

# Research on UAV formation signal location based on two-dimensional plane analysis

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**Abstract.** In recent years, the UAV technology has developed rapidly and is widely used in military, civil and scientific research fields. In order to avoid external electromagnetic interference, the bearings-only passive positioning method is usually used to adjust the position of the UAV to maintain the formation. In this study, according to the circular formation of Uavs, the positioning model of the received signal of Uavs on the excellent arc and the positioning model of the received signal of Uavs on the inferior arc are established. The Uavs with slightly deviated positions in the circular formation are effectively adjusted, and the positioning model of Uavs with the passive receiving signal is established. In this paper, the double-station cross-positioning method is used to calculate that in the case of circular formation, four Uavs are needed to transmit signals, among which one is located in the center of the circle and the other three are distributed on the circumference to realize the effective positioning of Uavs.

**Keywords:** Uav location, Optimal arc model, Poor arc model, Double station cross positioning.

## 1. Introduction

In recent years, with the rapid development of UAV technology, UAV has been more and more widely used in military, civilian, scientific research, and other fields. As a new type of technical means, UAV formation has the advantages of high efficiency, flexibility, and low cost, and has become one of the hotspots of current UAV technology research. In the application of UAV formation, achieving accurate positioning and navigation is the key to ensuring its efficient operation. Through the study of the azimuth pure passive positioning of circular UAV formation, this paper establishes the most simple and efficient positioning model in the UAV formation positioning, so that the application of convenient and accurate positioning and fast realization of the goal.

There are many researches on traditional UAV positioning methods. Meng Shengbo et al. studied the error of cross positioning based on the principle of cross positioning and other technologies[1]. Zhang Nan et al. studied the passive positioning of circular formation of Uavs through the double-station cross positioning method of "three points to determine the circle"[2]. Zhao Yanyang et al. applied bearings-only passive positioning technology to unmanned formation to complete accurate positioning by establishing polar coordinate equation and using triangulation positioning algorithm[3]. Liu Jia et al. established an elementary mathematical model according to the Angle relationship, and used the legacy algorithm to iteratively calculate the positioning problem of circular formation Uavs[4]. In this paper, through the superior and inferior arc model and the double station cross positioning method, from another idea, this paper explores a more simple and efficient bearings-only passive positioning technology of UAV.

Based on the UAV bearings-only passive positioning research, this study consists of a circular formation formed by 10 Uavs, 9 of which are evenly distributed on the circumference and 1 is located in the center of the circle. Uavs maintain the same altitude according to their perception. The UAV in the center of the circle and any two UAVs on the circumference are set to transmit signals, and the position of the UAV in the transmitted signal is not deviation. The positioning model of the UAV with the passive receiving signal is studied. The positioning model of the UAV with the receiving signal on the excellent arc and the positioning model of the UAV with the receiving signal on the poor arc are established, and the positioning of the UAV with the receiving signal in the two states is



discussed. The model was used to solve the positioning coordinates of the passively receiving signal UAV. At the same time, in the case of determining the center of the circle and one UAV transmitting signals on the circumference, several UAVs transmitting signals are also needed to realize the effective positioning of passive signal-receiving Uavs. In this paper, six types are classified according to the superior and inferior arc model, and the triangle composed of the transmitting signal UAV and the receiving signal UAV is classified. By using the double-station cross-positioning method, it is calculated that two more UAVs are needed to transmit signals to realize the effective positioning of the UAV.

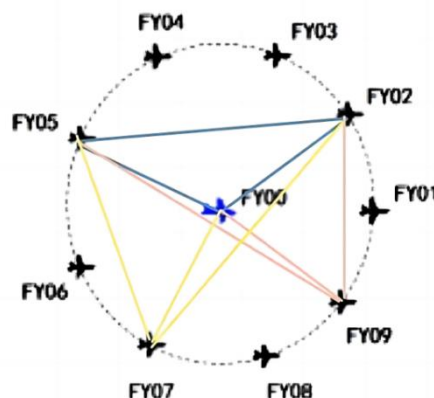
## 2. Research on the positioning model of UAV received signals on superior and inferior arcs of the circular formation

### 2.1. The received signal UAV positioning model is established

In the case of determining the center of circle (FY00) UAV, two signal transmitting UAVs are selected in the formation, and the positioning model of the receiving signal UAV is established under the condition that the position of the transmitting UAV is without deviation and the number is determined. In this paper, the UAV formation is abstracted into a two-dimensional plane for research. In the process of determining the signal-transmitting UAV, it is found that due to the restriction of uniform distribution of UAV on the circle, the connection line of any two UAVs in the distribution of signal-transmitting UAV on the circle is not the center of the circle. According to the analysis, In this paper, there are two cases, which are the positioning model of the UAV receiving signal on the excellent arc and the positioning model of the UAV receiving signal on the inferior arc [5].

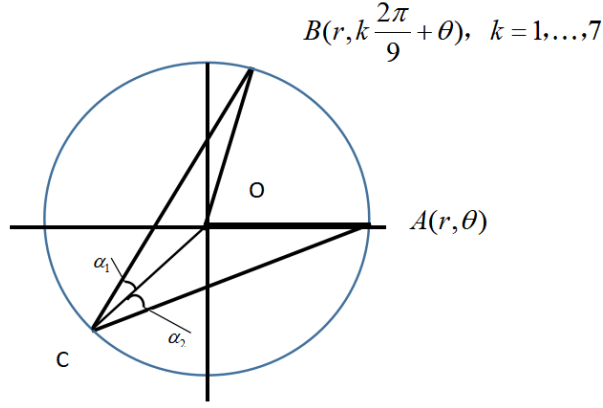
(1) The positioning model of the UAV received signal on the optimal arc

On the circumference of the UAV formation, any two UAVs are identified as signal-transmitting Uavs, and they are connected to form a triangle connected with the center of the circle (FY00). By the connection of two transmitting UAVs on the arc, a receiving signal UAV is taken from each of the large circular arcs, and then connected with the three vertices of the triangle respectively. The Angle analysis is carried out to establish the positioning model of any UAV on the optimal arc. Figure 1 is an example of FY05 and FY02.



**Figure 1.** Localization model diagram of the received signal UAV on the optimal arc

Figure 2 shows that the Uavs keep flying at the same altitude, the FY00 and any two Uavs in the formation at the center of the circle transmit signals with no deviation in position, and the Uavs at the remaining positions are slightly deviated. Then the polar coordinate system is established, and it is assumed that the coordinates of the two UAVs transmitting signals in the formation are two points A and B, and the coordinates of the positioning UAV are point C.



**Figure 2.** Analysis diagram of the target UAV on the inferior arc

Because there is no deviation in the position of two points A and B,  $A(r, \theta)$   
 $B\left(r, k \frac{2\pi}{9} + \theta\right), k = 1 \dots 7.$

$BC = l_1, AC = l_2,$  because C is biased,  $OC \neq r$  they, make  $OC = m; \angle BCO = \alpha_1,$   
 $\angle ACO = \alpha_2, \angle BCA = \alpha_3.$

When point C is on the optimal arc;

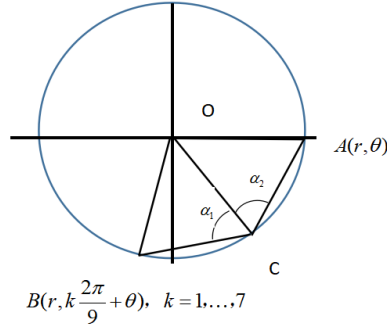
$$\begin{cases} \cos \alpha_1 = \frac{l_1^2 + m^2 - r^2}{2l_1 m} \\ \cos \alpha_2 = \frac{l_2^2 + m^2 - r^2}{2l_2 m} \\ \alpha_3 = k \frac{\pi}{9} \\ \cos \angle COB = \frac{m^2 + r^2 - l_1^2}{2mr} \end{cases} \quad (1)$$

Thus the point C coordinates can be expressed as:

$$C\left(m, \arccos \frac{m^2 + r^2 - l_1^2}{2mr} + k \frac{2\pi}{9} + \theta\right), k = 1 \dots 7 \quad (2)$$

(2) The positioning model of the UAV received a signal on the inferior arc

According to the UAV model analysis of the received signal on the excellent arc, the positioning model of the UAV with the received signal on the inferior arc is established in the relative position, the correlation between the distribution angles is constructed, and the formula of the UAV positioning model is derived. The following also takes FY05 and FY02 as examples to establish the positioning model diagram of the UAV with the received signal on the inferior arc.



**Figure 3.** Positioning model and analysis diagram of the UAV receiving signals on the inferior arc  
When point C is on the inferior arc;

$$\begin{cases} \cos \alpha_1 = \frac{l_1^2 + m^2 - r^2}{2l_1m} \\ \cos \alpha_2 = \frac{l_2^2 + m^2 - r^2}{2l_2m} \\ \alpha_3 = \arccos \frac{l_1^2 + m^2 - r^2}{2l_1m} + \arccos \frac{l_2^2 + m^2 - r^2}{2l_2m} \\ \cos \angle COB = \frac{m^2 + r^2 - l_1^2}{2mr} \end{cases} \quad (3)$$

Thus the point C coordinates can be expressed as:

$$C \left( m, \arccos \frac{m^2 + r^2 - l_1^2}{2mr} + k \frac{2\pi}{9} + \theta \right), k = 1 \dots 7 \quad (4)$$

## 2.2. Analysis of the results of the received signal UAV localization model

According to the received signal UAV positioning model, the positioning model of the UAV with excellent arc receiving signal and the positioning model of the UAV with poor arc receiving signal are established respectively.

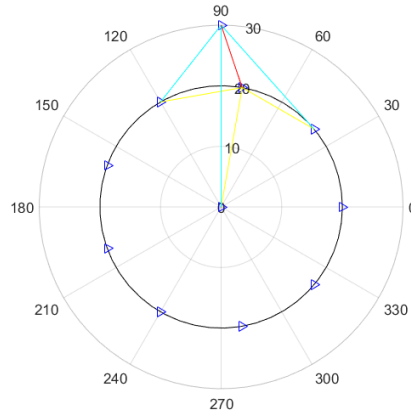
The positioning coordinates of the UAV with optimal arc receiving signal are as follows.

$$C \left( m, \arccos \frac{m^2 + r^2 - l_1^2}{2mr} + k \frac{2\pi}{9} + \theta \right), k = 1 \dots 7 \quad (5)$$

The positioning coordinates of the inferior arc receiving signal UAV are as follows.

$$C \left( m, \arccos \frac{m^2 + r^2 - l_1^2}{2mr} + k \frac{2\pi}{9} + \theta \right), k = 1 \dots 7 \quad (6)$$

In summary, no matter under that model, the UAV positioning coordinates of the received signals are the same. That is, the coordinates are the positioning coordinates of the received signal UAV. Take the positioning polar coordinates of the received signal UAV into the numerical simulation, assume that the offset polar coordinate value of FY03 is (30,90), and simulate the positioning of FY03 and the direction and distance adjusted to the ideal position. The yellow line is the positioning of the ideal target position, the cyan line is the positioning of the deviation position, and the red line is the adjustment path of the position. The simulation effect is shown in Figure. 4.



**Figure 4.** Simulation rendering of the receiving signal UAV

### 3. Research on effective positioning of circular formation Uavs based on double station cross positioning

#### 3.1. Effective localization based on the two-station cross-localization model

From the previous section, it can be known that uniform distribution requires at least 2 Uavs on the circumference. If the further optimization problem only needs one UAV and the position of the UAV cannot be determined, at least 2 UAVs are needed. Therefore, this study can introduce multi-station passive positioning, which requires multiple UAVs to work synchronously and transmit data, and can obtain more information than single-station passive positioning, which helps improve the positioning accuracy. Except for FY00 and FY01, the other numbers of the Uavs that can transmit signals on the circle are unknown, so the double-station cross-positioning calculation method can be used to determine the position of the Uavs. Suppose that the numbers of the other two aircraft are FYOK1 and FYOK2, then the UAVs that can transmit signals are FY00, FY01, FYOK1, and FYOK2. The UAV that passively receives the signal is P.

Firstly, the position of the UAV is slightly deviated and at a fixed altitude. Therefore, this study can ignore the deviation of the UAV in altitude and only consider the position and positioning of the UAV in the two-dimensional Cartesian coordinate system. The following figure shows the relative position of the UAV and the target position in the two-dimensional plane. Where "+" represents the target position of the UAV, and "." represents the passively received signal UAV position. To achieve the effective positioning of the UAV, it should be considered that the UAV should send out fewer electromagnetic wave signals and complete the formation as little as possible. Therefore, the UAV should cooperate to correct its position at the same time. As shown in Figure 5 below, there are many possibilities for the correction trajectory of the UAV passively receiving signals. The corrected trajectory with the smallest distance between the passively received signal UAV and the target position should be preferred.



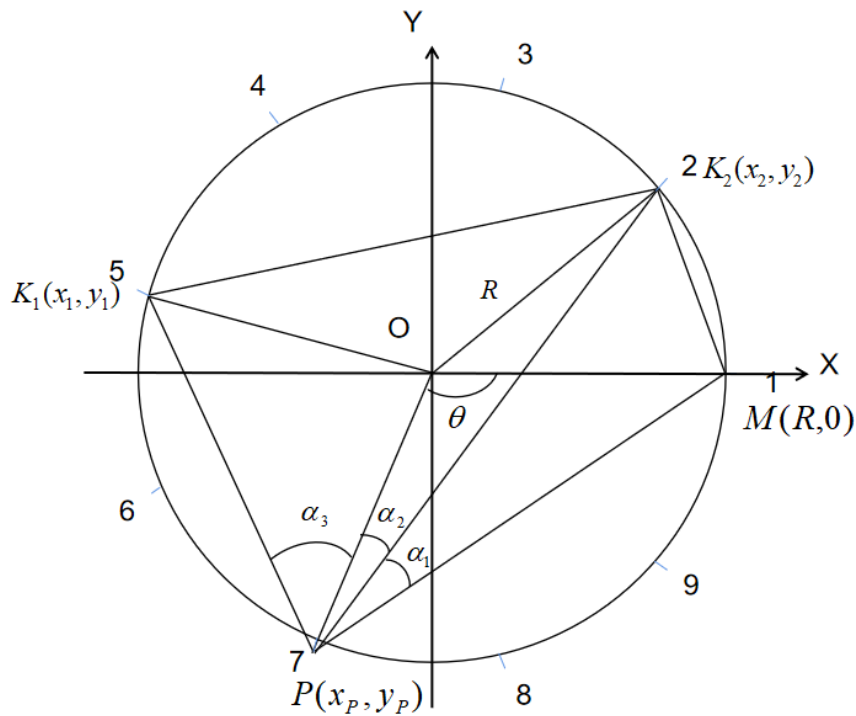
Where the positions of the signal transmitting Uavs FYOK1, and FYOK2 are known, then D is known. When  $\varphi_1, \varphi_2$  is determined,  $\beta_1, \beta_2$  can be obtained, and then the distance  $d_1, d_2$  between the passive receiving signal UAV and the transmitting signal UAV can be calculated.

$$\frac{d_1}{\sin \beta_2} = \frac{d_2}{\sin \beta_1} = \frac{D}{\sin(\beta_1 + \beta_2)} \quad (9)$$

$$d_1 = \frac{D \sin \beta_2}{\sin(\beta_1 + \beta_2)}, d_2 = \frac{D \sin \beta_1}{\sin(\beta_1 + \beta_2)} \quad (10)$$

### 3.3. Calculation of the model based on double station cross-location

Figure 7 shows the established plane rectangular coordinate system, taking the connection direction of UAV number 0001 as the X-axis, the coordinates of the signal transmitting UAV FY0K1 on the circle are  $(x_1, y_1)$ , the coordinates of FY0K2 are  $(x_2, y_2)$ , the center O is FY00, the coordinate is (0,0), M is FY01, the coordinate is (R,0), and the coordinate is (r,0). P is any passively received signal UAV with coordinates  $(x_p, y_p)$ .



**Figure 7.** Signal transceiver drone position analysis diagram

In the plane rectangular coordinate system, to adjust the position of the UAV with slight deviation[8], the target position of the UAV has been solved in the previous section. In this problem, only the position of the passively receiving signal UAV is determined, that is, the distance and Angle between the passively receiving signal UAV and the transmitting signal UAV need to be determined, so the position adjustment can be carried out. In the case of considering the positioning of edges and angles[9], the calculation method of the triangle cosine theorem can be used. There are six cases listed in the following table for the combination way of forming a triangle with the passively received signal UAV P.

**Table 1.** UAV number classification table

Classification	Uav number		
1	FYOP	FY01	FY0K2
2	FYOP	FY01	FY00
3	FYOP	FY00	FY0K2
4	FYOP	FY00	FY0K1
5	FYOP	FY01	FY0K1
6	FYOP	FY0K1	FY0K2

For the first case, the following equation can be obtained:

$$(PM)^2 + (PK_2)^2 - 2 \cdot PM \cdot PK_2 \cdot \cos \alpha_1 = (MK_2)^2 \quad (11)$$

In the second case, the following equation can be obtained:

$$(PM)^2 + (PO)^2 - 2 \cdot PM \cdot PO \cdot \cos(\alpha_1 + \alpha_2) = (MO)^2 \quad (12)$$

And so on, the equations for the remaining cases are as follows:

$$\begin{cases} (PO)^2 + (PK_2)^2 - 2 \cdot PO \cdot PK_2 \cdot \cos \alpha_2 = (OK_2)^2 \\ (PO)^2 + (PK_1)^2 - 2 \cdot PO \cdot PK_1 \cdot \cos \alpha_3 = (OK_1)^2 \\ (PM)^2 + (PK_1)^2 - 2 \cdot PM \cdot PK_1 \cdot \cos(\alpha_1 + \alpha_2 + \alpha_3) = (MK_1)^2 \\ (PK_1)^2 + (PK_2)^2 - 2 \cdot PK_1 \cdot PK_2 \cdot \cos(\alpha_2 + \alpha_3) = (K_1K_2)^2 \end{cases} \quad (13)$$

According to the above equations, the edge  $MK_2, MK_1, MO, OK_1, OK_2$  is known, its Angle  $\alpha_1, \alpha_2, \alpha_3$  is unknown, and  $PM, PO, PK_1, PK_2$  is unknown. In the case of 7 unknowings in the 6 equations, the solution cannot be obtained, and the equations with additional angles need to be added[10].

Let the Angle  $\angle MOP = \theta$  be, then the equation related to the Angle  $\alpha_1, \alpha_2, \alpha_3$  is as follows.

$$\begin{cases} \frac{\sin \alpha_3}{R} = \frac{\sin(\pi - \omega_1 \alpha - \theta)}{PK_1} \\ \frac{\sin \alpha_2}{R} = \frac{\sin(\theta + \omega_2 \alpha)}{PK_2} \\ \frac{\sin(\alpha_1 + \alpha_2)}{R} = \frac{\sin \theta}{PM} \\ \frac{\sin \alpha_1}{MK_2} = \frac{\sin\left(\frac{\pi - \omega_2 d}{2} + \pi - \alpha_1 - \alpha_2\right)}{PK_2} \end{cases} \quad (14)$$

Among them,  $\alpha_1, \alpha_2, \alpha_3$   $\angle MPK_2, \angle K_2PO, \angle OPK_1$  respectively.  $\theta$   $\angle MPO$  is,  $\alpha$  is  $\frac{2\pi}{9}$  with uniform distribution, and  $\omega_1, \omega_2$  is coefficient.

The edge can be converted into the following equation:

$$\begin{cases} PM = \sqrt{(x_p - R)^2 + y_p^2} \\ PK_2 = \sqrt{(x_p - x_2)^2 + (y_p - y_2)^2} \\ PO = \sqrt{x_p^2 + y_p^2} \\ PK_1 = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2} \end{cases} \quad (15)$$

Among them, the coordinates of the transmitted signal Uavs FY00, FY01, FYOK1, and FYOK2 are known, that is,  $x_1, x_2, y_1, y_2$  is known, and the radius R is known.

The coordinates of UAV P can be obtained simultaneously from the above equations, and the effective positioning of the UAV can be determined. Therefore, in addition to the signal-transmitting UAV FY00.FY01, another two signal-transmitting Uavs are needed to effectively locate the passive signal-receiving UAV P.

#### 4. Conclusions

In this paper, the positioning model of UAV received signal on the excellent arc and the positioning model of UAV received signal on the poor arc are established, and the UAV positioning of the received signal in the two states is discussed respectively. Through calculation, the polar coordinates of the UAV positioning of the received signal are

$$C \left( m, \arccos \frac{m^2 + r^2 - l_1^2}{2mr} + k \frac{2\pi}{9} + \theta \right), k = 1 \cdots 7$$

and according to the excellent arc and poor arc models, Six kinds of triangle combinations composed of transmitting signal Uavs and receiving signal Uavs are classified. By using the double station cross positioning method, it is calculated that two more Uavs are needed to transmit signals to realize effective positioning of Uavs.

In this paper, the formation of Uavs is adjusted according to the bearings-only passive localization method, and the positioning model of passive signal receiving Uavs in circular formation is established. Under this condition, the formation of UAVs requires at least several signal-transmitting Uavs to achieve effective formation localization. It is proved that the positioning model of UAV received signal on the excellent arc and the positioning model of UAV received signal on the inferior arc, as well as the double-station cross positioning method, the feasibility of solving the position deviation of UAV in the bearings-only passive positioning of UAV and the simplicity compared with other positioning methods are provided, which provides reference for the subsequent research on bearings-only passive positioning of UAV.

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