. While each method has its advantages and limitations. This article will focus on the two methods radial velocity measurement and transit observation. By analysing the corresponding data and synthesizing the theory knowledge of Kepler's laws and the Doppler effect, this text will show the progress of finding exoplanets and information measurement. The purpose of this paper is to deepen the reader's understanding of the existing detection methods and provide directions and ideas for future research.

2. Doppler Effect

The Doppler effect refers to the phenomenon in which the frequency received by the observer is not the same as the frequency emitted from the source when there is relative motion between the wave source and the observer [4].

The star can be seen as the source of the wave. When the light source moves, the corresponding wave crest moves at the same time. Hence, in the direction of the moving wave source, the distance between two adjacent crests is compressed, and in the contrary direction, the distance is expanded. The received wavelength λ' becomes different from the original frequency λ .

So, the moving velocity v of the wave source can be calculated by the change in wavelength $\Delta\lambda$.

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = -\frac{v}{c} \tag{1}$$

Where c is the light speed with the value of $3 \times 10^8\ \mathrm{ms}^{-1}$.

From the point of view of celestial motion, as the star passes toward the observer (see 3.1), the received frequency becomes higher, resulting in the line shift to the ultraviolet region in the emitted spectrum of the star. This is called the blueshift. Inversely, the wave frequency becomes lower when the star is moving away, which is called a redshift. The moving velocity v of the wave source can be denoted as radial velocity v_r, where v_r is the component of the velocity of a celestial body directed along the line of sight.

3. Radial Velocity Method

3.1. The Introduction and Formula Derivation

In the binary system, the orbital motion of the planet around its host star indicates that the planet is acted on by the force exerted by the star $\overrightarrow{F_{s\to p}}$. According to the Newton's third law, the planet also exerts a gravitational force on the star $\overrightarrow{F_{p\to s}}$, where $\overrightarrow{F_{planet\to star}} = -\overrightarrow{F_{star\to planet}}$.

Hence, the star also moves along an orbit. That means, in the binary system, it is both the planet and the sun orbiting around the centre of mass of the system but not just the planet moving around the sun. As the star is much more massive than the planet, the centre of mass is very close to the star, which means that the orbital radius of the star r_s is small. So it is difficult to directly detect the moving trajectory of the star.

The shift of the light wave can be detected because the star is moving and emitting the light. When the star is moving towards the observation point (the Earth), blueshift occurs, the lines on the spectrum shift to the violet part. When the star is moving far away from the Earth, the redshift occurs, and lines shift to the red part. The below Fig. 1 shows the motion of the binary system and the corresponding line spectrum. Where the red arrow shows the star is moving away from the Earth, the blue arrow shows the star is moving towards the Earth. The light that the star emits to the Earth is show by blue and red wavy lines. The three emitted spectrums of hydrogen show the corresponding changed lines when redshift and blueshift occur.

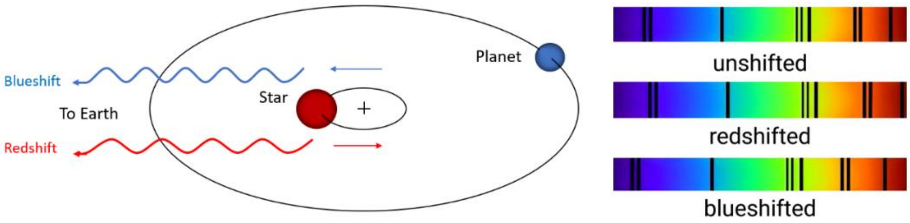

Fig. 1 The motion of the binary system and the corresponding Doppler effect. (Photo/Picture credit: Original)

Showing on the line spectrum, the lines shift back and forth with a period that equals the orbital period--when the star has done one circle, the same pattern repeats. With lines shifting, the change of wavelength $\Delta\lambda$ can be measured, then the $\Delta\lambda$ and time t curve can be obtained. The resulting data is presented as a sine curve. According to Doppler effect, $\frac{\Delta\lambda}{\lambda} = -\frac{v_r}{c}$, radial velocity v_r can be obtained from other three known values. Derived from the decomposition of velocity, the amplitude of v_r is equal to v_{star} (velocity of the star) in magnitude. Hence, v_s is obtained.

According to gravitational law, where a is the distance between the planet and the star, M_p is the mass of the planet.

$$v_s{}^2 = \frac{GM_p r_s}{a^2} \tag{2}$$

G is the gravitational constant, with the value of $6.67 \times 10^{-11} \mathrm{m^3 kg^{-1} s^{-2}}$.

As r_s is much smaller than r_p, $a = r_s + r_p$, a is approximately equal to r_p.

In the same way, $v_p{}^2$ can be obtained, where v_p is the velocity of the planet and M_s is the mass of the star.

$$v_p{}^2 = \frac{GM_s}{a} = \left(\frac{2\pi a}{T}\right)^2 \tag{3}$$

$$a = \left(\frac{GM_s T^2}{4\pi^2}\right)^{\frac{1}{3}} \tag{4}$$

Based on centre of mass equation $M_p r_p = M_s r_s$, v_s can be expressed in the rewrite form below:

$$v_s = \left(\frac{2\pi G}{T}\right)^{\frac{1}{3}} \frac{M_p}{M_s^{\frac{2}{3}}} \tag{5}$$

Then M_p can be calculated:

$$K = \left(\frac{2\pi G}{T}\right)^{\frac{1}{3}} \frac{M_p \sin i}{M_s^{\frac{2}{3}}} \tag{6}$$

Where $M_p \sin i$ is the projected mass on the plane of planet’s equator (see 4.3.2 inclination). (The following calculation progress is the same as that in 4.2 chapter.)

3.2. Limitation

3.2.1. Low-Mass Orbiting Planets Cannot be Detected

Because the star is moving around its orbit, its orbital radius depends on the location of centre of mass. The smaller the mass of the planet, the smaller the orbital radius r_s. Also, the acceleration of the star a_s is proportion to M_p and $a_s = \frac{v_s{}^2}{r_s}$, v_s decreases as a_s decreases. Hence, a less massive planet causes smaller v_s , which also is the amplitude of radial velocity. However, when the amplitude of the stellar Doppler shift is small below $1\ \mathrm{ms^{-1}}$, the star's vibrations will mask the Doppler shift [5]. For this reason, a less massive planet is unable to cause detectable Doppler effect of the star. Based on present technology, a planet with a mass less than $10M_{Earth}$ cannot be detected through the radial velocity method [6].

3.2.2. This Method is Only Sensitive to $M_p \sin i$

M_p cannot be directly measured through the radial velocity method unless the value of inclination i is obtained. Through direct imaging, inclination can be estimated. Thus, with the assistance of direct imaging, M_p can be obtained in theory. However, planets at 1 au from their host star are at the limit detecting range of the radial velocity method, but cannot be observed by direct imaging [7].

4. Transit Method

4.1. Transit Detections Developments

In 1999, the first transit planet HD 209458 b was observed [8]. In 2003, planet OGLE-TR-56 b was detected, it was the first exoplanet to be discovered through the transit method and confirmed through radial velocity [9]. After a few years, seven more OGLE planets were discovered [10]. As of August 2024, astronomers have discovered a total of 542 planets through the transit method [11].

4.2. The Introduction and Calculation Example

The transit method refers to the measurement of the parameter of an exoplanet through the periodic dimming of light caused by the exoplanet passing by its host star.

When the exoplanet passes by its host star, the planet blocks the light of the star. Displaying in the Kepler telescope's perspective, the planet leaves a black dot on the image of the star behind it, which is called transit. The brightness that the Kepler telescope received will decrease. Because the amount of light an exoplanet blocks depends on the planet's size, the depth of transit increases as the increase in planet's size.

Also, the radius of the star affects the transit depth. The same planet passing by a smaller star causes deeper transit than passing by a larger star. So the radius of the exoplanet Rp can be derived by transit depth and the star's radius R_s.

By illustrating the brightness received by the Kepler telescope, we can get the light curve of the star. As the planet passes in front of the star, the brightness of the star decreases, showing a dip in the light curve. The depth and width of the dip determine the radius of the planet and the transit duration. The length of time between each transit shows the orbital period T, shown by the time interval between two adjacent lowest flux points.

Figs. 2 and 3 below show the light curve of star HATS-11. The vertical axis in Figs. 2, and 3 is the relative flux, the horizontal axis shows the BJD time. The black curve shows the raw flux and the red curve represents the corrected flux.

$$T = 316066s \approx 3days\ 15h \tag{7}$$

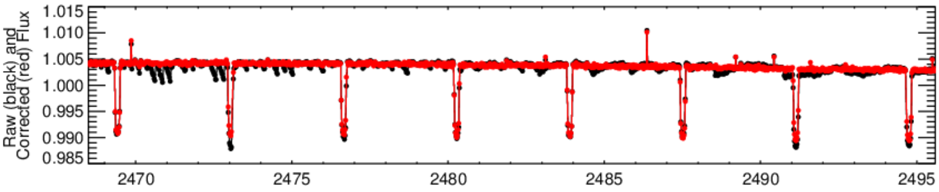

Fig. 2 The light curve of star HATS-11 (period 1 to 8) (K2SFF Search (stsci.edu))

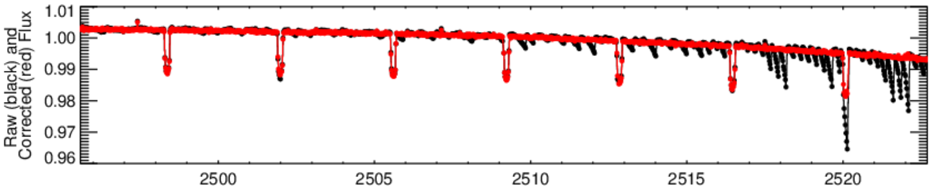

Fig. 3 The light curve of star HATS-11 (period 9 to 15) (K2SFF Search (stsci.edu))

Table 1 shows the exact value of relative flux in each period, where F_0 is the flux received from the star when there is no transit, $F_0 - \Delta F$ is that when there is a transit.

Table 1. Example processed data of relative flux change of star HATS-11

Period T	$\boldsymbol{F_0}'$/ F_0	$\boldsymbol{F_{min}/F_0}$	$\Delta F/F_0$
1	1.004	0.990750	0.013250
2	1.004	0.990308	0.013692
3	1.004	0.990510	0.013490
4	1.004	0.989981	0.014019
5	1.004	0.990279	0.013721
6	1.004	0.989874	0.014126
7	1.004	0.989430	0.014570
8	1.003	0.989358	0.013642
9	1.002	0.988560	0.013440
10	1.002	0.988294	0.013706
11	1.001	0.987791	0.013209
12	1.000	0.986492	0.013508
13	0.9992	0.985399	0.013801
14	0.9975	0.983545	0.013955
15	0.9952	0.981412	0.013788

The average relative change of flux $\frac{\Delta F}{F}$ is 0.0137 ± 0.0007. As F_0 is proportional to $\pi {R_{star}}^2$ and ΔF is proportional to $\pi {R_{planet}}^2$,

$$\frac{\Delta F}{F} = \left(\frac{R_p}{R_s}\right)^2 \tag{8}$$

The radius of star HATS-11is $0.683 \pm 0.009 R_{\odot}$, where $R_{\odot}$ is the radius of the sun. Thus Rp is $18.3 \pm 0.7 R_{Earth}$, where R_{Earth} is the Earth's radius.

According to Gravitational Law, where a is the orbital radius and M_s is the mass of the star with the value of 1.989×10^{30}kg.

$$ {a_p}^3 = \frac{G M_s T^2}{4\pi^2} \tag{9}$$

$$a_p = 6.95 \times 10^9 \mathrm{ms}^{-2} \tag{10}$$

Use the radial velocity formula, where K the amplitude of the radial velocity signal that can be obtained from the Doppler effect.

$$K = \left(\frac{2\pi G}{T}\right)^{\frac{1}{3}} \frac{M_p \sin i}{M_s^{\frac{2}{3}}} \tag{11}$$

Here, $\sin i$ is equal to 1.

$$M_p = 1.61 \times 10^{27} = 268.7 M_E \tag{12}$$

Hence, the mass density ρ can be calculated from known data.

$$\rho = \frac{M_p}{\frac{4}{3}\pi {R_p}^3} = 239.1 \mathrm{kgm}^{-3} \tag{13}$$

The value of mass density shows the main composition of the planets is probably gas.

4.3. Limitation

4.3.1. The Need for Complementary Measurement

As shown in the calculation progress, the radial velocity is needed to get the planet's mass. Not all parameters of the planet can be obtained only by the transit method. Hence, most times, transit is used to detect a planet and confirm the planet's radius and orbital period. Subsequently, the radial velocity method is required to assist in the measurement, to further confirm and identify the features of the transiting planet.

4.3.2. The Instability of Observations

In 2000, when observing the transit of planet HD 209458, the second transit was unobservable because of the longitude of APT, and the third transit was unobservable as well because of the block by cloud [8]. As can be seen from this example, the observation of transit is affected by such uncertainties---nebulae and inclination.

Inclination (α) is the angle between the line of sight and the plane of the planet's equator [12]. Transits occur in the centre of the star when the line of sight is even with the orbital plane, the inclination is near $0°$ in this case. If the angle of view is shifted up or down, the projection of the planet will be close to the edge of the star. That means instead of transiting in front of the equatorial plane, the planet is going to transit further down or up. So there is a maximum value of α to let the transit occur. For example, the inclination of HD209458 has an upper limit of $30°$ [13].

Because of the restricted range of inclination, the chance of seeing a planetary transit is slim. The range of inclination is determined by the planet's size and orbital radius.

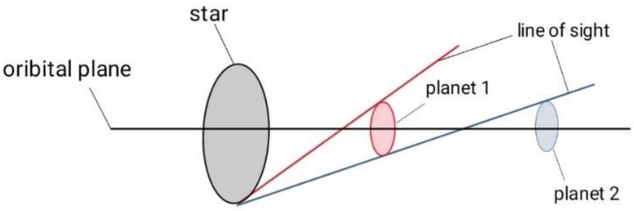

Fig. 4 A simple schematic diagram of the maximum inclination of a planet's transit. (Photo/Picture credit: Original)

For planets with the same size, the further one has a smaller range of inclination, as shown in Fig. 4. The three ellipses represent a star and two orbiting planets, where the red and blue lines represent the line of sight at the maximum inclination that allows the transits of planets 1 and 2 to occur respectively. That means planets further than a certain value may not be observed at a certain inclination. When observing the transit of planet HD209458, the inclination is 2.9±0.1°, implying planets further than 0.1 AU from the star will show no transits [14]. It can be seen that a subtle dispersion in inclination will directly affect the probability of finding the planet’s transit.

For planets of different sizes that are at the same distance, their maximum inclination is distinct as well. The larger one has a greater range of inclination as shown in Fig. 5 below. The red ellipse represents the smaller planet 1, the blue ellipse represents the bigger planet 3. Planets 1 and 3 have the same orbit, as shown by the coincident geometric centers. Just from the orbital radius, a fixed standard value cannot be obtained.

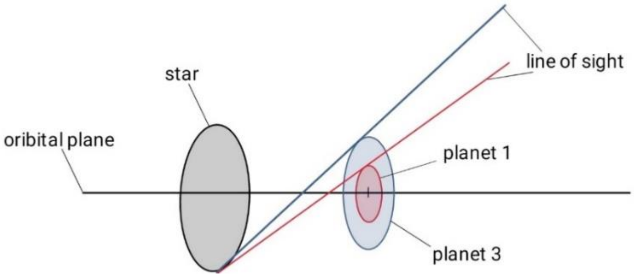

Fig. 5 A simple schematic diagram of the maximum inclination of transit of planets 1 and 3. (Photo/Picture credit: Original).

All these factors cause the instability of observation.

4.4. Advantage

Less massive planets that cannot be easily measured by the radial velocity method can be detected with photometry. For instance, in the HD 209458 system, the radial velocity data of M99 shows that there was no other massive planet [8]. However, the central transit of a Uranus-sized planet at 0.2AU can be observed by a 2m telescope [14]. Thus, the signal of an Earth-sized planet is attainable from the Earth’s perspective by transit method [14].

5. Conclusion

This article introduces two of the measurement methods of exoplanets include radial velocity method, transit method. Each has its advantages and limitations and is often used in combination to obtain a more comprehensive picture of planetary signatures. This article discovered the radial velocity method is based on the Doppler effect of the star. Only massive planets can be detected and secondary calculation is required to obtain its mass. Transit is based on direct photometric observation of periodic small changes in the brightness of a star, but only by combining the radial velocity method can the full parameters of the planets be obtained. Also, the probability of observing a planet is limited due to the limitation of inclination and other factors. Therefore, a single approach is not enough to achieve stable results, a combination of those method's advantages (including the direct imaging method and microgravitational lensing) is the best way. The radial velocity method and the transit method have been extensively applied in discovering and characterizing exoplanets, and they have played important roles in projects such as the Kepler Space Telescope, leading to the identification of thousands of exoplanetary candidates. This paper is helpful for readers to understand the principles and detailed calculation process of these two methods, and at the same time to know the general advantages and disadvantages of the methods. The stellar data of HATS-11 is used as an example in this paper and the derived parameters are limited due to the technical limitations at that time, thus the updated data can be used in subsequent studies. The development of instruments and telescopes will enhance the observing ability, and more detailed information on planets' atmospheres can be obtained, potentially leading to the discovery of habitable Earth-like planets.

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