

Research on Submersible Position Prediction Based on Kalman Filter Algorithm

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Abstract. The global situation of the world's submersible technology is experiencing a boom, with countries competing to develop advanced submersible equipment to explore the deep sea, undersea resources, and uncharted territories. With the wide range of applications of submersibles in various fields, the risks of communication loss and mechanical failures faced during their missions require the establishment of accurate predictive models of submersible positions to ensure safe and efficient rescue. In this study, a model containing equations of motion is developed from the motion characteristics of a submersible on the seafloor. Uncertainties such as sea currents, topography, and seawater density are considered, and a random walk model is used to simulate these effects to ensure that the model is closer to reality. Subsequently, combined with the Kalman filter algorithm, a time-dependent position prediction model is constructed in this study. The model can predict the position of the submersible in real-time, and after error analysis, it shows that the average variance and root mean square error are at a low level, which proves that its prediction is accurate and reliable, and provides strong technical support for the safe rescue of the submersible.

Keywords: Random Walk Model, Kalman Filtering Algorithm, Location Prediction Model, Error Analysis.

1. Introduction

With the deepening of marine resource development and scientific research, submersibles are increasingly widely used in such fields as ocean exploration, resource exploration, environmental monitoring, and scientific research. However, submersibles often face risks such as loss of communication with the main ship and mechanical failures during their missions. To ensure the safe operation and efficient rescue of submersibles, it is necessary to establish an accurate submersible position prediction model, so that the position of the submersible [3] can be quickly and accurately localized and rescue can be carried out in the event of emergencies.

To prevent the submarine from losing communication with the main ship and possible mechanical defects, we constructed a predictive model of the submarine's position on the seafloor based on the Kalman filtering algorithm [1]. The Kalman filtering algorithm, proposed by Kalman in 1960, is a powerful recursive algorithm designed to optimally estimate the state of a linear dynamic system based on measurement data containing noise. The algorithm is widely used in several fields, including deep sea exploration, autonomous and unmanned vehicles, aerospace, intelligent transportation systems, robotics, military and security, and logistics tracking. With the Kalman filtering algorithm, these fields can more accurately predict and control system states [2] and improve system performance and stability.

Firstly, a model of the submersible's motion on the seafloor is established using its equations of motion on the seafloor [5]. Considering that the submersible [6] is affected by uncertainties such as currents, seafloor topography, and seawater density, the effects of uncertainties on the submersible's motion are simulated by a random walk model [4], and finally, a predictive model of the submersible's position over time is established using the Kalman filtering algorithm. We infer from the parameters required for modeling that the submersible can periodically send current speed, surface current speed,



sea surface temperature, water temperature, salinity, and depth information to the main vessel to reduce uncertainty, and that obtaining this information requires the submersible to be equipped with an acoustic Doppler current profile, conductivity, temperature, and depth sensors, and drifting buoys.

2. Data Preparation and Uncertainty Analysis

The data for this study was obtained from <https://www.ncei.noaa.gov/news/the-underwater-world-bathymetric-data>.

Clearly define the objective of predicting the position of the submersible, including the time frame, accuracy requirements, etc., such as the maximum dive depth, maximum surface depth, and maximum velocity of the submersible.

2.1. Description of the state of motion of points in three-dimensional space

1. Position: expressed as a point in a three-dimensional coordinate system, such as (x,y,z) using the Cartesian coordinate system.

2. Velocity: Velocity is the rate of change of position concerning time, and is also a vector, indicating direction and magnitude. In three dimensions, velocity can be expressed as v_x , v_y , and v_z .

3. Acceleration: acceleration is the rate of change of velocity with time, also a vector.

For rotational motion, you can add: Angular velocity: angular velocity is a measure of the rate of rotation of an object, expressed as a vector, with the direction following the right-hand rule. Angular acceleration: angular acceleration is the rate of change of angular velocity with time, expressed as a vector.

2.2. Uncertainty analysis

In this paper, it is found that environmental factors and parameter uncertainties have a significant impact on submersible position prediction.

Currents and currents in the ocean can have an impact on the path of a submersible.

Changes in the density and temperature of seawater may also lead to changes in the position of the submersible.

The complexity and topographic relief of the seafloor terrain can also affect the location of submersibles, for example, the path of movement of submersibles in areas of submarine mountains or submarine canyons may be restricted.

Biological communities and biological activity in the ocean may also affect the movement path of the submersible, as some marine organisms may attract or influence the movement of the submersible.

This uncertainty may have a significant impact on the prediction of the position of the submersible. For example, the initial position and velocity of the submersible, the parameters of the marine environment (eg., seawater density, temperature, etc.), and the characteristics of the submersible itself (eg., the performance parameters of the mechanical system) may have some uncertainties. When there are errors or uncertainties in the values of these parameters, it will lead to a certain uncertainty in the results of the prediction of the position of the submersible as well. This uncertainty may bias the prediction results or even lead to inaccuracy of the prediction results.

3. Kalman Filter for State Estimation

3.1. Initial modeling

For the movement of the submersible in the seabed this paper simply use the velocity and time formula to determine the change rule of the submersible position with time.

$$\begin{cases} V_{XN} = V_X + a_X t \\ V_{YN} = V_Y + a_Y t \\ V_{ZN} = V_Z + a_Z t \end{cases} \quad (1)$$

$$\begin{cases} X_N = X + V_X t + 0.5 \cdot a_X t^2 \\ Y_N = Y + V_Y t + 0.5 \cdot a_Y t^2 \\ Z_N = Z + V_Z t + 0.5 \cdot a_Z t^2 \end{cases} \quad (2)$$

3.2. Normal Fault-Free Location Prediction Model

Equations of motion of a submarine: The equations of motion of a submarine are equations describing the relationship between the submarine's position, velocity, acceleration, angular velocity, and other physical quantities, and can be expressed in terms of Newton's second law and Euler's equation, as follows:

$$m \frac{dv}{dt} = F - mg - ma_r - D(v - v_w) \quad (3)$$

$$I \frac{d\omega}{dt} = M - \omega I_\omega - C(\omega - \omega_w) \quad (4)$$

Flow equations for ocean currents: The flow equations for ocean currents are equations describing the relationship between the physical quantities of seawater, such as velocity, pressure, density, temperature, etc., and can be expressed in terms of the Navier-Stokes equations and the continuity equations, as follows:

$$\rho \frac{Dv_w}{Dt} = (-Vp) + \rho g + \mu V^2 v_w \quad (5)$$

$$\nabla \cdot v_w = 0 \quad (6)$$

Topographic equation of the sea floor: The topographic equation of the sea floor is an equation describing the relationship between the height, slope, curvature, and other geometric quantities of the sea floor. It can be expressed by a partial differential equation as follows:

$$\frac{\partial^2 h}{\partial x^2} \frac{\partial^2 h}{\partial y^2} = -k \quad (7)$$

$$\frac{\partial h}{\partial x} \frac{\partial^2 h}{\partial x \partial y} - \frac{\partial h \partial^2}{\partial y \partial x^2} = r \quad (8)$$

To more realistically model the stochastic motion of a submarine under the influence of ocean currents. In this paper, it is assumed that the submarine can move in a random direction at each time step as a random perturbation of ocean currents.

The Kalman filter assumes that the state of a system at the current moment is transformed from the state and inputs of the previous moment, and of course, this process contains noise; in this paper, a sensor can be used to measure this state, but the measurement process is also noisy. Then, the system can be defined as follows:

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{u}(k) + \mathbf{w}(k) \quad (9)$$

$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{y}(k) \quad (10)$$

The situation now is that there are already measurements $\mathbf{z}(k)$ and one wants to estimate the true state of the system $\mathbf{x}(k)$.

First, establish the forecasting process:

$$\mathbf{x}(k|k-1) = \mathbf{A}\mathbf{x}(k-1|k-1) + \mathbf{B}\mathbf{u}(k) \quad (11)$$

$$\mathbf{P}(k|k-1) = \mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^T + \mathbf{Q} \quad (12)$$

The filtering process is then established:

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{H}^T \cdot [\mathbf{H}\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{R}]^{-1} \quad (13)$$

$$\mathbf{x}(k|k) = \mathbf{x}(k|k-1) + \mathbf{K}(k)[\mathbf{z}(k) - \mathbf{H}\mathbf{x}(k|k-1)] \quad (14)$$

$$\mathbf{P}(k|k) = [\mathbf{I} - \mathbf{K}(k)\mathbf{H}]\mathbf{P}(k|k-1) \quad (15)$$

3.3. Predictive Modeling of Position with Loss of Propulsion

Consider a situation where a submarine may lose propulsion. This could be due to mechanical failure, fuel exhaustion, etc.

If the submarine can rotate, consider the effect of rotation on its motion.

$$\frac{d\mathbf{v}_x}{dt} = -\mathbf{k}_r \cdot \mathbf{v}_x(t) \quad (16)$$

where \mathbf{k}_r is the parameter associated with the loss of propulsion.

Stochastic processes: Stochastic processes are introduced to represent uncertainties in the external environment. This can include randomness in ocean currents, uncertainty in water density, etc.

Uncertainty in model parameters: Consider parameter uncertainty in a dynamical model.

$$\begin{cases} \frac{dx}{dt} = f_x(x, y, z, t) + \epsilon_x(t) \\ \frac{dy}{dt} = f_y(x, y, z, t) + \epsilon_y(t) \\ \frac{dz}{dt} = f_z(x, y, z, t) + \epsilon_z(t) \end{cases} \quad (17)$$

Where $\epsilon_x(t), \epsilon_y(t),$ and $\epsilon_z(t)$ are stochastic processes representing uncertainty.

Position Information Sending Event: Model when the submersible will be able to send position information to the primary vessel. This may involve limitations in communications equipment, dive depth limitations, etc.

Communication Event Functions:

$$g(t) = \begin{cases} 1, & \text{If a communication event occurs} \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

The function may depend on time, depth, or other relevant factors.

4. Numerical Calculation and Visualization of Position Prediction Model

For such a complex position prediction model, the process of numerical computation and visualization of the above equations by computer can be done by discretizing the equations by finite difference method, finite element method, finite volume method, etc., and then solving the equations by iterative, direct, and relaxation methods, etc., and finally displaying the results by graphical software.

4.1. Simulation of the Ionian Seafloor

By processing the data in the Ionian Sea, the DEM display of the seafloor topography of the Ionian Sea was obtained using MATLAB as follows in Figure 1.

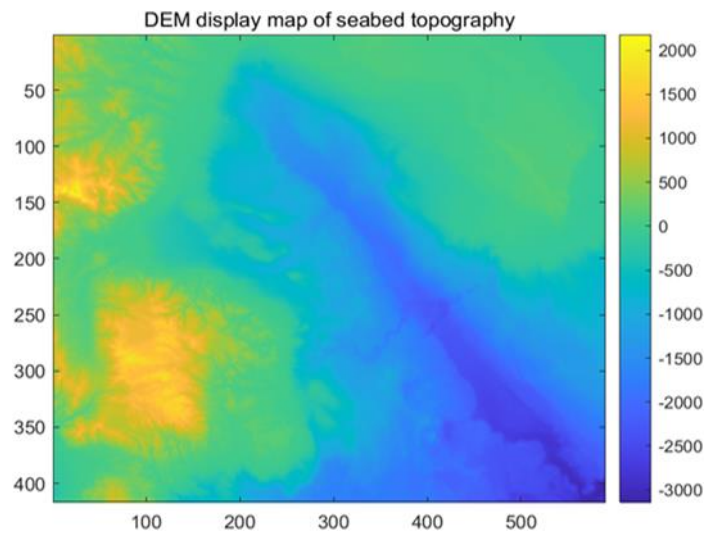


Figure 1. Neural network structure

Continuing with the 3D simulation of the seabed using MATLAB, the environment and conditions required for the simulation were first constructed to ensure the smooth progress of the following simulation. As shown in Figure 2:

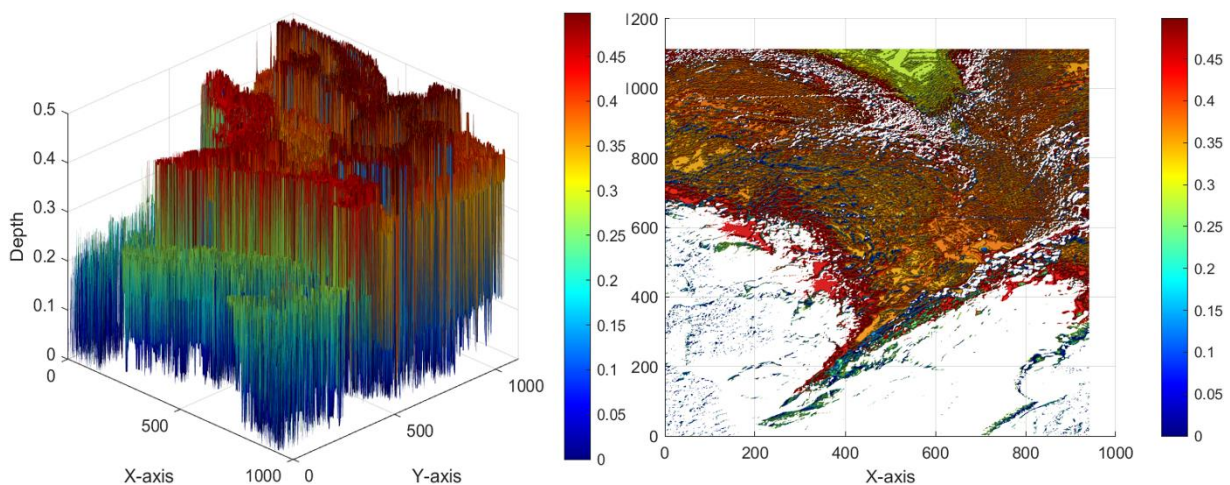


Figure 2. Submarine geomorphologic chart and Seafloor topographic map x-y

4.2. Simulation Modeling of Submersible Motion Position

Using the MCMC algorithm for stochastic simulation of currents, seawater density, and seafloor topography, a simulation model of the position of the submersible running on the seafloor over time can be obtained, and a random walk model is carried out with MATLAB to simulate the random effects of currents, and the simulation of the effects of the currents on the movement of the submersible to make its model closer to the real environment. The simulated model and the inverted diagram are shown in Figure 3.

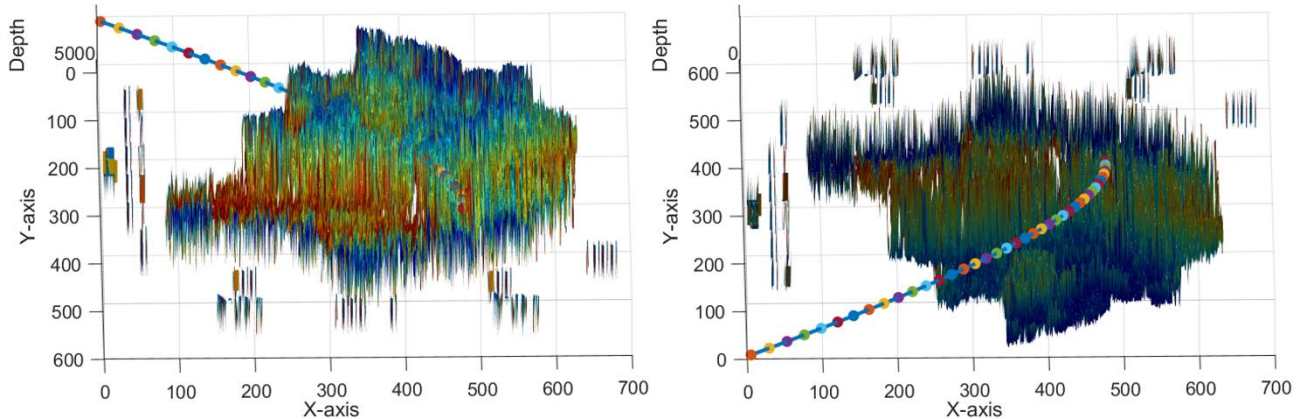


Figure 3 Three-dimensional motion diagram of the submarine of submarine (inverted)

4.3. Predictive modeling

The prediction of a submersible's trajectory on the seafloor can be performed based on the Kalman filter principle. Kalman filter is a recursive estimation algorithm that can be used to estimate the state of the system, especially in the presence of noise effects. In the prediction of the motion trajectory of the submersible, the Kalman filter can help this paper to estimate the position and velocity of the submersible based on the system model and the observation data, to realize the prediction and tracking of the motion trajectory of the submersible.

Specifically, the prediction of the motion trajectory of the submersible on the seabed based on the Kalman filter principle can be divided into the following steps:

- (1) Determine the system model: firstly, the motion model of the submersible needs to be established, including the state transfer equation and the observation equation. The state transfer equation describes the motion law of the submersible, and the observation equation describes how the position information of the submersible is observed from the sensor data.
- (2) Initialization of state estimates and covariance: the initial state estimates and covariance matrix need to be initialized through the Kalman filter.
- (3) Prediction step: based on the system model and the current state estimate, the state is predicted using the state transfer equation, and the covariance matrix of the state estimate is updated.
- (4) Update step: based on the observed data and the predicted state estimate, update the state using the observation equation to obtain a more accurate state estimate and covariance matrix.
- (5) Repeat prediction and update step: Based on the real-time observation data, the prediction and update step is repeated continuously to realize the continuous prediction and tracking of the motion trajectory of the submersible.

The three-dimensional model was built by MATLAB using Kalman filter algorithm and the results are shown in Figure 4 below, error analysis is carried out, and the results are shown in Figure 5

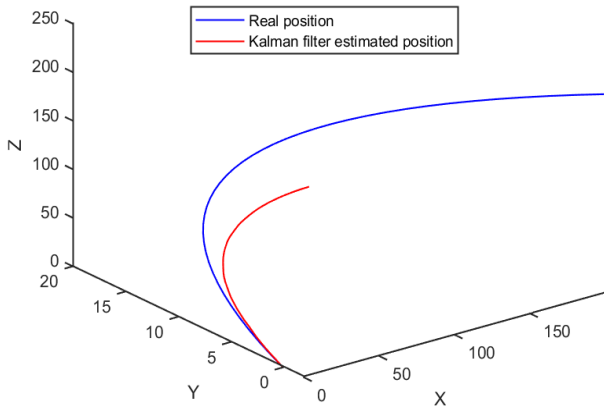


Figure 4 True and predicted trajectory diagrams

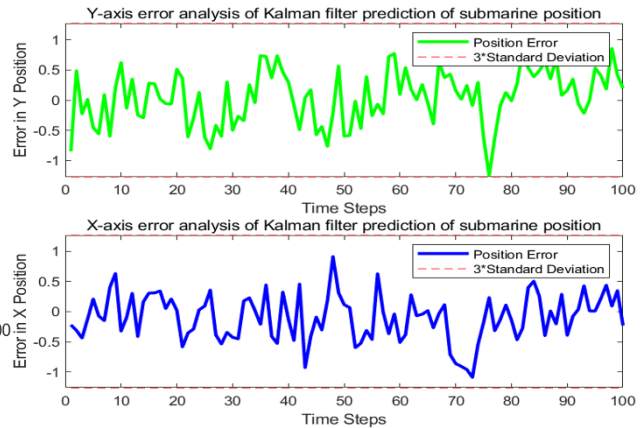


Figure 5 Error analysis

5. Conclusions

This paper details the construction and application of a submarine seafloor position prediction model based on the Kalman filter algorithm. By comprehensively considering the uncertainty factors such as sea currents, seafloor topography, seawater density, etc., and combining with the motion characteristics of the submersible, this study successfully establishes an efficient prediction model for real-time tracking of the submersible's positional changes on the seafloor. After rigorous error analysis, the model exhibits satisfactory prediction accuracy with an average variance of $[-0.0937, 0.0446, 0.0134]$ and a root mean square error of $[0.3928, 0.4437, 0.3791]$. To further improve the prediction performance, this paper proposes a strategy: the submersible will periodically send real-time environmental data including current velocity, sea surface flow rate, sea surface temperature, water temperature, salinity, and depth to the main vessel. These data will be accurately acquired by the submersible equipped with advanced equipment such as acoustic Doppler current profilers, conductivity, temperature and depth sensors, and drifting buoys. Through the implementation of this strategy, this paper expects to significantly reduce the influence of uncertainty factors on the prediction results, and thus significantly improve the accuracy and reliability of the submersible's position prediction, providing strong technical support for ocean exploration activities.

In this paper, we propose a research idea and framework based on Kalman filter algorithm and apply it to the field related to marine emergency rescue. By comprehensively considering the complexity and uncertainty of the seabed environment, a model capable of predicting the position change of a submersible in real-time is constructed in this study. The model not only demonstrates a high degree of prediction accuracy but also verifies its feasibility in practical applications. This research provides efficient and reliable technical support for the field of marine emergency rescue, which is expected to significantly improve the efficiency and success rate of emergency rescue and contribute to marine safety and development.

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