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Matrix Optics in Optical Communication Systems

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Abstract. The concept of Matrix Optics is explained in detail and some key matrices which are used in Optics are given, including ABCD matrices and Jones matrices. Some examples of matrices to model optical components are provided. The basic principles of Optical Communication Systems are explicated, along with their principal components. The key performance metrics are mentioned, each with a brief explanation: Bandwidth, signal-to-noise ratio, and bit error rate. The applications of Matrix Optics in Optical Communication Systems are investigated, as well as some advanced techniques and technologies. In this regard, bends causing signal propagation in optical fibers, optical amplifiers, and resonators are explained; advanced techniques such as adaptive optics in modulating the atmospheric turbulence, space division multiplexing in describing the scrambling of the spatial modes, and Non-linear Optical Effects in quantum systems are mentioned.

Keywords: Matrix Optics; ABCD matrices; Optical Communication Systems; Application of matrices.

1. Introduction

Optical communication systems use optical signals, such as light, to transmit information between 2 points. It can be visually or using electronic devices. The earliest forms of optical telecommunication were performed using visual techniques such as smoke signals, used as signals to transmit news or danger; heliograph, using a mirror to reflect sunlight to the observer; and visual approach indicator, used in military aircraft landing. In the modern era, optical communication systems are mostly electronic. Different electronic devices transmit and receive pulses of light in order to communicate. These systems are widely used nowadays and thanks to them we have the communication medium between continents, and we have access to the global communication system “The internet”. Most of these systems are described and modeled by matrices, which is a mathematical tool that gives an efficient way to make calculations [1].

Matrix optics is a powerful mathematical tool used to calculate the path of particles in a determined system. It consists of a 2 by 2 matrix that operates on a vector and can be used to calculate the outgoing light ray by describing the income ray. The paraxial approximation was essential for this technique to work, which requires the angle between the perpendicular direction to the wavefronts and the optical axis to be small such that $\sin\theta \simeq \tan\theta \simeq \theta$ is true. Matrix Optics can be used both in ray optics and propagation of Gaussian beams and is very commonly used nowadays in Optical Communication Systems as will be seen later. The objective of this paper is to explore the application of matrix optics in enhancing the performance of optical communication systems. The structure of this paper consists of a general explanation of Matrix Optics and Optical Communication Systems, some discussion of the applications of Matrix Optics in such systems including advanced techniques and technologies, and a conclusion at the end to summarize all the important concepts.

2. Fundamentals of Matrix Optics

2.1. Basic principles of matrix optics

The matrix optics is used to calculate the propagation of a light ray in an optical system that is characterized by its distance r from the optical axis and by its angle θ from the same axis. The angles are assumed to be small. The linear relation between the r and θ and how they are transformed by an optical element is described by the following matrix

$$\begin{pmatrix} cr' \\ \theta' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r \\ \theta \end{pmatrix} \# \tag{1}$$

Where the dashed quantities represent the beam after passing the optical element, and the undashed quantities represent before passing the same element. The matrix with the coefficients A, B, C, and D is called the ray transfer matrix and characterizes each optical element. The determinant of this matrix is generally unity and is always invertible.

The ABCD matrices are linear operators, therefore, we can multiply different individual ABCD matrices to give a concise ABCD matrix that describes the entire optical system

$$M = M_N \cdots M_2 M_1 \,. \tag{2}$$

Other key matrices used in optics include Jone matrices and Mueller matrices. The Jone matrix is often used to model the effect of a medium on a polarization state [2]. The polarization state can be defined as a 2D vector

$$\underline{v} = \frac{1}{v} \begin{pmatrix} v_x \\ v_y \end{pmatrix} \tag{3}$$

Where $\underline{v}$ is normalized to the length of unity and is known as the Jones vector. The Jones matrix A is used to transform the polarization state, from the input polarization v_1 to the output polarization v_2.

The relation between the Jones matrix and the Jones vector is defined to be

$$\underline{v_2} = A \, \underline{v_2} \,. \tag{4}$$

The Jones matrix is defined as

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}. \tag{5}$$

2.2. Some examples of ABCD matrices of optical elements

Table 1. Examples of ABCD matrices.

Elements	Matrix	Symbols explanation
Propagation in vacuum	$\begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}$	d is the distance.
Lens	$\begin{pmatrix}1 & 0\\ -1/f & 1\end{pmatrix}$	f is the focal length.
Reflection from a curved mirror	$\begin{pmatrix}1 & 0\\ -\frac{2}{R} & 1\end{pmatrix}$	R’ is the curvature radius. Horizontal direction case: R=R’$\cos\theta$ Vertical direction case: R=R’$/\cos\theta$ R’ > 0 concave mirror.
Refraction flat interference	$\begin{pmatrix}1 & 0\\ 0 & \frac{n}{n'}\end{pmatrix}$	n stands for the initial refractive index and n’ for the final refractive index.
Refraction curved interference	$\begin{pmatrix}1 & 0\\ \frac{n-n'}{R\cdot n'} & n'\end{pmatrix}$	R stands for the radius of curvature and R > 0 for the convex mirror. n stands for the initial refractive index and n’ for the final refractive index.

3. Optical Communication Systems

3.1. Overview of Optical Communication Systems

The main components of an Optical Communication System are a transmitter, a receiver, and an optical glass or silica fiber which have the function of transmission medium [3]. The transmitting principle of a transmission medium such as optical glass is that it consists of the total internal reflection. Whenever a ray passes through 2 mediums and the angle with respect to the second medium is very small, the ray will be reflected by the second medium and remain in the first medium. Optical transmitters, however, are devices that transform electrical signals into optical signals by creating light waves [4]. Receivers are devices that convert optical signals to electrical ones using photodetectors or photosensors.

There are different types of optical communication systems: fiber optics, free-space, and integrated photonics. The most common one is the fiber-optics, which uses the optical fiber as a channel for optical communications. In free-space optical communication system compared to the fiber-optics, uses air or vacuum to transmit optical signals. The transmission can be achieved in both ways: either emitting an incoherent light in the area or emitting directly to the receiver [5].

3.2. Performance Metrics

It is of vital importance to use performance metrics since the capacity of optical communication systems has shown an increasing trend nowadays and the constructions of optical networks are getting more labyrinthine [6]. Each carrier carries a huge amount of data and hence a slight interruption can cause catastrophic consequences. Therefore, the monitoring systems need to be implemented as a prophylactic measure providing precise information about how healthy each channel is. Some of the key performance metrics include Optical power, signal-to-noise ratio (SNR), and bit error rate (BER).

Optical power is the most fundamental parameter to be monitored since because of the attenuation of fiber and lose due to fiber breaks, it can decrease drastically [7]. Regarding the signal-to-noise ratio (SNR), some optical amplifiers such as erbium-doped fiber amplifiers (EDFAs) which losses in the transmission are balanced thanks to the implementation of optical networks. However, amplified spontaneous emission (ASE) was added to the signals which is not wanted. Consequently, there will be an excess of ASE noise and hence the ASE noise is also crucial to be quantified and monitored [8]. The bit error rate (BER) evaluates how an optical network has performed on average by giving a number indicating the quality of the optical link. However, it does not give any insights into how different impairments have contributed to the deterioration of system performance.

4. Application of Matrix Optics in Optical Communication Systems

4.1. Signal propagation in Optical Fibers

One of the applications of Matrix Optics is in signal propagation. For example, bends in optical fibers that cause signal propagation in guided modes are expected to be lost by joining guided modes to radiation modes because of the presence of misalignments. The process of joining is called coupling. Matrix elements have the utility to describe the coupling strength of a bend in the corner between different modes. The mean propagation constant can be found using a sum rule at the corner bend. Such information can be utilized to determine some of the properties and changes of the propagation constant [9].

Another usage of ABCD transfer matrices, recall equation (1), is to model the Fabry-Perot (FP) Etalons. The etalon matrix needs to be first defined and then multiplied by the ABCD transfer matrices, recall equation (2), representing optical components to deliver light to or from the FP Etalons to model the system [10]. In a system that consists of 2 cascaded FP Etalons, comprising a series of mirrors and separated into two parallel cavities of the same material, and a pair of thin lenses, the system matrix is obtained by multiplying different transfer matrices representing individual FP Etalons in terms of mirror reflectivity, thickness and refractivity, and matrices representing lenses. The matrix of lenses is indicated in the second row of Table 1. For the cases when the mirror is curved or flat, the transfer matrices are summarized in the third, fourth, and fifth rows of Table 1.

4.2. Optical Amplifiers

The transfer matrix method (TMM) can be used to analyze different characteristics of semiconductor laser optical amplifiers. Even though the complexity of the computation is one of the most fundamental problems that obstruct analyzing the semiconductor laser amplifier (SLA), the TMM is a very useful way to approach this problem, giving an efficient algorithm to analyze SLA and reducing significantly the computing time [11].

The Jones matrix method recall equations (3)-(5) can be used to analyze optical coherence tomography which is a technique to measure the power of light reflected from the tissue. Jones and Mueller matrices can compare the polarization states of the reflected light from the surface and a biological sample, using alternating incident polarization to ensure the detection of tissues. This method is independent of the optical fiber components.

4.3. Optical Resonator

The optical resonators especially those that are fiber couplers dependent have been investigated thoroughly for their abilities to achieve higher frequency selectivity and greater power storage. The theory that has been used classically to study this type of resonator is not so effective for greater complexities devices. However, using the formulation of matrices the calculus is substantially simplified and able to consider all the inputs and outputs of arms and optic fiber compounds [12].

5. Advanced Techniques and Technologies

5.1. Adaptive Optics and Wavefront Correction

Adaptive optics are used to modulate or remove atmospheric turbulence that affects the spatial resolution of large telescopes. The matrix can be used to relate the wavefront measurements and the adaptive mirror, which are the components of the adaptive system. When the actuators of the mirror are successfully driven, the responses from the wavefront computer are measured and given the columns in the form of a matrix, which is called the interaction matrix. This matrix gives spatial modelization validity of the system.

5.2. Multiplexing

This subsection centralizes in the space-division multiplexing (SDM). SDM with spatial paths in multi-core fiber increases the transmission system capacity. However, because of the closeness of the spatial modes, they can mix during the process which leads to scrambling. This scrambling can be represented by a transfer matrix with N rows and M columns and can be unfasten using multiple electrical inputs and outputs. The matrix eigen-analysis can also enable the removal of the system's losses, including insertion loss and mode-dependent loss [13].

5.3. Nonlinear Optical Effects

Matrices in nonlinear optics have also shown great importance, especially in studies of nonlinear properties in optical quantum systems. The quantum wells which are caused by diminutive energy separation and dipole moment of considerable size that corresponds to the intersubband transition have been shown substantially large non-linear optical properties. Density matrix approximation was helpful to measure linear and non-linear electric susceptibility, and hence the refractive index and absorption coefficient can be later determined [14].

6. Conclusion

Both Matrix Optics and Optical communication systems are briefly explained. The main concepts and principles of both are mentioned, as well as some key matrices used in optics and some basic components of communication systems. The applications of matrices in Optical Communication Systems are investigated including some advanced techniques and technologies. The significance of matrices in enhancing Optical Communication systems is observed, as was discussed in sections 4 and 5, Matrix Optics was useful in signal propagation in Optical Fibers, analyzing different characteristics of optical amplifiers, and in optical resonators. Some advanced techniques include matrices in adaptive optics, space-division multiplexing, and non-linear optical properties in quantum wells.

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