

Research on rapid SAR solution for submarines based on Particle Swarm Optimization

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Abstract. This paper uses particle swarm and neural network algorithms to construct an efficient submarine search and rescue (SAR) solution based on an unmanned underwater detector cluster, aiming to provide stronger safety guarantees for tourists who experience the submarine project. Specifically, this article constructs a position prediction model and a search optimization model respectively. The former will accurately simulate the submarine's motion trajectory and various parameters through a combination of interpolation fitting and neural network learning. The latter will use the above data to Particle swarm algorithm optimizes the group behavior of unmanned detectors to derive the optimal search and rescue plan. Finally, upon analyzing the success rate of this strategy, it's surprised to find that its increasing trends complies with Logistic growth law. This indicates that the strategy has promising advantages in terms of time efficiency and accuracy. The research conducted in this article provides an application direction for the particle swarm algorithm and a feasible model for the construction of deep-sea search and rescue strategies, which has a great use value.

Keywords: Particle Swarm Optimization; Neural Networks; Machine Learning; Logistic Grow.

1. Introduction

The evolution of technology has brought submarines out of the realm of the unknown and into the lives of everyday individuals. While the rise of submarine tourism offers exciting possibilities, it also presents previously overlooked safety concerns. Among these, communication disruptions stand as the most significant cause of submarine accidents. According to a business report [1], the global civilian submarine market reached USD 373.7 million in 2021, with a projected six-year growth of 192.8% to reach USD 1,094.24 million by 2027. However, this growing popularity highlights the need to address emerging safety hazards, particularly those related to communication breakdowns. Incomplete data suggests that submarine accidents claim dozens of lives on average, a stark reminder of the urgency to improve safety measures in this sector. There are two main directions to enhance civilian submarine safety. First and foremost, we can simply strengthen submarine itself. This involves reinforcing communication systems and essential structural components to improve resilience and reliability. In addition, we can establish comprehensive Search and Rescue Plans: Implementing a well-defined and thorough search and rescue protocol is crucial for ensuring a swift and effective response during an incident. With the main safety concerns and potential solutions identified for civilian submarines, constructing the corresponding mathematical models and engineering solutions becomes a crucial and achievable next step.

2. Submarine positioning and trajectory prediction

2.1. Model preparation

To define specific modelling methods and obtain accurate results, it was necessary to query and collect relevant real-world data, including data on ocean currents and seabed topography in the Ionian Sea [2] just like Fig 1, as well as the technical parameters of tourist submarines which shows in Table.1.

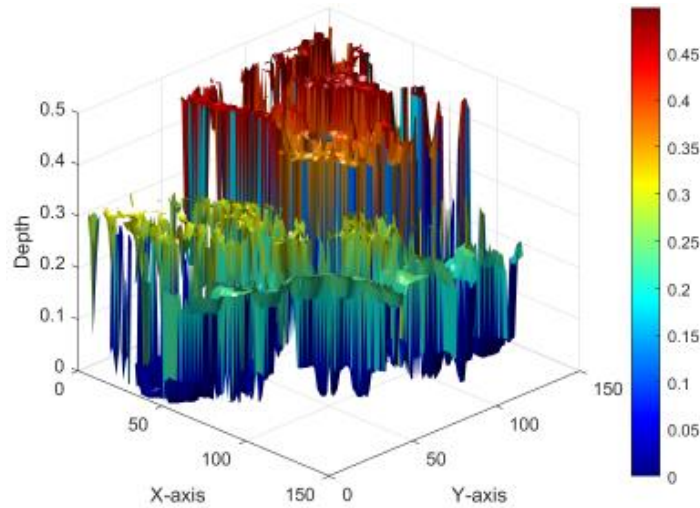


Figure 1. Ionian seabed

Table 1. A typical tourist submersible's Parameters

Trait	Value
Passengers	6
Operating depth	1000 meters
Size	4.55m (l) 3.6m (w) 2.5m (h)
Weight	11 tons
Cruising speed	3 knots
Autonomy	12 hours
Emergency endurance	10+hours

2.2. Analysis of modeling goals and uncertainty factors

The movement of a submarine in the ocean is essentially a Markov stochastic process, and its movement trajectory is affected by the submarine's own route planning and environmental factors such as ocean currents.

Since the submarine may not be restrained by the main ship after being released from the initial position, and will lead passengers on adventure activities to find sunken ships on the seabed, the submarine's route planning is random, and the main ship cannot obtain its information in real time. In addition, the marine environment will also cause certain disturbances to the submarine's position. Since the ocean is a complex dynamic system whose status changes rapidly, major environmental factors including ocean currents, seawater density distribution, water temperature, and seafloor graph cannot be accurately predicted for a long time.

In this way, this thesis advises the submarine to carry certain data collection and transmission equipment to collect information for the fuzzy predict model. Therefore, solve the uncertainty in the prediction process mentioned above. In the second place, we construct an interpolation prediction

model to combine the data sent back by the submarine and perform interpolation simulation, finally it will fit the precise trajectory of the submarine.

2.3. Interpolation prediction model

In this model, the submarine has not yet failed, its power system is good, and collected data is regularly sent to the host ship. From now on, we call the interval between the n th and $n-1$ th data transmissions by the submarine as the n th data collection process, and the position of the submarine at time t during the n th data collection process is recorded as $\rho_n(t)$.

First, according to static model analysis, the process of submarine navigation in deep sea should satisfy the following dynamic equation [3]:

$$m \frac{d^2 r}{dt^2} = mg + F_{buoancy} + F_{power} + f + F_{current} \quad (1)$$

Afterwards, combined with the explanation in the problem analysis module, we perform differential processing on the mechanical equations to discretize them, and obtain the formula as follows.

$$\rho_n(t) = \begin{pmatrix} x_n(t) \\ y_n(t) \\ z_n(t) \end{pmatrix} = \begin{pmatrix} x_{n-1} + V_x(t) + A_x(t) + \Gamma_x(t) + \varepsilon_x(t) \\ y_{n-1} + V_y(t) + A_y(t) + \Gamma_y(t) + \varepsilon_y(t) \\ z_{n-1} + V_z(t) + A_z(t) + \varepsilon_z(t) \end{pmatrix} \quad (2)$$

Since the steering operation of the submarine will bring additional increments to the uniform displacement components of the submarine in both directions on the horizontal plane, the interpolation prediction model requires the following discussion of the reported information

$$V_i(t) = v_i(n-1) \cdot [T_n(t) \cdot \eta_n(t_0) + |T_n(t) - 1|t] \quad (i = x, y) \quad (3)$$

$$V_z(t) = \begin{cases} v_z(n-1)t & , \text{if } v_z(n-1)t \leq z_n - z_{n-1} \\ 0 & , \text{if } v_z(n-1)t > z_n - z_{n-1} \end{cases} \quad (4)$$

At the same time, the submarine's steering and acceleration operations also depend on the submarine's own decision-making, and therefore also rely on the n th data report.

$$A_n(t) = \begin{cases} 1 & , (\text{input} = \text{true}) \wedge (t > \mu_n(t_0)) \\ 0 & , \neg((\text{input} = \text{true}) \wedge (t > \mu_n(t_0))) \end{cases} \quad (5)$$

$$T_n(t) = \begin{cases} 1 & , (\text{input} = \text{true}) \wedge (t > \eta_n(t_0)) \\ 0 & , \neg((\text{input} = \text{true}) \wedge (t > \eta_n(t_0))) \end{cases} \quad (6)$$

Therefore, according to simple physical laws and geometric relationships, the acceleration displacement increment and steering displacement increment of the submarine in the n th data collection stage can be accurately written respectively. $\delta(a)_i$ ($i = x, y, z$) means the random

simulation function of the acceleration in the i direction of the submarine during the n th data recording process

$$A_i(t) = A_n(t) \cdot \frac{\delta(a)_i}{2} (t - \mu_n(t_0))^2 \quad (i = x, y, z) \quad (7)$$

$$\Gamma_x(t) = T_n(t) \cdot |r_n \sin(\theta_n + \omega(t - \eta_n(t_0))) - r_n \sin(\theta_n)| \quad (8)$$

$$\Gamma_y(t) = T_n(t) \cdot |r_n \cos(\theta_n + \omega(t - \eta_n(t_0))) - r_n \cos(\theta_n)| \quad (9)$$

$$\left\{ \begin{array}{l} \theta_n = \frac{\pi}{2} + \arctan\left(\frac{v_y(n-1)}{v_x(n-1)}\right) \\ \omega = \frac{[\vec{v}_{xy}(n-1) \times \vec{v}_{xy}(n)]_z}{|[\vec{v}_{xy}(n-1) \times \vec{v}_{xy}(n)]_z|} \cdot \bar{\omega} \\ r_n = h_n \cot\left(\frac{1}{2} \arccos \frac{\vec{v}_{xy}(n-1) \cdot \vec{v}_{xy}(n)}{|\vec{v}_{xy}(n-1)| \cdot |\vec{v}_{xy}(n)|}\right) \\ h_n = \left| \|\rho_n - \rho_{n-1}\| \cdot \frac{\sin \langle \rho_n - \rho_{n-1}, \vec{v}_{xy}(n) \rangle}{\sin \langle \vec{v}_{xy}(n-1), \vec{v}_{xy}(n) \rangle} \right| \end{array} \right. \quad (10)$$

Finally, we consider random errors caused by environmental factors such as ocean currents, which is described in formular (11)

$$\varepsilon_i(t) = v_{current}(i, n) + \epsilon_i(t) \quad (i = x, y, z) \quad (11)$$

2.4. Fuzzy prediction model

When a submarine has an accident, it will no longer send information to us. At this time, the submarine's status can be in two situations: first, the submarine's communication system is good but the power system fails; second, the submarine artificially shuts down the power system due to communication system failure.

In the first case, the submarine will report its current position information to us in real time, so the only uncertainty factor is the disturbance of the submarine's position caused by ocean currents and other environmental variables. But the good news is that we can make good predictions of this disturbance based on information acquired by the submarine's data acquisition equipment.

$$\left\{ \begin{array}{l} dx = \vec{u} dt + \sqrt{2D_x} dW_x \\ dy = \vec{v} dt + \sqrt{2D_y} dW_y \\ dz = \vec{w} dt + \sqrt{2D_z} dW_z + \frac{\partial D_z}{\partial z} dt \end{array} \right. \quad (12)$$

For the second case, we cannot obtain the data information after the submarine failure occurs, so we cannot perform trajectory interpolation. Therefore, we use machine learning algorithms to perform deep learning on the submarine's historical trajectory characteristics [4], to make good predictions

about the autonomous decision-making behavior of the submarine when it does not malfunction, and predict the trajectory of the submarine during the data collection process when it malfunctions [5].

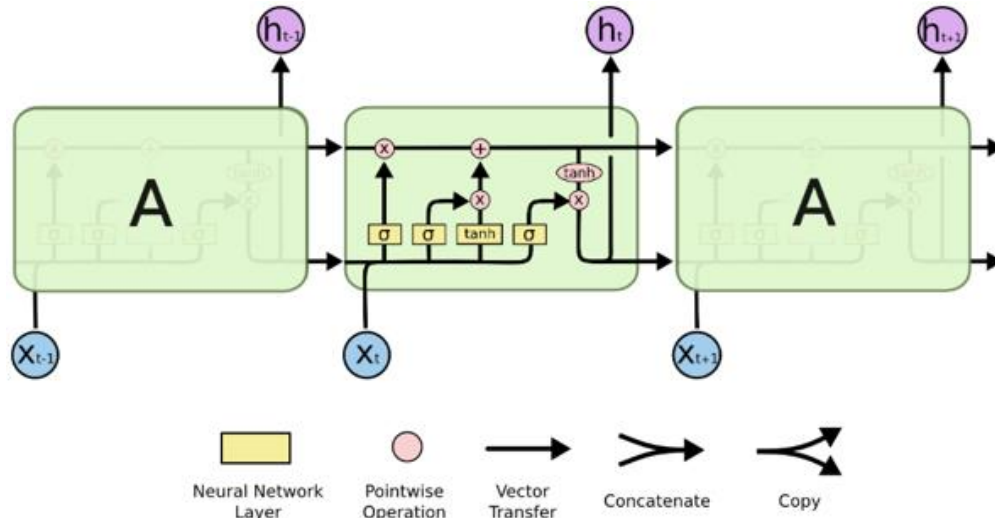


Figure 2. The repeating module in an LSTM contains four interacting layers

In the Fig 2, each line carries an entire vector, from the output of one node to the inputs of others. The pink circles represent pointwise operations, like vector addition, while the yellow boxes are learned neural network layers. Lines merging denote concatenation, while a line forking denotes its content being copied and the copies going to different locations. Based on these principles, the submarine trajectory information will be cyclically analyzed in the LSTM network.

2.5. Calculations and conclusions

For the interpolation prediction model, we wrote the above mathematical model into MATLAB code, and randomly generated several dual-value weighted points in the three-dimensional space (specifically, one point has two independent random 0-1 assignments) to simulate the normal behavior of the submarine. Information transmission points on the running trajectory and the acceleration and rotation operation information they convey. The code will read the double assignment of the above points and simulate the trajectory according to the above mathematical rules. Fig 3 is a schematic diagram we drew based on the simulation information.

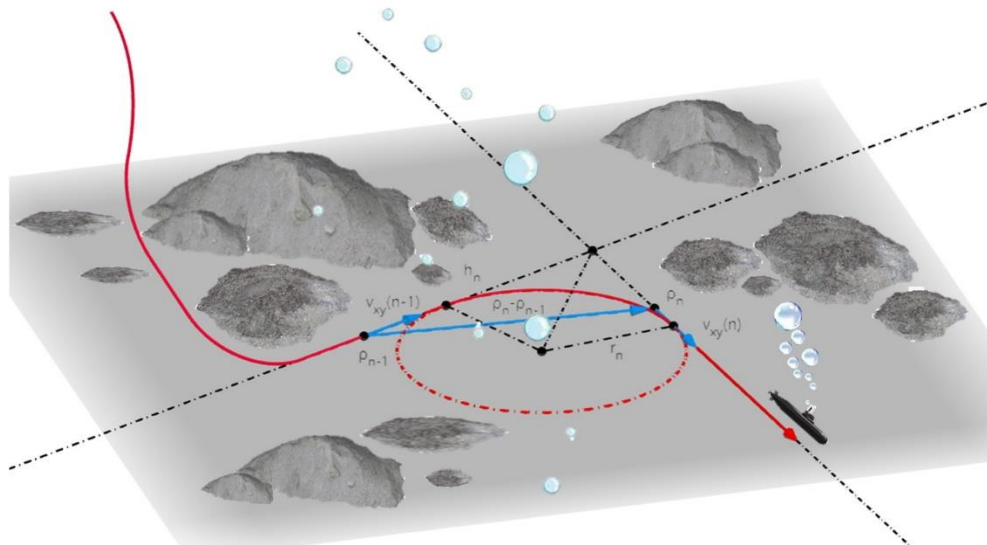


Figure 3. Simulate submarine trajectory

For the fuzzy prediction model, we first artificially set the trajectory preference of a submarine. The implementation is that we let the submarine perform random steering and acceleration operations

(controlled by a random number generator), and then once the submarine approaches a fixed special area within the simulation range, the probability of its steering or acceleration will increase (randomly the probability density function of the number generator changes).

After that, we obtained a large amount of precise route data through about a hundred simulations, which were used as data feed for training the LSTM network to generate cognitive inertia as shown in the Fig 4. Finally, we generate a new submarine trajectory according to the above preferences, let the LSTM network make independent predictions, and compare the prediction results. Like Fig 5 and Fig 6.

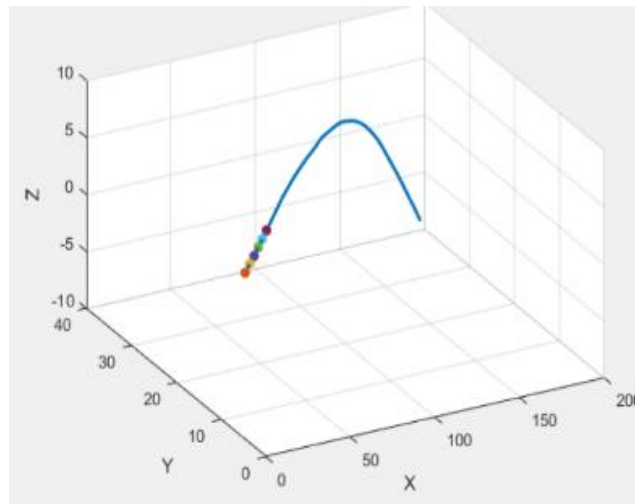


Figure 4. Trajectory simulation

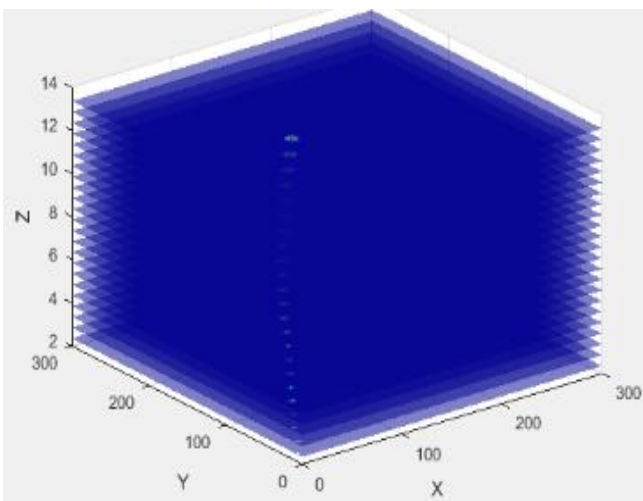


Figure 5. Probability density

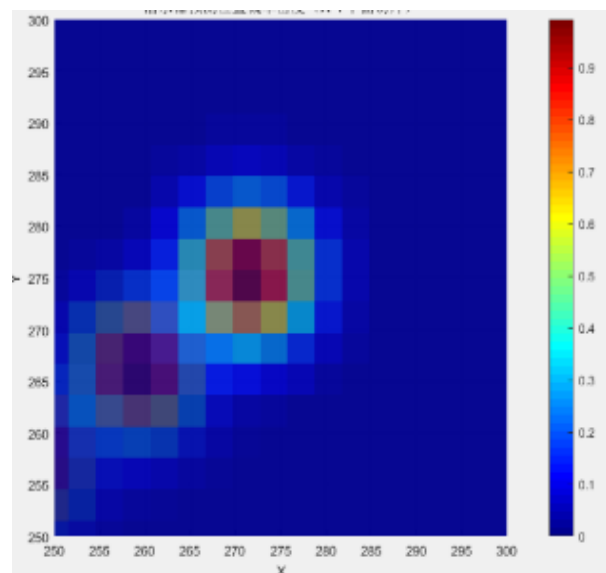


Figure 6. x-y slice

3. Search and Optimization Model

3.1. Model construction

Particle swarm optimization is essentially a heuristic optimization algorithm [6]. In this model, we regard small unmanned detectors as independent particle individuals. After a submarine malfunctions, it will broadcast a distress signal in the sea area where it crashed in real time. At this time, each the detector pair will sense the strength of the signal. Under initial conditions, its perception is random. Then we set the iteration index and update conditions so that each detector updates its position

according to this index. The index includes the optimal solution found by a single detector now, and the comprehensive optimal solution of a group behavior [7].

Assume that in a D-dimensional target search space, the radial vector marking each detector is

$$\rho_k = (x_1, x_2, \dots, x_D) \quad (k = 1, 2, \dots, N) \quad (13)$$

For each particle, we assign it an independent velocity vector

$$V_i = (v_{i1}, v_{i2}, v_{i3}, \dots, v_{iD}) \quad (i = 1, 2, 3, \dots, N) \quad (14)$$

So, we can write its position update equation

$$V_{id} = \omega V_{id} + C_1 \text{random}(0, 1) (P_{id} - X_{id}) + C_2 \text{random}(0, 1) (P_{gd} - X_{id}) \quad (15)$$

For the selection of optimization indicators, we use the test function method to select the two random probability functions described in formular (16) and (17).

Therefore, after n-step algorithm iterations, our detector particles will eventually converge to the submarine's crash location.

$$\zeta(\lambda) = \exp\left(\sum_{p=1}^{\infty} \Pi(p) \lambda^p / p\right) \quad (16)$$

$$\min f(x_i) = \sum_{i=1}^N \frac{x_i^2}{4000} - \prod_{i=1}^N \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \quad (17)$$

3.2. Calculations and conclusions

We used Python code to implement the calculation and simulation of the above submarine rescue model based on particle swarm algorithm. First, we simulated the process of how the position distribution of the detector in the ocean changes with time. The distribution trend can be clearly seen in the Fig 7.

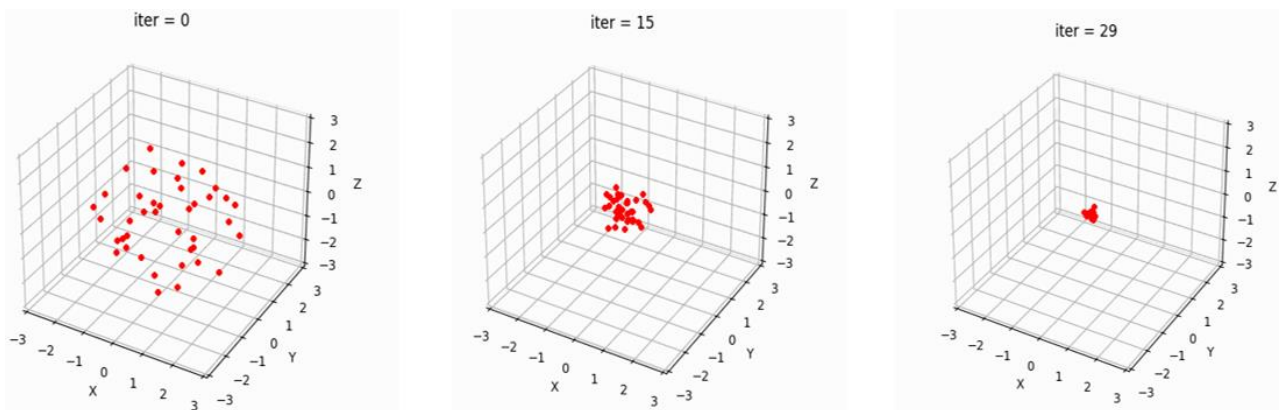


Figure 7. Detector search trajectory simulation

As the simulation time step increases, the detector obtains more and more effective data, so the collective coordination strategy can recursively produce a more accurate probability optimization

function, and the individual detectors also perceive the submarine's position information more and more accurately, thus they are all moving quickly towards the true location of the submarine crash. According to the change of detector aggregation degree with time, under the optimization of this algorithm, its convergence speed is also quite fast [8].

After that, this paper simulated the search and rescue success rate of the search strategy provided by the algorithm as it changes over time. The basic situation is shown in the Fig 8.

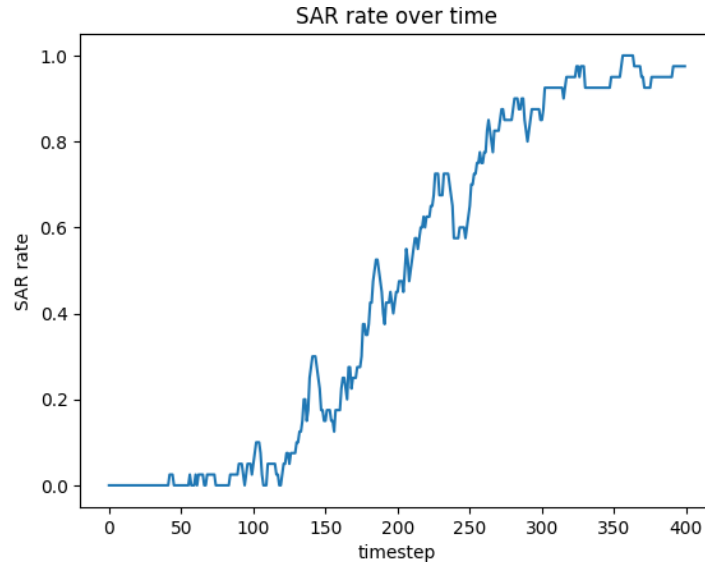


Figure 8. Preliminary simulation of SAR success rate relative to time distribution

To further verify our conjecture and find an accurate probability density time function to provide the search and rescue team with reliable estimates of search and rescue cost and search and rescue process time, we conducted more simulations of the above particle swarm search model, as shown in the Fig 9. The Logistic growth [9] of the probability time function is becoming more and more obvious.

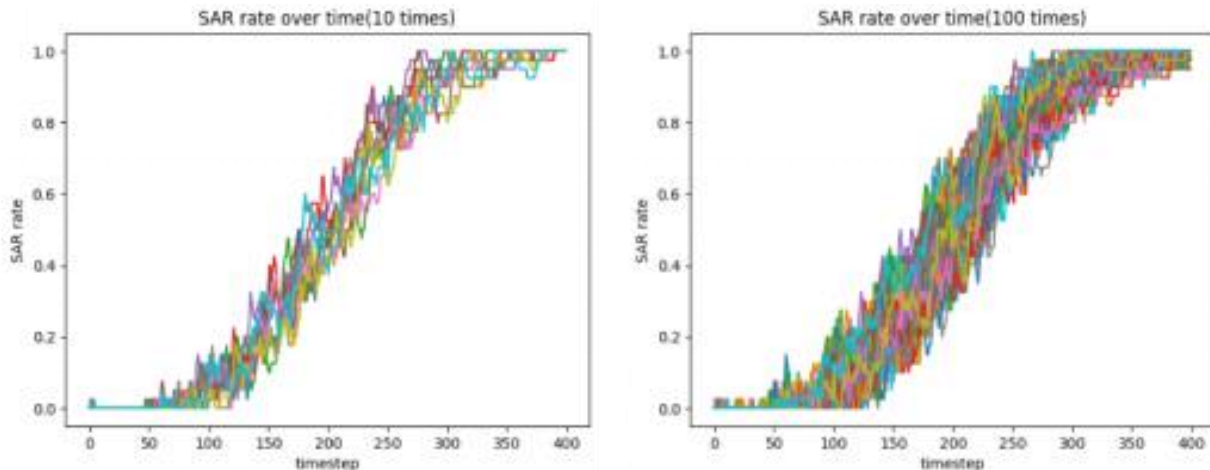


Figure 9. Search and rescue success rate over time

Therefore, according to the reduction convergence rule in statistics, we can conclude that the Logistic equation dominates the generation principle of the curve. Subsequently The probability inference analysis through the Bayesian inference model has indeed proved this point. The random process belongs to quasi-Logistic growth, and its probability time function satisfies the Logistic equation described in formular (18).

$$\frac{df(x)}{dx} = \frac{k}{a} f(x)(a - f(x)), \quad f(0) = \frac{a}{1 + e^{kr}} \quad (18)$$

Finally, we fitted the accurate probability time function in formular (19), which can perfectly describe the distribution of the search and rescue success rate over time of the search and rescue plan we developed based on the particle swarm algorithm.

$$P(t) = \frac{KP_0e^{rt}}{K + P_0(e^{rt} - 1)} = \frac{K}{1 + \left(\frac{K - P_0}{P_0}\right)e^{-rt}} \quad (19)$$

3.3. Sensitivity Analyses

In this paper, we adjusted the population size, kept other parameters constant, and repeated the experiment. In our model, Pop is set to 40. So, we test $Pop \in [10, 110]$ in steps of 20 to get an approximate range, and then test $Pop \in [30, 50]$ in steps of 2 to determine the exact threshold. The results shown in Fig 10. We can conclusion that stable results can be obtained when $Pop > 30$.

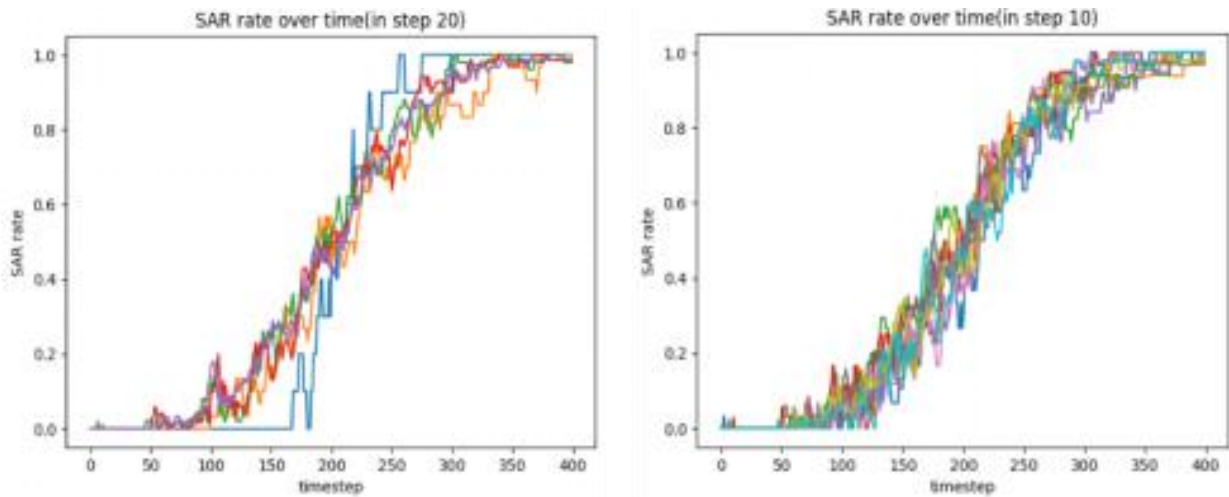


Figure 10. Sensitivity analysis of Pop

We have set the position of submarine \vec{P} at the origin in previous experiments. Here we randomly change its value to test the model. The result shown in Fig 12. It's clearly that this model is stable to changes in \vec{P} .

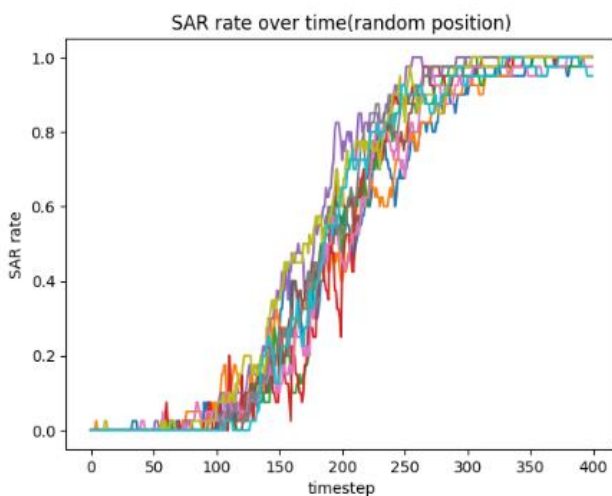


Figure 11. Sensitivity analysis of P

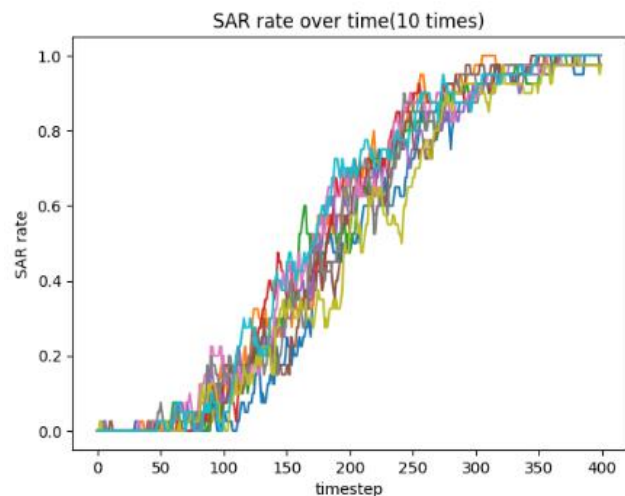


Figure 12. Sensitivity analysis of T

We model the seafloor topography as a set of constraints of PSO [10] and randomize these as constraints (e.g. $Ax^2 + By^2 + Cz$) for repeated experiments. The results shown in Fig 11. It can be noticed that the convergence rate of our model is slightly reduced, but the overall convergence trend is still significant.

4. Conclusion

Through the above results and evaluation, the adaptability of this model is very strong, and the rapid search of a single target is far from allowing us to touch the limit of its strength. The model constructed in this article has many advantages. The most important one is the PSO-based SAR model can intelligently and efficiently search for a wrecked submarine and can locate it within a definable SAR area in only about 3 hours, much less than the emergency endurance of a tourist submarine, allowing the SAR team plenty of time to spend on the rescue. Also, attributed to the scalability of PSO, the model is not difficult to extend to scenarios with multiple submarines and more complex seas.

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