

Study and Application of Optimization Model on Survey Line under Multibeam Bathymetric System

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Abstract. In this paper, the definition of overlap rate is deduced so as to clarify the influence of overlap rate between the horizontal plane and the slope on the multibeam line measurement problem, and the artificial correction is calculated by taking the average value of the coverage widths of the two times before and after. The expressions for the coverage widths of different lines and their corresponding water depths, slope inclination angles, and transducer opening angles are derived from the functional relationship by combining numerical and morphological methods. The resulting expression of the coverage width W was corrected, averaged and brought into the definition of the overlap rate, resulting in the final expression to calculate the corresponding value, and the resolution and accuracy of the data results were high.

Keywords: Multibeam Bathymetric System; Transducer Opening Angle; Coverage Width.

1. Introduction

Multibeam bathymetric system is a popular underwater measurement technology at present. It is different from four-beam bathymetric systems and single-beam bathymetric systems in that it is more efficient for seafloor geoid surveying [1]. Scholars have conducted a lot of research on the error analysis of multibeam systems. Zhao et al. used BP neural network to eliminate the roughness of multibeam bathymetry data [2]; Liu et al. proposed the method of uniformly adopting the relative error as the assessment index of the bathymetry accuracy [3]; and Yang et al. proposed the method of attitude compensation based on the bottom measurement data [4]. However, multibeam bathymetry accuracy assessment is often carried out only for a single project, and a unified accuracy assessment model has not yet been formed, and there is a lack of comprehensive evaluation methods [5]. In this paper, in view of the susceptibility of edge beams to multiple errors, we propose to use the overlapping region of adjacent survey lines to evaluate the accuracy of edge beam data and establish a mathematical model for the purpose of optimization.

2. Model Building

2.1. Derivation and Calculation

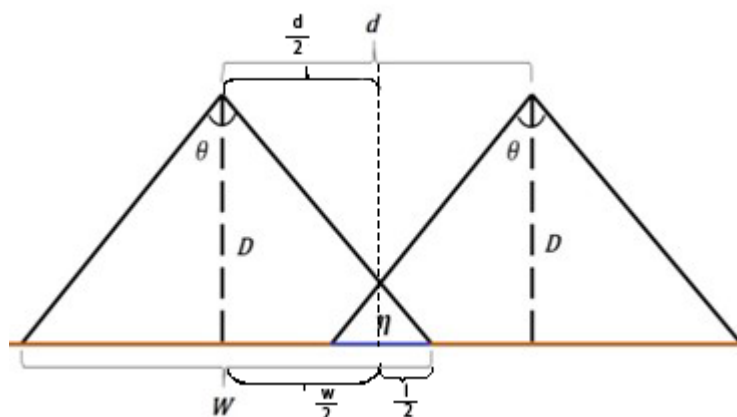


Figure 1. Overlap rate derivation chart

$$W_1 = \frac{2D \tan \frac{\theta}{2} - 2d_1 \tan \alpha \tan \frac{\theta}{2}}{1 - (\tan \alpha)^2 (\tan \frac{\theta}{2})^2} \quad (7)$$

$$W_2 = \frac{2D \tan \frac{\theta}{2} - 2d_2 \tan \alpha \tan \frac{\theta}{2}}{1 - (\tan \alpha)^2 (\tan \frac{\theta}{2})^2} \quad (8)$$

The average of the corrected values sought:

$$\bar{W} = \frac{W_1 + W_2}{2} = \frac{2D \tan \frac{\theta}{2} - d_1 \tan \alpha \tan \frac{\theta}{2} - d_2 \tan \alpha \tan \frac{\theta}{2}}{1 - (\tan \frac{\theta}{2})^2 (\tan \alpha)^2} \quad (9)$$

The corrected coverage widths are carried over into the formula for the overlap ratio to obtain the corrected η_{new} :

$$\eta_{new} = \frac{2(\frac{\bar{W}}{2} - \frac{d}{2})}{\bar{W}} = 1 - \frac{d}{\bar{W}} = 1 - \frac{d - d (\tan \frac{\theta}{2})^2}{2D \tan \frac{\theta}{2} - d_1 \tan \alpha \tan \frac{\theta}{2} - d_2 \tan \alpha \tan \frac{\theta}{2}} \quad (10)$$

2.2. Solving the Model

Based on the calculations, the results obtained are shown in Table 1 below.

Table 1. Solution results

Distance of the line from the center/m	Depth of sea water/m	Width of coverage/m	Overlap with the previous line/%
-800	90.9487	315.7051	—
-600	85.7116	297.5256	34.7717
-400	80.4744	279.3460	30.6605
-200	75.2372	261.1665	25.9962
0	70	242.9870	20.6591
200	64.7628	224.8074	14.4923
400	59.5256	206.6279	72.8622
600	54.2884	188.4484	-1.2463
800	49.0513	170.2688	-11.5085

3. Model Optimization and Refinement

3.1. Improvements to the Model

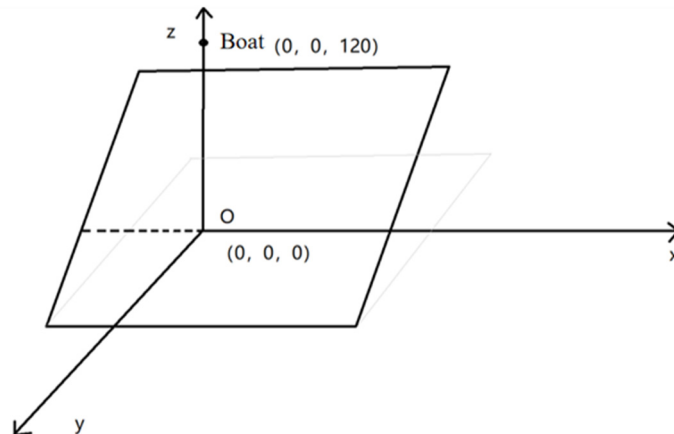


Figure 3. Finding the coordinate system of α_{new}

By analyzing the geometry after the addition of the angle of β , it is easy to obtain the slope inclination angle that needs to be constructed for the new slope triangle to be sought, which is the line-plane angle between the line perpendicular to the plane of the survey line and the slope and the horizontal plane. The new slope inclination angle is denoted as α_{new} , and its expression is obtained by establishing a coordinate system.

Let a ray perpendicular to the horizontal plane be made with the center of the sea as its endpoint, and note the intersection of the ray with the slope as the origin O of the coordinate system, as shown in Figure 3.

Because the depth of seawater is 120m at the center of the sea, and there is a survey boat right on the water at this time, so the coordinates of the boat can be recorded as (0,0,120). Let the coordinates of the normal vector \vec{n} of the slope be (A, B, C), because the y-axis is parallel to the line of intersection of the slope and the horizontal plane, and this intersection line is perpendicular to the normal vector \vec{n} , so the y-axis is perpendicular to the normal vector \vec{n} . Let the direction vector of the y-axis be (0,1,0). Set the normal vector \vec{m} of the horizontal plane to (0,0,1). Since the face-to-face angle between the slope and the horizontal plane is α , the equation can be linked based on the above two conditions:

$$\begin{cases} \vec{n} * (0,1,0) = 0 \\ \frac{\vec{n} * \vec{m}}{\sqrt{A^2+B^2+C^2} * 1} = \cos \alpha \end{cases} \quad (11)$$

Solve for the normal vector:

$$\vec{n} = (\sqrt{1 - (\cos \alpha)^2}, 0, \cos \alpha) \quad (12)$$

The projection vector \vec{n}_t of the normal vector \vec{n} on the XOY plane is:

$$(\sqrt{1 - (\cos \alpha)^2}, 0, 0) \quad (13)$$

The angle between the direction vector \vec{p} of the line of measurement and the projection vector of the normal vector \vec{n} on the xoy plane is β , set the projection vector of the direction vector of the line of measurement on the xoy plane $\vec{p}_t = (X, Y, 0)$, according to the angle between the two vectors β can be listed in the equation:

$$\frac{X(1 - (\cos \alpha)^2)}{\sqrt{X^2 + Y^2} \sqrt{(1 - (\cos \alpha)^2)^2}} = \cos \beta \quad (14)$$

Setting X to unit 1, then:

$$Y = \sqrt{1 - (\cos \beta)^2} \quad (15)$$

Therefore, the projection vector \vec{p}_t of the measured line vector is equal to:

$$(\cos \beta, \sqrt{1 - (\cos \beta)^2}, 0) \quad (16)$$

Calculation of the cover width also requires knowledge of the corresponding water depth as well as α_{new} . For α_{new} , the equation of the intersection line is obtained by associating the slope surface with the surface perpendicular to the survey line, and then calculating the line angle with the horizontal surface; for the depth of water, the above obtained equation of the intersection line is associated with the equation of the survey line, and the difference (120-z) is calculated to represent the depth of water. In the following, the slope equation is associated with the equation of the face perpendicular to the line of measurement:

$$\begin{cases} X \cos \beta + \sqrt{1 - (\cos \beta)^2} Y = 0 \\ \sqrt{1 - (\cos \alpha)^2} X + z \cos \alpha = 0 \end{cases} \quad (17)$$

Denote the normal vector of the horizontal plane as (0,0,1), so the line surface angle $\sin \alpha_{new}$ is:

$$\frac{\frac{-\sqrt{1-(\cos \alpha)^2}}{\cos \alpha}}{\sqrt{1+\frac{(\cos \beta)^2}{1-(\cos \beta)^2}+\frac{1-(\cos \alpha)^2}{(\cos \alpha)^2}}} \quad (18)$$

When $\beta=0$, $a_{\text{new}} = 1.5^\circ$. Since the depth relationship is determined by the difference in coordinates of the z-axis, the depth associated with the z-coordinate of a point on the slope surface is $(120-z)$. To calculate the depth expression for each corresponding point on the slope surface, it is necessary to first solve for the normal vector \vec{l} of the plane perpendicular to the survey line. It is easy to know from simple geometric relations that the direction vector $(0,0,1)$ of the z-axis is perpendicular to \vec{l} and the direction vector \vec{p}_t of the survey line is perpendicular to \vec{l} . Set $\vec{l} = (x, y, z)$ so can be associated with the relationship:

$$\begin{cases} z = 0 \\ x \cos \beta + y \sqrt{1 - (\cos \beta)^2} = 0 \end{cases} \quad (19)$$

Assigning unit 1 to x, then $\vec{l} = (\frac{-\sqrt{1-(\cos \beta)^2}}{\cos \beta}, 1, 0)$.

The equation of the plane in which the beam is located can be found by the Equation (19).

$$x \frac{-\sqrt{1-(\cos \beta)^2}}{\cos \beta} + y = 0 \quad (20)$$

Combined with the slope equations associatively:

$$\begin{cases} x \frac{-\sqrt{1-(\cos \beta)^2}}{\cos \beta} + y = 0 \\ x \sqrt{1 - (\cos \alpha)^2} + z \cos \alpha = 0 \end{cases} \quad (21)$$

Solve the expression for the z-coordinates of the points on the slope surface:

$$z = \frac{-\sqrt{1-(\cos \alpha)^2}}{\cos \alpha} x \quad (22)$$

Therefore, the depth of seawater D is:

$$D = 120 + \frac{\sqrt{1-(\cos \alpha)^2}}{\cos \alpha} x \quad (23)$$

3.2. Results of the Optimized Solution

Bringing the given data into the above equation separately, the equation for x versus the distance L from the center of the sea given in the table below is:

$$x = L \cos \beta \quad (24)$$

Table 2. Calculation result

Coverage width/m	Nautical mile from the center of the sea by the survey vessel/Distance								
	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	
Measuring line direction angle/ $^\circ$	0	451.132	505.828	560.523	615.219	669.915	724.610	779.306	834.002
	45	415.520	451.143	486.765	522.388	558.010	593.633	629.256	664.878
	90	415.348	415.348	415.348	415.348	415.348	415.348	415.348	415.348
	135	415.520	379.897	344.275	308.652	273.030	237.407	201.784	166.162
	180	415.692	365.293	314.894	264.496	214.097	163.698	113.299	62.900
	225	415.520	379.897	344.275	308.652	273.030	237.407	201.784	166.162
	270	415.348	415.348	415.348	415.348	415.348	415.348	415.348	415.348
	315	415.520	451.143	486.765	522.388	558.010	593.633	629.256	664.878

The calculation results are shown in Table 2.

4. Conclusion

Through the derivation of the definition of the overlap rate, the influence of the horizontal plane and the slope on the overlap rate in the multibeam line measurement problem is clarified, and it is concluded that the slope will affect the coverage width of the beam so that it is no longer a fixed value like the horizontal plane, and the coverage widths of the two times before and after the beam will be changed in a certain way so that they are different, and then it is necessary to take the average of the two times before and after the coverage widths by human correction for the calculation. Through the combination of numerical and morphological methods, the coverage width W is calculated as a function of slope inclination angle α , transducer opening angle θ and water depth D , and as a function of the water depth D_i of different measurement lines and the spacing d_i between the two measurement lines. Integration of the two functional relationships yields expressions for the coverage widths of the different lines and their corresponding water depths, slope inclination angles, and transducer opening angles. The resulting expression for the coverage width W is corrected, averaged and brought into the definition equation for the overlap rate η , resulting in a final expression to calculate the corresponding value.

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