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Study on the Influence of Sex Ratio on Ecology Based on Improved Lotka-Volterra Model

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Abstract. In a pioneering study of lamprey population dynamics, we employ an enhanced Lotka-Volterra model to illuminate the profound influence of sex ratio shifts on ecosystems. This exploration delves into how such changes affect resource availability, ecosystem equilibrium, and sustainability. We employ dynamic differential equations to simulate population fluctuations among males, females, and predators, incorporating elements like birth rate, mortality rate, competition coefficient, mating success rate, and predation rate. The differential equations are numerically resolved using the Runge-Kutta method, enabling us to analyze population dynamics across various scenarios. Concurrently, we construct Jacobian matrices to appraise ecosystem stability and equilibrium points. The refined Lotka-Volterra model effectively encapsulates the impact of sex ratio variations on reproductive and predation processes, offering profound insights into predator-prey interactions and ecosystem dynamics. Our pivotal discoveries underscore the influence of sex ratio modifications on reproductive capacity, population growth rates, social structure, genetic diversity, and predator-prey relationships. This study emphasizes the criticality of comprehending the repercussions of sex ratio shifts on ecosystems and communities.

Keywords: Lampreys, Sex ratio model, Lotka-Volterra model.

1. Introduction

Changes in the sex ratio are widespread in the animal kingdom, and changes in the adaptive sex ratio are a significant phenomenon. Sex ratio variation is important in influencing communities and ecosystems [1]. Take lamprey as an example. Under the influence of food supply, the sex ratio in the larval stage also changes, which is characterized by changes in the adaptive sex ratio. This paper established a model to study the impact of adjusting the sex ratio of lamprey populations on resource availability, and investigated the impact of this phenomenon on the ecosystem from various aspects.

Lampreys are the top predators in deep-sea ecosystems, and its sex ratio dynamics have been attracting much attention. Changes in the sex ratio may be influenced by factors such as food availability, but its specific impact on the ecosystem is unknown. Therefore, this study aims to gain insight into the ecological impact of sex ratio changes on lampreys and their habitats. While some animal species exist outside of the usual male and female sexes, most species are essentially female or male. This is known as adaptive sex ratio changes. In some lake habitats are thought to be parasites that have a significant impact on the ecosystem, and lampreys are also food sources in some parts of the world [2], such as Scandinavia, the Baltic Sea, and some indigenous peoples of the Pacific Northwest of North America.

The sex ratio of marine lampreys may vary depending on the external environment [3]. The growth rate of marine lampreys during the larval stage determines whether they become female or male [4]. The growth rate of these larvae is affected by the food supply. We are concerned with the sex ratio and its dependence on local conditions, particularly for marine lampreys.

Lampreys, as apex predators, play a crucial role in shaping ecosystems. Their ability to adjust sex ratios has a profound impact on population dynamics, influencing not only lampreys themselves but also the broader ecosystem. These changes in sex ratios trigger fluctuations in population sizes, which, in turn, affect the species that interact with lampreys, leading to alterations in the overall stability of the ecosystem. Studying the mechanisms of sex determination in lampreys can provide valuable insights into the underlying principles of sex determination in other vertebrates. This knowledge can lead to innovative approaches for managing species where they are valued or when they become invasive [5]. While various models have been used to understand the dynamics of predator-prey relationships, the Rosenzweig-MacArthur model has been associated with the risk of over-predation [6]. On the other hand, the MSOMs model, although widely used, has limitations due to its statistical framework, including computational challenges with Bayesian inference and issues with model selection [7]. In contrast, the sex ratio change model employed in this paper, coupled with the improved Lotka-Volterra model, offers several advantages. It is a simple model that requires minimal data, yet it effectively simulates population dynamics under different scenarios. This approach provides a valuable tool for studying and predicting the impacts of sex ratio changes in lampreys and their cascading effects on the larger ecosystem.

2. Sex ratio model

2.1. Dynamic differential equations

Differential equations are mathematical equations that describe the relationship between variables in a natural phenomenon or process. In ecology, differential and functional differential equations also play an important role [8], they are widely used to describe the dynamics of biological populations, such as changes in population size over time and interactions between different individuals within a population. For this purpose, we constructed dynamic differential equations for population changes of males, females, and predators in an ecosystem The model contains three main populations: males (M), females (F), and predators (P), and the growth rate of each population is affected by the natural birth rate, the natural death rate, the natural mortality rate, the coefficient of competition, and other factors (e.g., the mating success rate and the predation rates).

$$\frac{dM}{dt} = r_M M - d_M M + b_M \alpha M \tag{1}$$

$$\frac{dF}{dt} = r_F F - d_F F + b_F \alpha F \tag{2}$$

$$\frac{dP}{dt} = r_P P - aP(M + F) \tag{3}$$

where r is birth rate, d is death rate, b is competition coefficient, α is mating success, a is predation rate, t is time.

After constructing the differential equation, the equation needs to be solved to obtain the pattern of population change. The Runge-Kutta method is a method used to numerically solve a group of ordinary differential equations, the most commonly used of which is the classical fourth-order Runge-Kutta method. This method approximates the solution of the differential equation by stepwise approximation to obtain a numerical solution. The Runge-Kutta method approximates the solution of the differential equation by calculating multiple slopes. By using different slope estimates, the truncation error can be reduced, thus improving the accuracy of the numerical solution. The basic idea of the Runge-Kutta method is to divide the process of solving a differential equation into several small steps, each of which calculates a slope to estimate the rate of change at that point. Then, by weighting these slopes, the solution for the next time step is obtained. This process is repeated until the desired time point or interval is reached. The fourth-order Runge-Kutta method, the most commonly used variant, uses four different slope estimates. First, calculate a slope at the beginning of the current time step. Then, use that slope to estimate the slope of the intermediate time point and calculate the corresponding solution. Next, use that solution to calculate the slope of the intermediate time point and get a more accurate solution. Finally, use this more accurate solution to calculate the slope at the end of the current time step and get the final solution. By using the fourth-order Runge-Kutta method, more accurate solutions can be obtained than other numerical methods, and it has a small truncation error when approximating the true solution. This makes the Runge-Kutta method a common tool for solving differential equations, especially in numerical simulation and scientific calculation.

By using four slope estimates (K1, K2, K3, K4) [9] and combining them with specific weights, the fourth-order Runge-Kutta method is able to achieve a high degree of accuracy. The steps are shown in the Figure1.

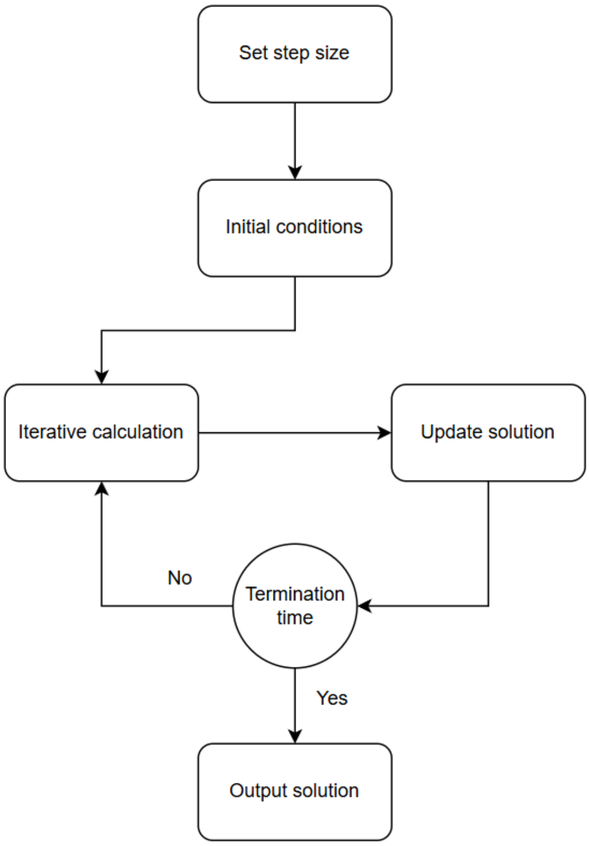

Figure 1. Runge-Kutta method solution steps

By solving the equation with ode45 [10] we can simulate population changes under different resource conditions. This ensures the accuracy and efficiency of the simulations and helps us to understand how resource availability affects sex ratios and their wider ecosystem impacts. By further analyzing the simulations generated by the ode45 method, changes in population dynamics under different parameter settings can be assessed in detail. This includes understanding how changes in sex ratios affect the reproductive capacity of populations, competition for resources, and the balance of predator-prey relationships.

2.2. Jacobian matrix and ecosystem balance points

Jacobian matrices play an important role in the nonlinear stability analysis of deterministic and stochastic systems [11]. Jacobian matrix is a matrix which contains all the first order partial derivatives of a multivariate function.

Further ecosystems are analyzed for stability based on the established ecological differential equations, and in order to assess the stability of the system, it is necessary to find the equilibrium points of the system, i.e., the values of the variables in the system when the right-hand side of all the differential equations is zero. These equilibrium points represent the steady state that the system may reach.

The Jacobian matrix is then constructed near the equilibrium points by linearizing the system. The Jacobi matrix contains local information about the dynamics of the system, and by analyzing its eigenvalues it is possible to understand the stability of the system near the equilibrium point. Jacobi Matrix

$$\begin{bmatrix} \alpha * b_M - d_M + r_M & 0 & 0 \\ 0 & \alpha * b_F - d_F + r_F & 0 \\ -a * P(t) & -a * P(t) & -a * \left(F(t) + M(t)\right) + r_P \end{bmatrix} \tag{4}$$

A negative real part of the eigenvalue indicates that the system is stable at that equilibrium point, while a positive real part of the eigenvalue indicates that the system is unstable. Stability analysis can help predict the behavior of the system such as stability, oscillations or divergence.

2.3. Improved Lotka-Volterra model

Lotka-Volterra [12] equations alias predator-prey equations. Consists of two first-order nonlinear differential equations. Often used to describe the dynamics of a biological system in which a predator interacts with its prey, i.e., the increase or decrease in the size of their community.

$$\frac{dx}{dt} = \alpha x - \beta xy \tag{5}$$

where $\frac{dx}{dt}$ is the rate of change in prey population size, α is the natural growth rate of prey, and βxy is the reduction in prey size due to predation.

$$\frac{dy}{dt} = \delta xy - Yy \tag{6}$$

where $\frac{dy}{dt}$ denotes the rate of change in predator population size, δxy denotes the increase in predator size due to food capture, and Yy is the natural mortality rate of predators.

Combining the Lotka-Volterra equation with the dynamic differential equation constructed above yields an improved Lotka-Volterra equation, which adds the effect of sex ratio compared to the Lotka-Volterra equation population dynamics of predators and prey. The effect of sex ratio on reproduction and predation processes in Lampreys or other organisms can be better captured.

$$\frac{dF}{dt} = \beta F \left(1 - \alpha \frac{M}{F}\right) \tag{7}$$

$$\frac{dM}{dt} = \beta M \left(1 - \alpha \frac{F}{M}\right) \tag{8}$$

$$\frac{dP}{dt} = r_P P - a_P(M + F) \tag{9}$$

where, β is the birth rate of prey, α is the coefficient of influence of competition between male and female prey.

Five differential equations were also used to describe the dynamics of predator and prey populations, including sex ratio

$$\frac{dF}{dt} = \beta F\left(1 - \alpha\frac{M}{F}\right) \quad (10)$$

$$\frac{dM}{dt} = \beta M\left(1 - \alpha\frac{F}{M}\right) \quad (11)$$

$$\frac{dF}{dt} = r_F F - d_F F + b_F \alpha F \quad (12)$$

$$\frac{dM}{dt} = r_M M - d_M M + b_M \alpha M \quad (13)$$

$$\frac{dP}{dt} = r_P P - aP(M + F) \quad (14)$$

where, F and M represent the number of females and males in the predator population, f and m are the number of females and males in the prey population, respectively, and P is the total number of prey. β stands for predator fecundity, α represents the sex ratio influence factor, r_M and r_F represent the natural mortality of female and male preys, d_M and d_F represent female and male prey reproduction rates, a represents predation efficiency, r_P represents the growth rate of the prey population.

3. Results

3.1. Sex ratio model

By inputting a dynamic differential equation model of the Lampreys population into ode45, we can simulate population changes under different resource conditions. The method is applied to calculate step size, which ensures the accuracy and efficiency of the simulation and helps us to understand how resource availability affects sex ratio and its wider ecosystem impacts.

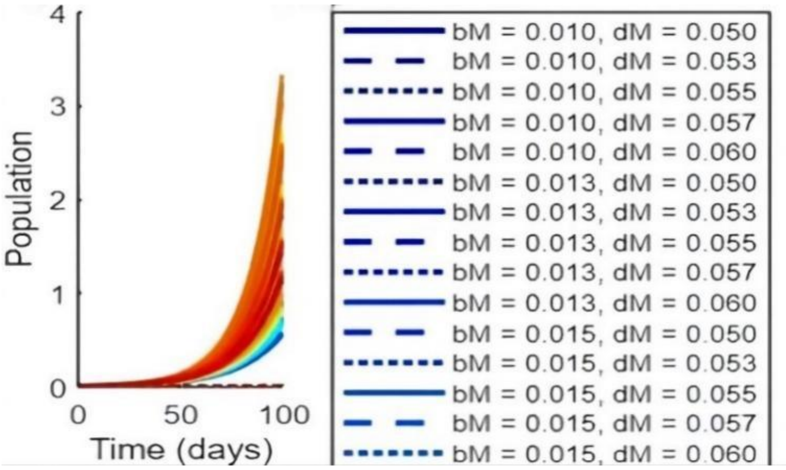

Figure 2. Population Dynamics in Changing Environments

Based on the model and Figure 2, numerical changes in hypothetical populations over time are shown for different competition coefficients (bM) and natural mortality (dM) conditions. At low values of bM and dM, population growth peaks and then declines slowly, reflecting the fact that populations can be maintained for a period of time under certain conditions of resource competition and mortality. However, as bM and dM values increased, the population declined rapidly after peaking, indicating that higher competitive pressures and mortality rates accelerated the population decline, which may indicate that populations are more susceptible to negative impacts and less likely to stabilize over time in resource-limited environments.

The model further suggests that the peak and subsequent decline rates of a population are closely related to the magnitude of the coefficient of competition and natural mortality. An increase in the coefficient of competition implies an increase in the intensity of competition between individuals for resources.

3.2. Ecological balance points

In real ecosystems, we usually look for non-trivial equilibria where the population of at least one species is not zero. In order to find such an equilibrium, we usually need more information, such as specific values for growth and mortality, or at least the proportionality of these parameters.

To continue the analysis, we need to set some specific values for the model parameters. But here, we can assume the existence of such a non-trivial equilibrium point and continue our analysis with the local stability of this point. To do this, we need to compute the Jacobi matrix and analyze its eigenvalues. We will first construct the symbolic representation of the Jacobi matrix and then assign specific values to the computed parameters.

Given parameters set parameters. r_M= 0.1 % male birth rate parameter. d_M= 0.05% male natural mortality rate parameter. b_M= 0.01; % male coefficient of competition parameter. α= 0.75 % mating success parameter. r_F= 0.1 % female birth rate parameter. d_F= 0.05 % natural female mortality rate b_F= 0.01 % individual female competition coefficient r_P= 0.1 % predator birth rate = 0.1 % predation rate to resolve the initial value.

The trend of mass data in power system provides a basis for load characteristic analysis and prediction model establishment, but the classical load forecasting method cannot afford such a huge time and computing resource consumption. The problem of over fitting in large sample set will affect the prediction accuracy. In this paper, a power load forecasting model is built by using the BP neural network model, making full use of the powerful data processing function of Clementine and preventing the over fitting function. The experimental results show that the BP neural network model has good predictability and robustness, and has a certain practical application value.

$$\begin{bmatrix} 0.0575 & 0 & 0 \\ 0 & 0.0575 & 0 \\ -a * P(t) & -0.1 * P(t) & -0.1 * F(t) - 0.1 * M(t) + 0.1 \end{bmatrix} \tag{15}$$

Changes in sex ratios can have significant impacts on ecosystems, especially for species where sex ratios play an important role in reproductive success. In the simulation above, we can see that the number of males and females fluctuates randomly over time, and these fluctuations may have an impact on predator populations The following are some of the possible impacts of changes in sex ratio:

Changes in fecundity: if there is a significant decrease in the number of individuals of one sex, this may lead to a decrease in the fecundity of the population as a whole, as it becomes more difficult to find mates.

Decrease in population growth rate: When the number of individuals of one sex decreases, the population growth rate may be affected, especially if the decrease in the number of females is more pronounced, as the number of females usually limits the number of offspring.

Changes in social structure: In some species, imbalances in sex ratios may lead to changes in social structure and behavior, such as increased or decreased competition for mates.

Reduced genetic diversity: A significant reduction in the number of a species may lead to reduced genetic diversity, which may reduce the ability of populations to adapt to environmental changes.

Changes in predator-prey relationships: Changes in sex ratios may affect predator-prey relationships by decreasing reproductive rates and population sizes, which in turn may affect the structure and function of the overall food web.

3.3. Improved Lotka-Volterra model

The conclusion of ecosystem simulation based on model is as follows.

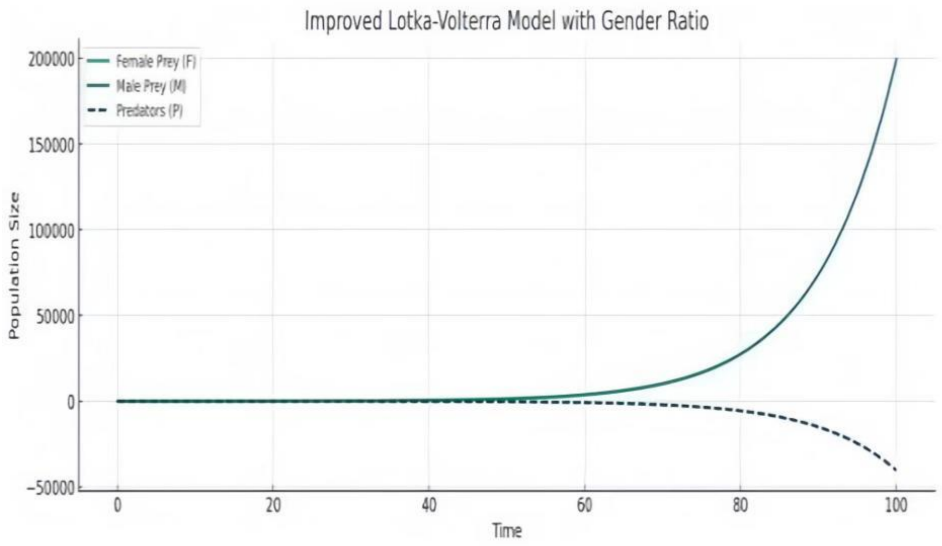

Figure 3. Sex Ratio Impact on Prey Population Dynamics

From the Figure 3 we can analyze the possible effects of changes in sex ratio on population dynamics. Changes in the sex ratio can affect prey fecundity, as it is often assumed that reproduction requires the involvement of both sexes. In the model, prey fecundity (β) is related to the sex ratio (male-female ratio), which can be interpreted as an imbalance in the sex ratio that may limit the reproductive potential of the population.

If the number of females decreases. (the sex ratio is unbalanced), then the overall fecundity of the prey population will decrease, even if the number of males remains the same, because fewer females will be available for breeding. Conversely, if the number of males decreases, then overall fecundity will also suffer. In the long term, this could lead to a reduction in prey populations, which would reduce the food source for predators and adversely affect predator populations.

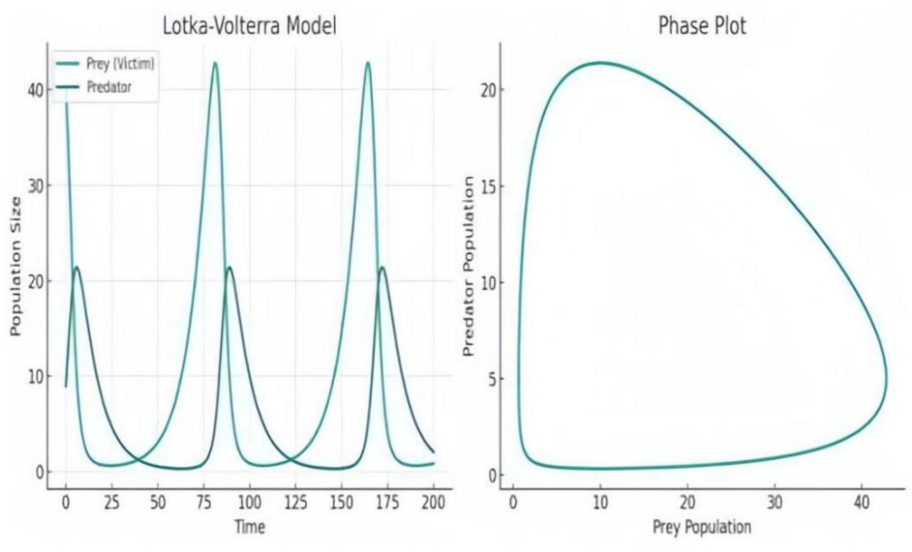

Figure 4. Dynamic Predator-Prey Population Simulation

Figure 4. shows a dynamic simulation of a predator and prey population based on the Lotka-Volterra model. The graph on the left shows the number of predators and prey for the predator, and the graph on the right is a phase diagram showing the relationship between the prey and predator populations.

In the Figure 4. on the left, we can see periodic changes in the number of predator (green) and prey (cyan) populations. An increase in the number of prey usually precedes an increase in the number of predators; similarly, a decrease in the number of prey precedes a decrease in the number of predators. This is because the abundance of prey directly affects the food source and the ability of predators to survive. ... When there is an abundance of prey, predators have more food sources, which leads to an increase in the number of predators. Conversely, when the amount of prey decreases, food becomes scarce and the number of predators decreases.

In the phase diagram on the right, we can observe a closed trajectory, which suggests that the dynamics of the predator and prey populations are cyclical and repetitive

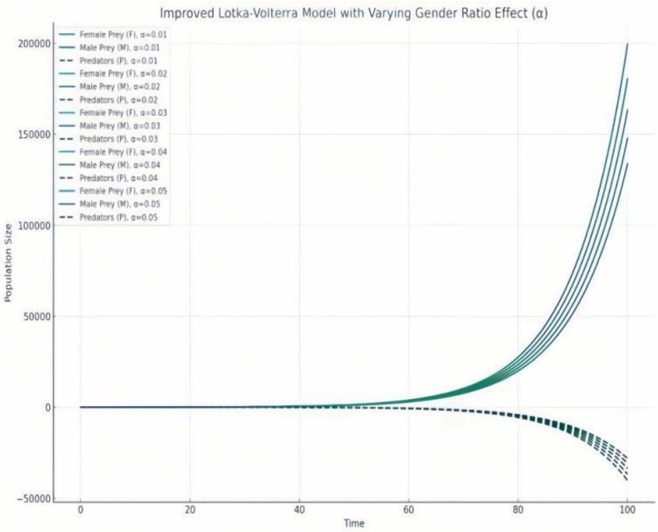

Figure 5. Sex Ratio Effects on Population Dynamics

This Figure 5 shows the effect of sex ratio effects (α) on population dynamics in a modified Lotka-Volterra model. The figure includes changes in female prey (F), male prey (M) and predator (P) populations for different values of α.

It can be seen that as the sex ratio influence coefficient increases, the number of prey populations (both female and male) undergoes a significant change in the number of prey populations, and the number of predator populations undergoes a change in the number of predators. Each curve represents the population dynamics at different values.

It can be seen that the population size tends to increase with increasing values, indicating that the sex ratio has a significant effect on the population size. At lower values, there is a rapid increase in the number of prey, followed by excessive predation pressure, which leads to a sharp decrease in the number of prey, followed by a decrease in the number of predators.

This pattern suggests that changes in sex ratios are important for maintaining ecosystem balance. An unbalanced sex ratio can lead to erratic fluctuations in prey or predator populations, which can be harmful to other species in the ecosystem. In contrast, a stable sex ratio helps to maintain population stability, which benefits the ecosystem as a whole.

4. Conclusion

In this study, we constructed a basic framework based on the Lotka-Volterra model, to which we added the effect of sex for exploring the interactions of the Lampreys with other species in its ecosystem, and considered the effects of changes in sex ratio on the ecosystem. Our model provides a simple but effective mathematical form capable of fitting the population dynamics of the Lampreys and the ecosystem as a whole by adjusting parameters. Although the model has some limitations, such as ignoring complex factors in the ecosystem and assumptions based on equilibrium states, it is important as a starting point for preliminary studies of ecosystem interactions.

In future studies, we can further improve the model by considering more complex factors, such as spatial structure, seasonal variations and time-lag effects on resources, to make the model more realistic and comprehensive. In addition, we can also explore the application of the model to a wider range of areas, such as environmental protection, resource management and biodiversity conservation. The ecological model can provide predictive tools for environmental protection to help managers better understand and manage natural resources, as well as support biodiversity conservation by assessing risks and designing effective conservation measures.

Overall, the ecological model constructed based on the Lotka-Volterra model has the potential to explore ecosystem interactions. At the same time, Parasitic lampreys occupy relatively high trophic levels in aquatic ecosystems can help us to have a better understanding of other populations with the help of this model. Future research can further improve the model, expand the application field, and provide more useful information and support for research and practice in the fields of ecology and environmental science.

References

[1] Fryxell D C, Arnett H A, Apgar T M, et al. Sex ratio variation shapes the ecological effects of a globally introduced freshwater fish[J]. Proceedings of the Royal Society B: Biological Sciences, 2015, 282(1817): 20151970.

[2] Hanel L, Andreska J, Dyldin Y V. Lampreys in human life, their cultural and folklore importance[J]. Human Soc Sci, 2022, 10: 300-315.

[3] Ren Y, Zeng J, Pan J. Modeling the Impact of Lamprey Population Dynamics on Aquatic Food Webs: A Leslie and Lotka-Volterra Approach[J]. Highlights in Science, Engineering and Technology, 2024, 98: 489-497.

[4] Johnson N S, Swink W D, Brenden T O. Field study suggests that sex determination in sea lamprey is directly influenced by larval growth rate[J]. Proceedings of the Royal Society B: Biological Sciences, 2017, 284(1851): 20170262.

[5] Johnson N S, Swink W D, Brenden T O. Field study suggests that sex determination in sea lamprey is directly influenced by larval growth rate[J]. Proceedings of the Royal Society B: Biological Sciences, 2017, 284(1851): 20170262.

[6] Lin X, Xu Y, Gao D, et al. Bifurcation and overexploitation in Rosenzweig-MacArthur model[J]. Discrete Continuous Dyn. Syst. Ser. B, 2023, 28: 690-706.

[7] Devarajan K, Morelli T L, Tenan S. Multi-species occupancy models: Review, roadmap, and recommendations[J]. Ecography, 2020, 43(11): 1612-1624.

[8] DiffeRihan F A. Delay differential equations and applications to biology[M]. Singapore: Springer, 2021.

[9] Sabawi Y A, Pirdawood M A, Sadeeq M I. A compact fourth-order implicit-explicit Runge-Kutta type method for solving diffusive Lotka–Volterra system[C]//Journal of Physics: Conference Series. IOP Publishing, 2021, 1999(1): 012103.

[10] Mathworks. Solve Nonstiff Differential Equations–Medium Order Method-MATLAB Ode45[J]. 2022.Weckwerth W. Toward a unification of system-theoretical principles in biology and ecology—the stochastic lyapunov matrix equation and its inverse application[J]. Frontiers in Applied Mathematics and Statistics, 2019, 5: 29.

[11] Tsukamoto H, Chung S J, Slotine J J E. Contraction theory for nonlinear stability analysis and learning-based control: A tutorial overview[J]. Annual Reviews in Control, 2021, 52: 135-169.

[12] Wang Z A, Xu J. On the Lotka–Volterra competition system with dynamical resources and density-dependent diffusion[J]. Journal of Mathematical Biology, 2021, 82: 1-37.