

# Research on the Impact of Sex Ratio Changes on the Population Dynamics of Lamprey Based on the Lotka-Volterra Algorithm

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**Abstract.** This study employs mathematical modeling to investigate the impact of sex ratio changes on the sea lamprey population dynamics. Utilizing differential equations and specific models for population growth, reproduction success, and resource utilization efficiency, the study examines the effects of sex ratio on population dynamics, reproduction success rates, and resource usage efficiency. Diagrams illustrating variations in sex ratio and population quantities offer insights into their effects on population growth and ecological dynamics. The findings suggest that altering the sex ratio significantly influences reproductive success, resource efficiency, and overall ecosystem dynamics, potentially leading to changes in reproduction rates, population structure, genetic diversity, and predator-prey relationships. By analyzing the boosted Lotka-Volterra model, the study demonstrates that as the sex ratio effect ( $\alpha$ ) increases, significant fluctuations occur in prey populations, contributing to environmental instability. Moreover, predator populations exhibit greater variations and a sharp decline, indicating the intricate interaction between sex ratio, prey stability, and ecosystem dynamics. The conclusion is that changes in sex ratio have a significant impact on the ecosystem. Possible issues that might arise include changes in reproduction rate and population growth rate, changes in population structure, reduced genetic diversity, and changes in predator-prey relations. These results highlight the critical role of sex ratio in population dynamics and ecosystem stability, emphasizing the need for further exploration and management strategies in marine ecosystems.

**Keywords:** Lamprey; Gender Ratio; Stability of The Ecosystem; Lotka-Volterra.

## 1. Introduction

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Use italic for emphasizing a word or phrase. Do not use boldface typing or capital letters except for section headings (cf. remarks on section headings, below).

## 2. Introduction

The lamprey, a parasitic fish native to the North Atlantic and Western Atlantic regions, was introduced to the Great Lakes of North America in the early 19th century, primarily through the construction of the Welland Canal in 1829, linking Lake Ontario and Lake Erie and enabling lampreys to access the Great Lakes system [1]. While the lamprey is considered an invasive species in the Great Lakes, it is also endangered in its original habitats in North America and Europe [2]. Since their introduction, lampreys have had a significant impact on native freshwater fish species in the Great Lakes, particularly those of ecological and economic importance like salmon, trout, and sturgeons [3].

Various population control methods, including biological control measures such as male sterility release, have been developed to manage lamprey populations due to their gender-dependent impact on population size [4-6]. However, the traditional Lotka-Volterra models used for ecological estimation may have limitations such as overlooking ecological factors like parasitism, sex ratios, and



environmental changes, and being primarily designed for describing competition between two species rather than multiple species or under complex environmental conditions [7-9]. This study seeks to enhance the Lotka-Volterra model by considering the influence of lamprey sex ratios on population size and incorporating the effects of lamprey predators and prey on population dynamics. The research aims to investigate how lamprey sex ratios impact intra- and interspecies interactions and the role of lampreys in local ecosystems, determining whether changes in their sex ratio can affect ecosystem stability [10].

### 3. The fundamental of Improved Lotka Volterra model

Data source: <https://figshare.com>

#### 3.1. The Expansion of the improved Lotka Volterra model

Based on the population characteristics of lampreys in the traditional Lotka Volterra model, and expanding its upper and lower predatory relationships.

(1) Population growth model (Logistic model)

$$\frac{dN}{dt} = rN \left( 1 - \frac{N}{K} \right) \quad (1)$$

Among them:  $N$  is the population number.  $T$  is the time the time.  $r$  is the intrinsic growth rate of the population.  $K$  Is the amount of environmental capacity (the maximum amount that a population can support in the environment).

The analytical solution of the model is as follows:

$$N(t) = \frac{K}{1 + \left( \frac{K - N_0}{N_0} e^{-rt} \right)} \quad (2)$$

Among them,  $N_0$  is the initial population number (number at  $t=0$ ).

(2) Model of reproductive success:

$$S(R, A) = \alpha R(1 - R)A \quad (3)$$

Sto present a reproductive success rate,  $A$  Indicates the amount of resources,  $\alpha$  is the coefficient, The model assumes that reproductive success is proportional to the product of resource amount and sex ratio rate.

(3) Resource utilization efficiency model:

$$E(R) = \beta \frac{R}{1-R} \quad (4)$$

$E$  represents the resource utilization efficiency,  $\beta$  is the efficiency coefficient, and the model assumes that the resource utilization efficiency is proportional to the ratio of sex ratio.

(4) Population dynamics of Lampreys:

$$\frac{dN_{\text{tamprey}}}{dt} = r_{\text{lamprey}} N_{\text{lamprey}} \left( 1 - \frac{N_{\text{lamprey}}}{K_{\text{lamprey}}(R)} \right) - p_{\text{lamprey}} N_{\text{lamprey}} N_{\text{predator}} \quad (5)$$

(5) Prey Population Dynamics:

$$\frac{dN_{\text{prey}}}{dt} = r_{\text{prey}}N_{\text{prey}} \left(1 - \frac{N_{\text{prey}}}{K_{\text{prey}}}\right) - c_{\text{prey}}N_{\text{lamprey}}N_{\text{prey}} \quad (6)$$

(6) Other key population dynamics (for example, predators):

$$\frac{dN_{\text{predator}}}{dt} = r_{\text{predator}}N_{\text{predator}} \left(1 - \frac{N_{\text{predator}}}{K_{\text{predator}}}\right) + e_{\text{predator}}N_{\text{tamprey}}N_{\text{predator}} \quad (7)$$

Among them,  $N_{\text{lamprey}}$ ,  $N_{\text{prey}}$ ,  $N_{\text{predator}}$  Representing the population sizes of Lampreys, prey, and predators respectively,  $r_{\text{lamprey}}$ ,  $r_{\text{prey}}$ ,  $r_{\text{predator}}$  Representing their respective natural growth rates,  $K_{\text{lamprey}}$ ,  $K_{\text{prey}}$ ,  $K_{\text{predator}}$  Depicting the impact of sex ratio  $R$  on the environmental carrying capacity of Lampreys, as well as the environmental carrying capacity of both prey and predators.  $p_{\text{lamprey}}$ ,  $c_{\text{prey}}$ ,  $e_{\text{predator}}$  Indicate the predation rate and efficiency coefficient.

### 3.2. The Improved Lotka Volterra Model Analysis Method

One powerful theoretical tool for analyzing which factors contribute to the stability of a large system of many interacting constituents, such as a complex ecosystem, is random matrix theory (RMT). This is the method that was employed by May in his seminal work. May started from the Jacobian of a hypothetical ecosystem, linearized about an "equilibrium." He assumed that the entries of this Jacobian matrix were in-dependent and identically distributed random variables. This allowed for the deduction of a stability criterion using a result from RMT: Girko's circular law for the distribution of eigenvalues of large independent and identically distributed random matrices. May's initial and somewhat austere model has been extended in recent years, accounting for additional features of ecosystems such as food web structure, spatial dispersal, alternative interpretations of "interaction strength", and varying self-regulation. This article attempts to generalize the influence of gender ratio on lampreys based on this.

The relationship between the initial gender ratio ( $p_0$ ) and the number of males and females in a population, as well as the fluctuations in the gender ratio ( $p$ ). Logical regression is conducted using the following formula:

$$P(\text{Male} = 1) = 1 / (1 + e^{-(\beta_0 + \beta_1 * \text{Location} + \beta_2 * \text{Type}_{\text{Encoded}} + \beta_3 * \text{Years})}) \quad (8)$$

It signifies the probability of the 'Male' category, that is, the probability that the model predicts a sample belongs to the 'Male' category.  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  are the parameters of the model, representing the model's intercept and the weight coefficients of the features respectively. Location, Type\_Encoded, and Years are the input feature variables, representing the features of the model. 'e' represents the base of the natural logarithm (approximately 2.71828).

When developing stability indications, the Analytic Power Structure Refine (AHP) can be utilized to determine the relative importance of various indicators, thus constructing an extensive security evaluation model. To start with, mathematical fitting is performed on the information to obtain functions reflecting biodiversity and types consistency modifications with time. Then, fix the Jacobian matrix of these features to find the system's balance factor. Next, by introducing a tiny perturbation, we observe the system's reaction to analyze its stability.

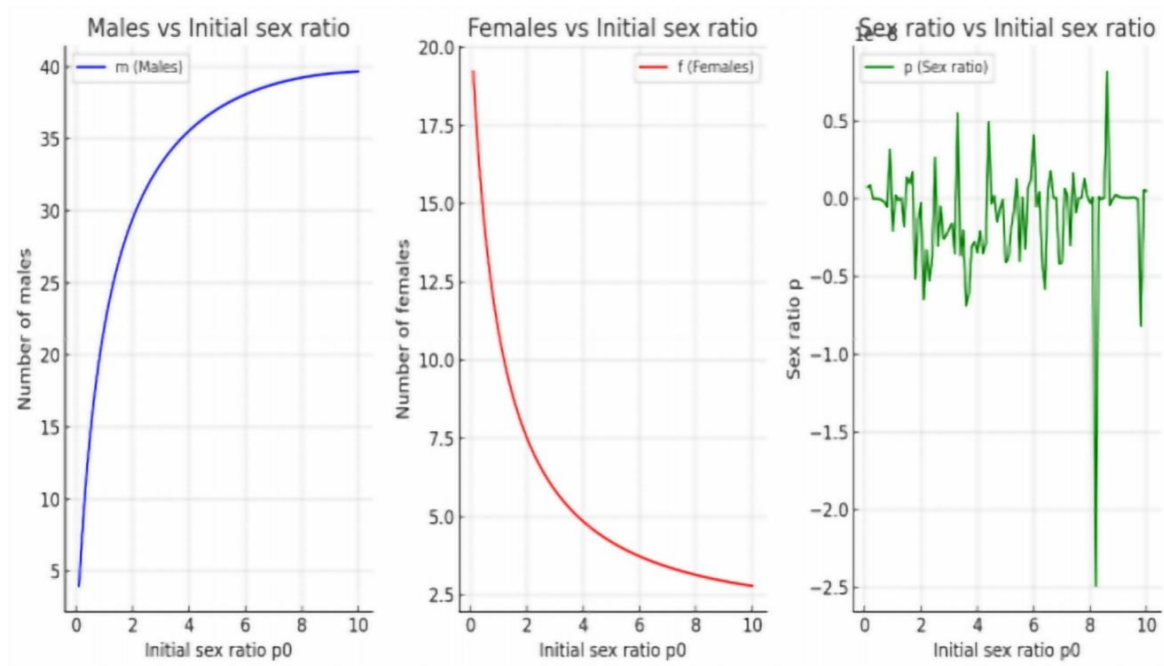
To analyze the system's stability, it is first required to find the system's stability factor, i.e., when all formulas on the right-hand side amount to zero, and the values of variables M, F, and P. After that, linearize the system, calculate the Jacobian matrix near the balance factor, and analyze its eigenvalues to evaluate the stability.

Details procedures include establishing parameters, locating the system's balance point, then computing the Jacobian matrix and examining the stability. In actual communities, we generally try to

find non-trivial equilibrium points, i.e., a minimum of one varieties' amount is non-zero. This needs even more details, such as details worths of growth rate and death price, or the proportion of these specifications. Then, construct the symbolic depiction of the Jacobian matrix and execute calculations with specific numerical values appointed to the parameters.

## 4. Results

### 4.1. The impact of gender ratio changes



**Figure 1.** The Impact of Initial Gender Ratio on Gender Distribution

The mating system of lampreys is primarily polygynandrous. Our working hypothesis is consistent with studies that linked male-biased sex ratios in lampreys to high larval density.

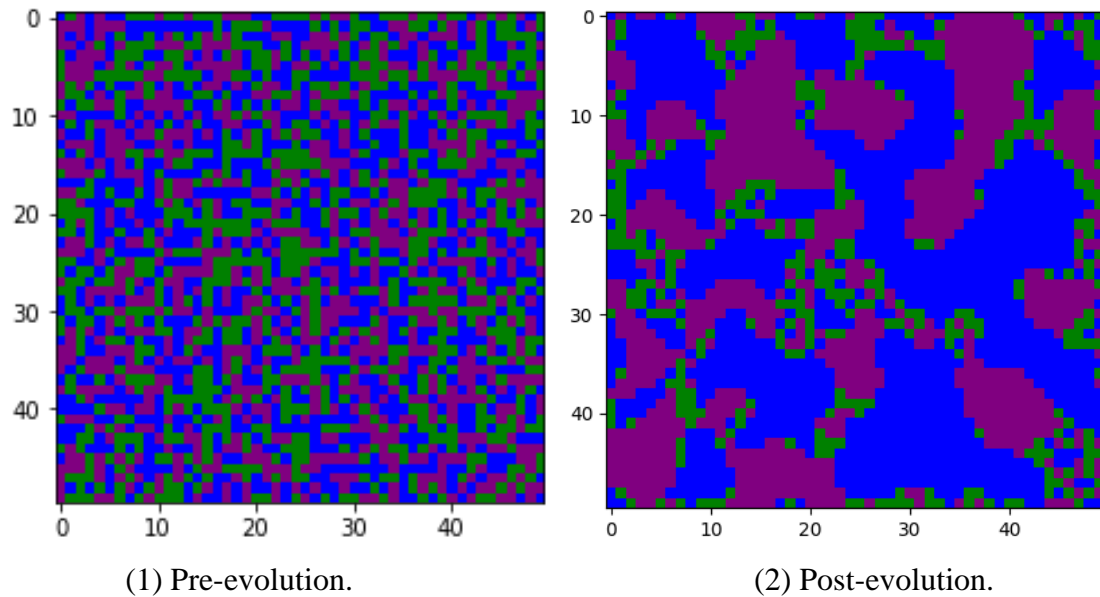
Figure 1 presents us with the following findings:

**Male population and initial sex ratio trend:** When the initial sex ratio ( $p_0$ ) is close to zero, the number of males increases rapidly, indicating that even a small number of males can mate with multiple females in female-dominated populations, leading to a quick rise in male population. However, as the initial sex ratio reaches a certain threshold, the growth of male population slows down, approaching a maximum level. This could represent an ecological saturation point, indicating the optimal number of males that can be sustained in the population before resource competition, reduced mating opportunities, etc., occur.

**Female population and initial sex ratio trend:** The decrease in female population significantly corresponds to the increase in the initial sex ratio, suggesting that females are responsive to the initial sex ratio. In male-dominant populations, the decline in female numbers may impact the reproductive capacity of the population, as each male's mating opportunities are limited by the available number of females.

**Sex ratio variation trend:** The graph displaying sex ratio fluctuations demonstrates drastic changes, particularly when the initial sex ratio approaches a specific threshold. These variations may stem from non-linear dynamics and complex ecological interactions within the population, including mating strategies, resource allocation, and feedback effects on population density.

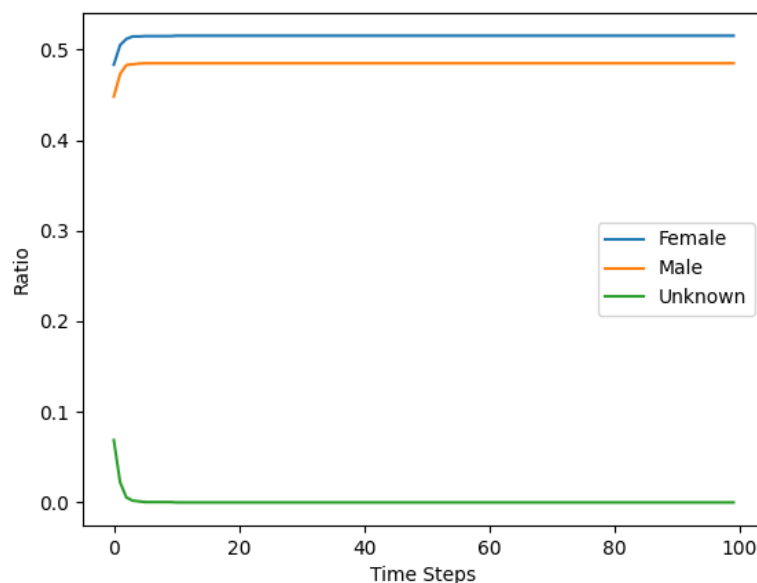
#### 4.2. The impact of differentiated individual gender of lampreys on the gender of surrounding undifferentiated individuals



**Figure 2.** The Interference of Gender-Differentiated Individuals on the Evolution of Surrounding Individuals.

In this study, we utilized a color coding scheme where purple represents cells of undetermined gender, blue represents female cells, and green represents male cells (as illustrated in Figure 2), effectively portraying the evolutionary dynamics of cells within the population. This visual approach has furnished us with a potent instrument to observe and analyze cellular transitions, offering a fresh perspective on investigating the process of cellular evolution. It is evident that through this color coding scheme, a profound convenience is conferred upon the comprehension of cellular state alterations and evolutionary processes. The surviving larvae had less competition for habitat and food, grew faster, and were more likely to be female.

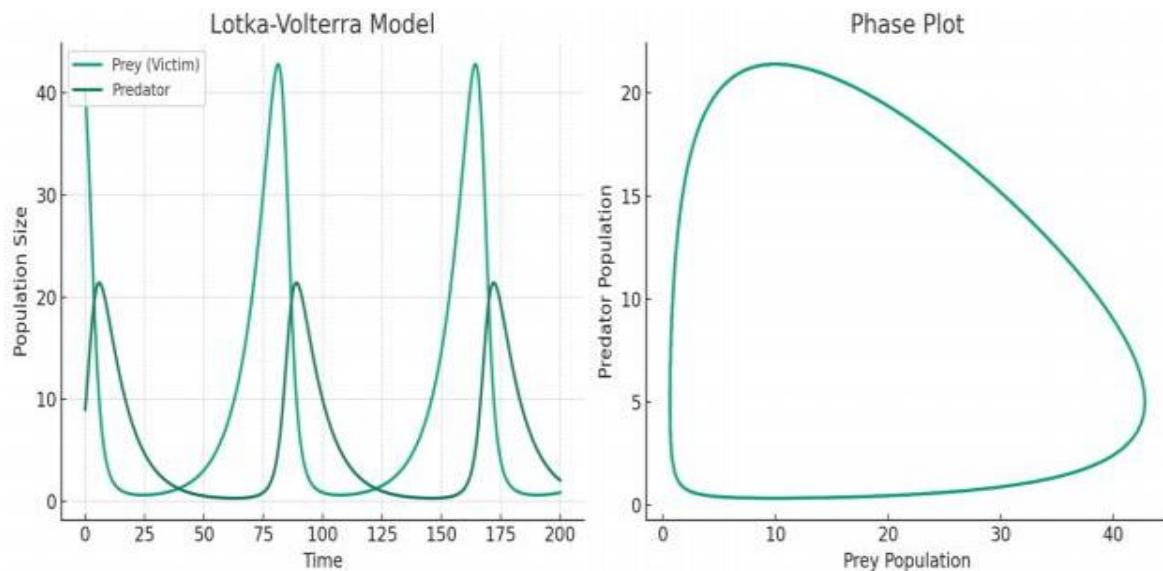
#### 4.3. Population size changes without introducing the influence of gender ratio



**Figure 3.** Traditional Lotka Volterra model

In the Figure 3, we observed arbitrary changes in population numbers, however no clear pattern demonstrating how changes in the sex ratio directly impact the number of predators. This may be because the version does not straight link the sex proportion to reproduction success or killer's victim choice.

#### 4.4. Population size changes under the influence of gender ratio



**Figure 4.** Improved Lotka-Volterra Model with population

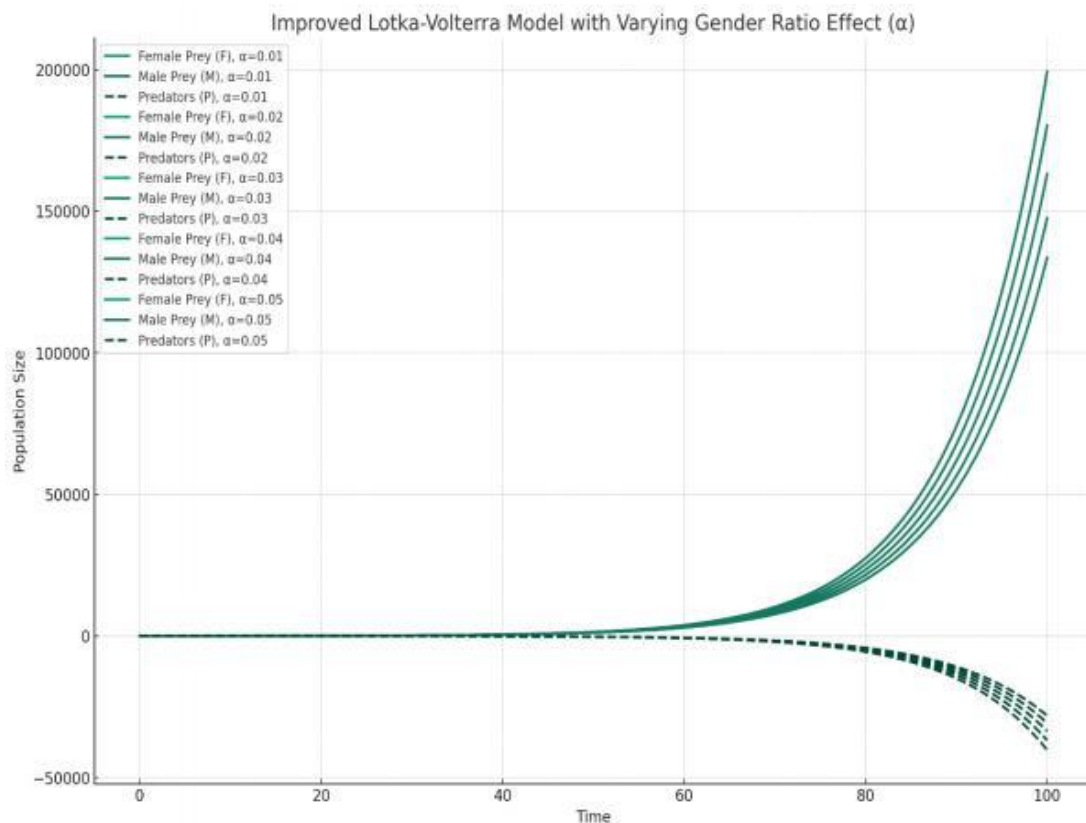
In a productive stream environment where environmental features were conducive to growth and condition, larval sea lamprey quickly recovered from the tagging effect, sex ratios becoming less skewed towards males. Conversely, in unproductive lentic environments, environmental features resulted in further skewing of sex ratios. Previous population demographic work showed that larvae reared in lentic environments grew two to four times slower and metamorphosed into the parasitic stage at smaller sizes than larvae from the stream environments.

The Figure 4 depicts a dynamic simulation of predator and prey populations based on the Lotka-Volterra model. The left chart illustrates fluctuations in the population sizes of predators (in green) and prey (in cyan) over time, while the right chart is a phase diagram illustrating the interplay between prey and predator populations.

In the left chart, we observe cyclical changes in both predator and prey populations. Changes in prey numbers typically precede changes in predator numbers, and declines in prey populations also occur before declines in predator populations. This pattern emerges because the abundance or scarcity of prey directly impacts the food supply and stability of predators. When prey populations are abundant, predators have ample food resources, leading to an increase in predator numbers. Conversely, when prey populations are scarce, food becomes limited, causing a decrease in predator numbers.

In the phase diagram on the right, we see a closed trajectory, indicating that the dynamics of predator and prey populations are cyclic and repeat consistently. The trajectory's shape suggests that various combinations of prey and predator population sizes will continue to cycle. It also highlights the non-linear relationship between prey and predator population sizes, where a small population of prey can sustain a large number of predators. However, an overabundance of prey leads to a reduction in the predator population growth rate.

#### 4.5. The impact of the sex ratio result (alpha) on populace dynamics in the boosted Lotka-Volterra model.



**Figure 5.** Improved Lotka-Volterra Model with Varying Gender Ratio Effect (a)

The Figure 5 reveals that variations in prey populations, both females and males, become significant as the sex ratio effect (alpha) increases. Higher alpha values lead to more drastic fluctuations, indicating that sex ratio has a substantial impact on the stability of prey population size. A greater sex ratio effect can result in significant population size fluctuations and increased instability.

In the case of predator populations, as alpha increases, their numbers also show greater variability and drop sharply at a certain point in time. This could be attributed to unstable food supply due to fluctuations in prey populations. This suggests that an increase in sex ratio effect not only affects the stability of prey populations but also significantly influences predator population numbers, adding uncertainty to the ecosystem.

When alpha is low, there is minimal fluctuation in population numbers over time, demonstrating consistent growth. However, with high alpha values, population numbers increase rapidly and then decline abruptly, indicating heightened system instability as the sex ratio effect intensifies. This highlights the essential role of the sex ratio effect in population dynamics and ecological community stability. A higher sex ratio may increase environmental instability, thereby impacting the overall health and functioning of the entire community.

## 5. Conclusion

In our innovative approach within the Lotka-Volterra model, we introduced a convex function related to gender ratios to simulate the impact of sex ratio on the surrounding ecosystem of lampreys and analyze the characteristics of this unique population. While traditional Lotka-Volterra models typically show monotonic and uninteresting population growth, our modifications led to surprising results that aligned more closely with real-world dynamics. We observed a dynamic interplay of growth and decline, reflecting a more realistic simulation where lampreys and their adjacent predatory species

formed a interconnected network. The lampreys and their predator-prey relationships truly intertwined, showcasing the intricate and interconnected nature of population dynamics under the influence of gender ratios. Changes in sex ratio in population ecology have both advantages and disadvantages. Increasing the number of males can enhance population reproductive capacity when there is a high number of females, as males can mate with multiple females, promoting genetic diversity and population growth. However, excessive male numbers can lead to intensified resource competition, reduced mating opportunities, and over-exploitation of females, negatively impacting population long-term stability. A rapid increase and subsequent stabilization in male numbers may indicate sufficient males for reproductive activities, but further increase beyond a threshold could lead to intra-specific competition, uneven resource allocation, and societal structure disruption.

A significant decline in female numbers suggests that male dominance can make females a limiting resource, creating a bottleneck for population growth and breeding. A decrease in females could limit reproduction opportunities, affecting long-term population sustainability. Sex ratio volatility may indicate complex population interactions concerning breeding strategies, success rates, competition, and survivability. Extreme variations suggest high sensitivity to environmental changes and internal dynamics, leading to significant population fluctuations under uncertain environmental conditions. These studies emphasize the substantial impact of gender ratio on population dynamics, particularly when the ratio is imbalanced. An excessively skewed male to female ratio could negatively impact the ecosystem, accelerating food species depletion and causing instability in predator populations.

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