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Research on the Sex Ratio Variation of Lampetra japonica and Its Reciprocal Altruism Effects Based on the Logistic and Lotka-Volterra Models

Xingyu Bao [1, *], Shiya Li [2], Zihan Chen [3]

[1] Beijing University of Civil Engineering and Architecture, School of Civil and Transportation Engineering, Beijing, China,100044

[2] Shiya Li, Beijing University of Civil Engineering and Architecture, School of Urban Economics and Management, Beijing, China,100044

[3] Zihan Chen, Beijing University of Civil Engineering and Architecture, School of Civil and Transportation Engineering, Beijing, China,100044

* Corresponding author: xingyu_bao0813@163.com

Abstract. The lampreys, a parasitic fish native to the Atlantic Ocean, is an invasive non-native species in the Laurentian Great Lakes of North America and an endangered species in most of its native regions in North America and Europe. This paper addresses the study of demographic changes in the population of lampreys in the Great Lakes basin of North America, which is of reference significance to the control of the population of lampreys in the Great Lakes basin of the United States and the protection of its populations in other regions. Since the sex ratio of lampreys changes according to the external environment, its growth rate in the larval stage is fast or slow to determine its final sex, and its larval growth rate is affected by food availability, this paper adopts the logistic model and the Lotka-Volterra model, and simulates the natural growth of the lampreys population in an ideal state by building a mathematical model and the sex ratio changes, and introduced the intra-species competitive role and inter-species competitive role of lampreys population, through the changes in the number of lampreys population, and further analyzed the advantages, disadvantages and such changes of the sex ratio changes on the evolution of the lampreys population itself and the effects of the evolution of the population on the ecosystem and its stability.

Keywords: Lampreys; Stocks, Ecosystems; Logistics regression; Lotka-Volterra systems.

1. Introduction

The lampreys are an invasive, non-native species in the Great Lakes basin of North America [1]. This study addresses the factors that influence the growth rate of the lampreys population and the effects of changes in the growth rate of the lampreys population on the population itself and the ecosystems in which it is found, and is informative for the control of the lampreys population in the U.S. Great Lakes Basin and the conservation of the population in the rest of the region.

It has been shown that the sex ratio of lampreys ultimately determines the growth rate of their larvae [2], so this paper is based on this premise to study the lampreys population. Among the existing studies, Brittney G. Borowiec et al. [3] studied the response of lampreys to chemical substances from the perspective of their own physiological characteristics, so as to control the invasive population of lampreys in the Great Lakes, and to rationalize the protection of the local population. Margaret F. Docker et al. [4] focused on the effects of regional differences in phenotypic and quantitative characteristics (e.g., sex ratio, body size) of lampreys on the dispersal of the population to provide a scientific basis for management and control of the population; and Diogo Ferreira-Martins [5] and Ronald E. Thresher [6] have studied the natural growth of lampreys from a genetic and hereditary point of view, respectively, with the aim of controlling the population in the Great Lakes basin of North America. However, most of these studies have focused on the lampreys population itself, but less on the impacts of the lampreys population on the ecosystems in which it is located.

In this context, it is known that invasive species themselves evolve in response to their interactions with native species and also in response to new abiotic environments [7]. Unlike previous studies that mostly focused on the internal characteristics of lampreys populations and their general impacts on the ecosystems, this paper innovatively adopted logistic modeling and Lotka-Volterra equations to systematically In this paper, the dynamics of the population growth rate and its far-reaching effects were investigated, so as to provide a scientific basis for the development of more precise and effective ecological interventions.

The innovations of the study are mainly reflected in the following aspects: firstly, by combining modern ecological theories and mathematical models, this study comprehensively and systematically analyzes the influencing factors of the population growth rate of lampreys, which fills in the deficiencies of the existing studies. Secondly, this paper not only analyzes the impacts of lampreys population growth on its own population, but also discusses its long-term impacts on the diversity and stability of the Great Lakes ecosystem, which provides a new perspective for understanding the interactions between invasive species and the ecosystem. Finally, the management and control recommendations put forward in this study are based on comprehensive scientific research and provide a theoretical basis and technical guidance for relevant policy formulation and practical operation, which are of great practical significance and application value.

2. Sources of data

The data in this paper are mainly obtained from the official website of the North American Great Lakes Fishery Commission (http://www.glfc.org/sea-lamprey.php), while the data in this paper have been optimized to simplify the calculation, and some of the data have been supplemented based on relevant references [8].

3. A logistic model-based sex ratio change model

In this paper, we first consider that under the ideal state, the natural growth of lampreys population is only affected by the environmental capacity, and its growth conforms to the logistic model [9] curve, the expression of which is shown in equation (1):

$$\frac{dx_i}{dt} = r_i x_i (1 - \frac{x_i}{N_i}) \tag{1}$$

Where x_i denotes the sex ratio of lampreys, r_i denotes the intrinsic growth rate, and N_i denotes the maximum number of lampreys in a given environmental capacity.

According to this model, this paper further expands it for the specificity of lampreys population, which is used to analyze the effect on the population sex ratio under different food conditions, so as to further explore the effect of the change of the sex ratio of the lampreys population on the change of its population itself.

3.1. Sex ratio change model of lamprey’s population under different food conditions

Data from the North American Great Lakes Fishery Commission surfaces that sex ratios of lampreys are related to food availability, with a lower percentage of females when lampreys are in the larval stage and in environments where food is less available. In contrast, the proportion of females was higher in environments where food was more readily available. To investigate the functional relationship between food availability and the sex ratio of lampreys, a logistic model was constructed in this paper:

Assuming that the sex ratio of lampreys is determined by food abundance, the relationship between food availability and the sex ratio of females is shown in equation (2):

$$p(x) = \frac{1}{1+e^{-k(x-x_0)}} \quad (2)$$

Where k controls the trend of the curve, reflecting the sensitivity of the change in food level to the sex ratio, and x_0 is the critical point of the change in the sex ratio of males and females, from which the following image can be obtained, as shown in Figure 1 below:

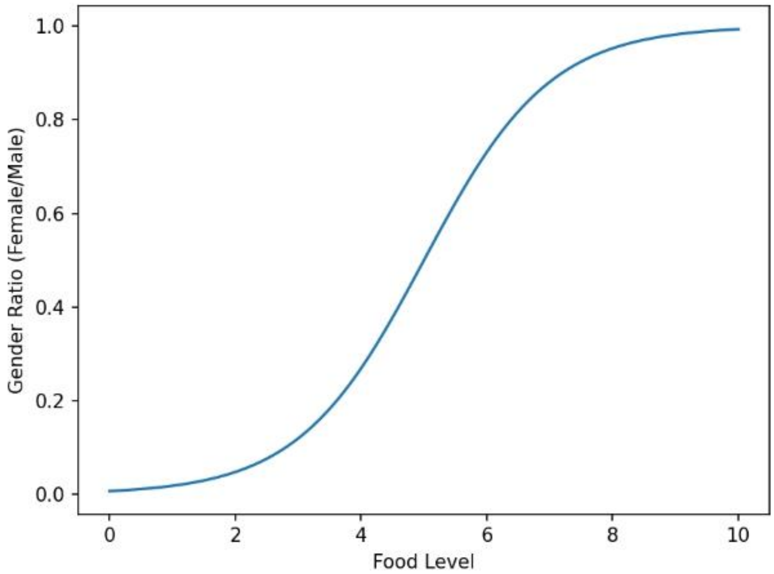

Figure 1. Trend plot of sensitivity of changes in food levels to sex ratio

On this basis, the concept of population generations is introduced in this paper because the growth rate of larvae changes. However, due to the rapid growth of population size, this paper only considers the change of sex ratio with generation within 10 generations under different food sufficiency conditions.

To simplify the model, the following assumptions are made in this paper:

1. food levels can be expressed using continuous values and are variable over a range;

2. the rate of increase of the sex ratio is 1 when food is sufficient; when food is not sufficient, the rate of increase of the sex ratio decreases with increasing number of generations.

Meanwhile, this paper introduces the sigmoid function to reflect the effect of food level on sex ratio:

The general form of the sigmoid function [10] is known as follows:

$$f(x) = \frac{1}{1+e^{-ax}} \quad (3)$$

Where x is the input value and a (also denoted by k) is a parameter that controls the steepness of the function. As the value of a increases, the function image becomes steeper; the sigmoid function is the standard logistic function when $a = 1$. e is the base of the natural logarithm and has a value equal to approximately 2.71828.

First, define f_{effcet} as a sigmoid function that responds based on food level (f_{level}):

$$\boldsymbol{f_{effcet}} = \frac{1}{1+e^{-k(\boldsymbol{f_{level}}-\boldsymbol{x_0})}} \quad (4)$$

The food effect was then used as input to another sigmoid function to calculate the male sex ratio($male_{ratio}$):

$$male_{ratio} = \frac{1}{1+e^{-k(\boldsymbol{f_{effect}}-x_{\mathbf{0}})}} \times \boldsymbol{g_{rate}} \tag{5}$$

Accordingly, software simulations were run to derive the sex ratio over generations under different food sufficiency conditions. To simplify the image, the data averaged over multiple generations is taken, which can be further simplified into Figure 2 below:

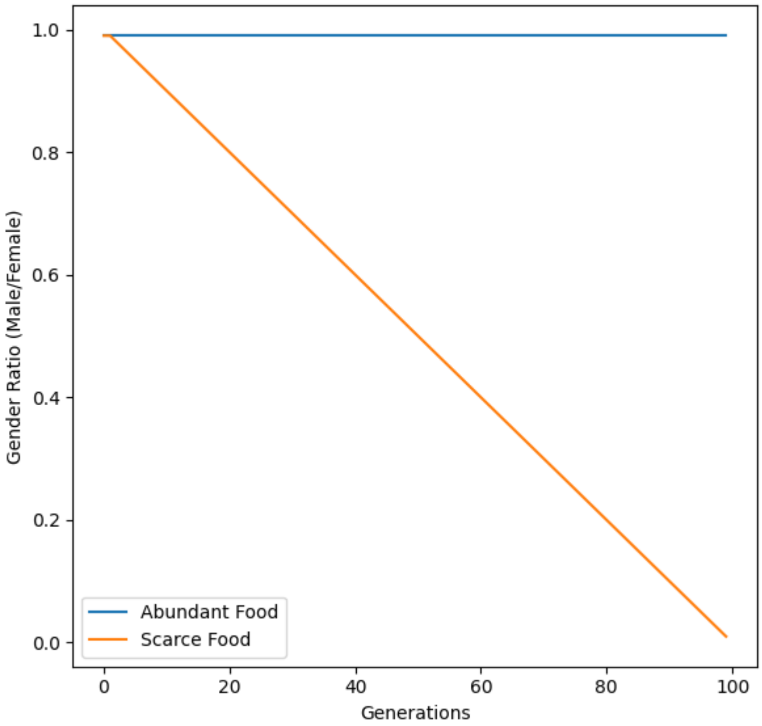

Figure 2. Simplified trend graph of sex ratio with generation change

Conversely, according to this, the same relationship of population size with sex ratio is known, i.e., from the data, the population size decreases with the increase in the number of males and increases with the increase in the number of females.

3.2. A dynamic model of reproductive success in a lamprey population with a fluctuating sex ratio

Considering that changes in the sex ratio of the population may increase the competitive pressure of males competing for females or the lack of sufficient mating opportunities for females, it will lead to a decrease in the reproduction rate, and therefore further lead to a decrease in the population size. Based on this judgment, this paper decided to study the effect of the change in sex ratio on the reproductive success of the population.

In order to simplify the model, this paper makes the following assumptions:

1. the sex ratio of lampreys population and reproductive success will interact with each other, and the effects of reproductive success and food availability on the sex ratio of the population are independent;

2. an increase in the sex ratio of females in lampreys populations will increase the reproductive success of the population.

Without considering the effects of environmental ecological factors such as temperature, predators and parasites on the sex ratio of the population, the following equation was obtained:

$$R(f,m) = \frac{1}{1+e^{-k(\boldsymbol{f_{level}}-x_{\mathbf{0}\boldsymbol{f}})}} \times \frac{1}{1+e^{-k_m(\boldsymbol{f_m}-x_{\mathbf{0}\boldsymbol{m}})}} \tag{6}$$

Calculations based on data from the official website of the North American Great Lakes Commission yielded the results shown in Figure 3 below:

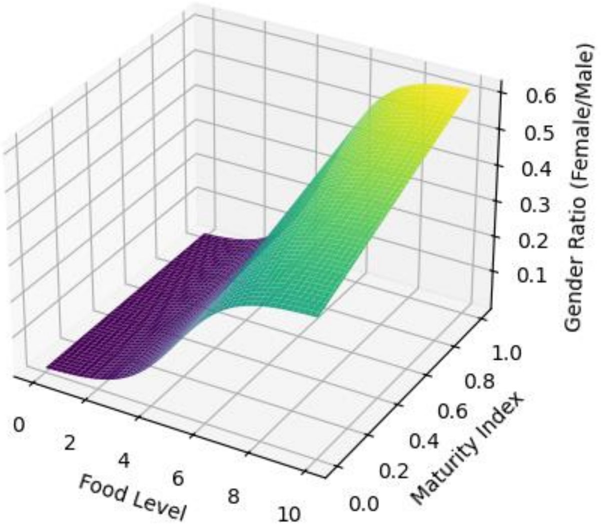

Figure 3. Trends in reproductive success of lampreys populations under ideal conditions

As can be seen from Figure 3, environmental conditions and sex ratio of the population have a potential regulatory effect on the reproductive success of lampreys populations: as food level, maturity index, and male proportion continue to increase, the reproductive success of the population continues to increase.

In order to be more relevant, this paper continues to introduce the effects of temperature and predation rate on the reproductive success of populations. To this end, based on the water temperature changes in the Great Lakes of North America in recent years, and with reference to the predation rate of lampreys populations, the following assumptions are made in this paper:

1. The effects of food level, maturity index, temperature, and predation rate on reproductive success are linearly independent, i.e., each factor independently produces an effect, which is multiplied together to produce total reproductive success;

2. The effect of each influencing factor follows a logistic growth function, i.e., as each factor increases, its contribution to reproductive success increases rapidly and then levels off;

3. Each influencing factor has a critical point beyond which the increase in its positive effect on reproductive success slows down.

On this basis, the following equation can be developed:

$$R(f,m,T) = \frac{1}{1+e^{-k(f_{level}-x_0)}} \times \frac{1}{1+e^{-k_m(f_m-x_{0m})}} \times \frac{1}{1+e^{-k_T(f_T-x_{0P})}} \tag{7}$$

By software operation, the result can be obtained as shown in Figure. 4:

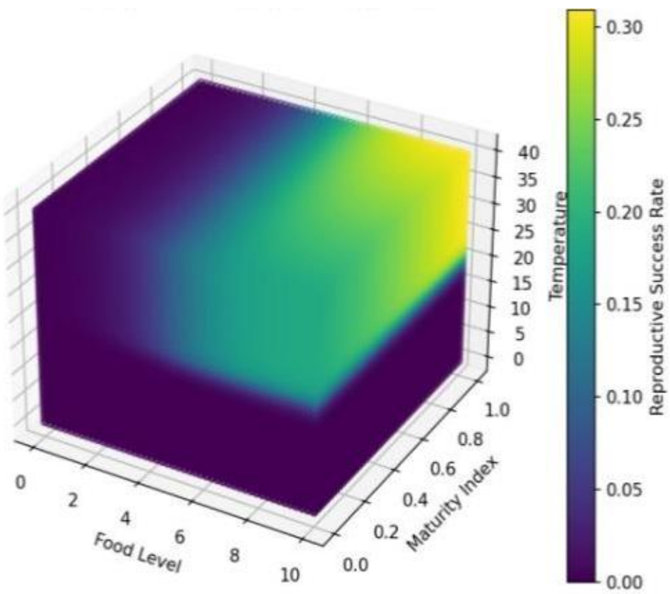

Figure 4. Trends in reproductive success of lampreys populations considering the effects of temperature and predation rates

As can be seen in Figure 4, changes in any single factor within a certain range may have a significant effect on reproductive success, all else being equal. However, once the factor reaches a certain threshold, the effect on reproductive success may reach a saturation point, at which point further increases in the factor may not result in a significant increase or decrease in reproductive success. Accordingly, this paper predicts that the ideal breeding environment may be one in which food is abundant, individual maturity is high, temperatures are favorable, and predation rates are low.

4. A Lotka-Volterra model-based Ecosystem impact model

After considering the evolution of the lampreys population itself due to the change of the sex ratio of the population itself, based on the logistics model, this paper further considers the interspecific relationships within the ecosystem. It is well known that there are four kinds of interspecific relationships in ecosystems: predation, competition, parasitism and symbiosis. Accordingly this paper is based on the Lotka-Volterra equation [11].

4.1. Interspecies relationship model of predator (lamprey) and prey (lake trout)

First of all to simplify the calculations, in this subsection this paper only discusses the predation-prey relationship. It is known that lampreys prey on most large fish in the Great Lakes, such as lake trout and brown trout. Based on data from the Great Lakes Fishery Commission, the following hypotheses are made for the predator-prey pair of lampreys-lake trout selected for this paper:

1. the growth of both lampreys and lake trout follows a logistic model,and the predation rate of lake trout is much greater than the natural mortality rate,so its natural mortality rate is neglected;

2. the survival and growth of lampreys is mainly dependent on the lake trout population, i.e., lake trout are the main food source for lampreys;

3. the sex ratio of lampreys populations directly affects their population growth rate.

Accordingly, the following equation was obtained:

Rate of change in lake trout population:

$$\frac{dN}{dt} = r_t \times N \times (1 - \frac{N}{K_t}) - a \times N \times P \tag{8}$$

Rate of change in lampreys population due to sex ratio:

$$\frac{dP}{dt} = r_l \times P \times (1 - \frac{P}{K_l}) - s \times P + b \times N \times P \tag{9}$$

Where $r_l = r_{lb} \times sex, 0 < sex < 1$;

Using the software to solve the above equations, the following graphs are obtained in this paper, as shown in Figure. 5:

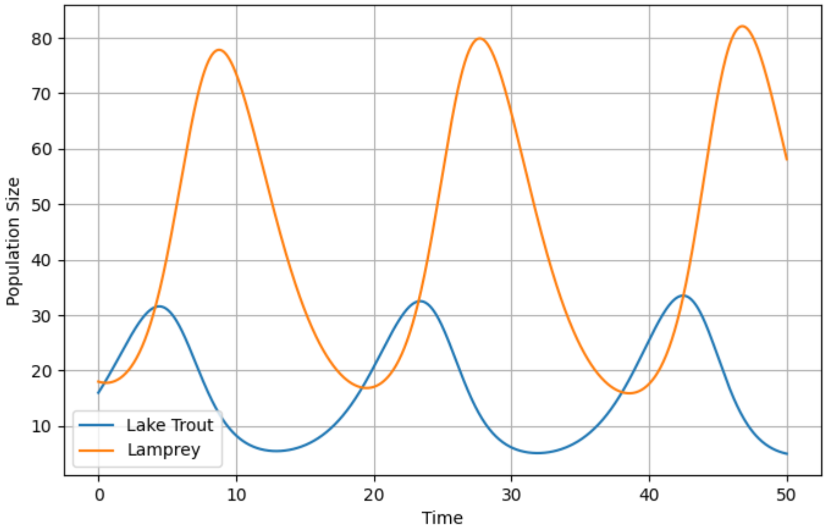

Figure 5. Schematic of changes in lampreys populations and lake trout populations

Accordingly, it can be seen that the population changes of lampreys and lake trout show fluctuating growth, with both species experiencing distinct peaks and troughs in what appears to be an inverse relationship. As the lake trout population declines, the lampreys population begins to surge, and as the lampreys population declines, the lake trout population begins to recover. The number of lampreys increases as the lake trout population decreases, probably due to a reduction in predation pressure. Conversely, lake trout populations may rebound when lampreys numbers decrease. This suggests a delicate ecological balance between predators and prey, where predator and prey populations are interrelated and influence each other's dynamics.

Meanwhile, it can be seen in this figure that the number of lampreys as predators was even larger than the number of lake trout prey, which is not in line with the energy pyramid principle of ecology, which may be related to the setting of model parameters. However, the Lotka-Volterra model is a simplified mathematical model that only considers interactions between populations and does not take energy transfer efficiency or other ecological factors into account. The basic form of this model assumes unlimited resources, does not take into account environmental capacity constraints, and assumes that the growth rate and interaction coefficients for each population are constants.

4.2. Interspecies relationship model with the introduction of competitors, symbiotics and parasites

To address the limitations mentioned in the previous section, this paper continues with the introduction of competitors, symbionts, and parasites, which are similarly modeled through the Lotka-Volterra equation. Again, the following assumptions are added to the previous section:

1. there is no growth in the number of competitors, symbionts and parasites;

2. the natural mortality rates of lampreys, competitors, symbionts and parasites are the same;

3. competitors have a negative effect on lampreys, while symbionts have a positive effect and parasites have a negative effect on lampreys, and that these effects satisfy a linear relationship.

First, considering the model with no growth in the populations of competitors, symbionts, and parasites, the following equation can be obtained:

Rate of change in the population of lake trout:

$$\frac{dN}{dt} = r_t \times N \times (1 - \frac{N}{K_t}) - a \times N \times P \tag{10}$$

Rate of change in the number of competitors:

$$\frac{dCom}{dt} = -s \times Com \tag{11}$$

Rate of change in the number of symbionts:

$$\frac{dSym}{dt} = -s \times Sym \tag{12}$$

Rate of change in the number of parasitizers:

$$\frac{dPar}{dt} = -s \times Par \tag{13}$$

Rate of change in lampreys population due to sex ratio:

$$\frac{dP}{dt} = r_l \times P \times \left(1 - \frac{P}{K_l}\right) - s \times P + b \times N \times P - C \times com + S_b \times Sym \tag{14}$$

Where $r_l = r_{lb} \times sex, 0 < sex < 1$;

The trend of population size over time within the ecosystem including lampreys, lake trout, competitors, symbionts, and parasites was simulated by software algorithms, as shown in Figure 6 below:

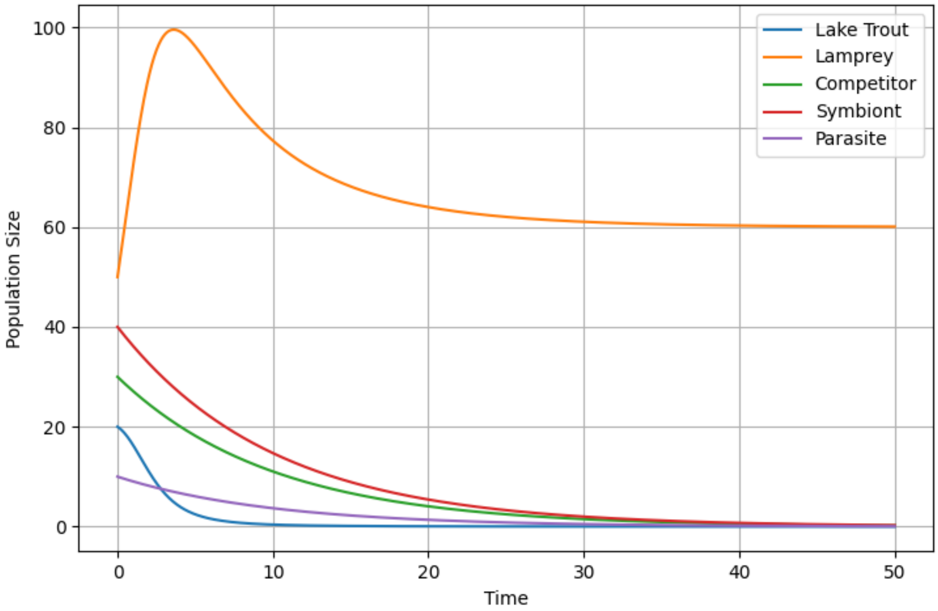

Figure 6. Trend of population size within the ecosystem over time

As shown in the figure, the lake trout and lampreys populations initially changed substantially over time and then stabilized, while the other three populations showed a declining trend.

This figure clearly shows how lampreys populations may be characterized as a typical invasive species in the Great Lakes of North America: a rapid increase followed by a rapid decline and ultimately reaching a new, more stable state. This pattern is often associated with explosive growth of an invasive species in a new environment where there are no natural enemies and resources are abundant, followed by a rapid decline due to a variety of factors (e.g., depletion of resources, spread of disease, predator acclimatization, etc.).

Of course, this graph also clearly shows that the gradual invasion of lampreys populations, over time, eventually tends to extinction of the remaining populations. The implications of this consequence are dire and will lead to a dramatic decrease in ecosystem stability.

However, since this section begins with the assumption that there is no growth in the number of competitors, symbionts, and parasites, this could also be the cause of the eventual extinction of the remaining populations. Therefore, in what follows, it is assumed that all modeled growth is consistent with the logistic model, and to minimize the error, natural mortality due to natural factors such as old age, disease, and other natural causes of death are added for lake trout in addition to energy gained due to predation, and the rest of the assumptions remain unchanged.

Based on the modified assumptions, the following equation can be obtained:

Rate of change in lake trout populations:

$$\frac{dN}{dt} = r_t \times N \times \left(1 - \frac{N}{K_t}\right) - a \times N \times P - s \times N \tag{15}$$

Rate of change in the Number of competitors:

$$\frac{dCom}{dt} = r_{com} \times Com \times \left(1 - \frac{Com}{K_{com}}\right) - s \times Com \tag{16}$$

Rate of change in the Number of symbionts:

$$\frac{dSym}{dt} = r_{Sym} \times Sym \times \left(1 - \frac{Sym}{K_{sym}}\right) - s \times Sym \tag{17}$$

Rate of change in the Number of parasitizers:

$$\frac{dPar}{dt} = r_{Par} \times Par \times \left(1 - \frac{Par}{K_{par}}\right) - s \times Par \tag{18}$$

Rate of change in the Population of lampreys:

$$\frac{dP}{dt} = r_l \times P \times \left(1 - \frac{P}{K_l}\right) - s \times P + b \times N \times P - C \times com + S_b \times Sym \tag{19}$$

Continuing with the software simulation, a trend graph can be obtained as shown in Figure 7:

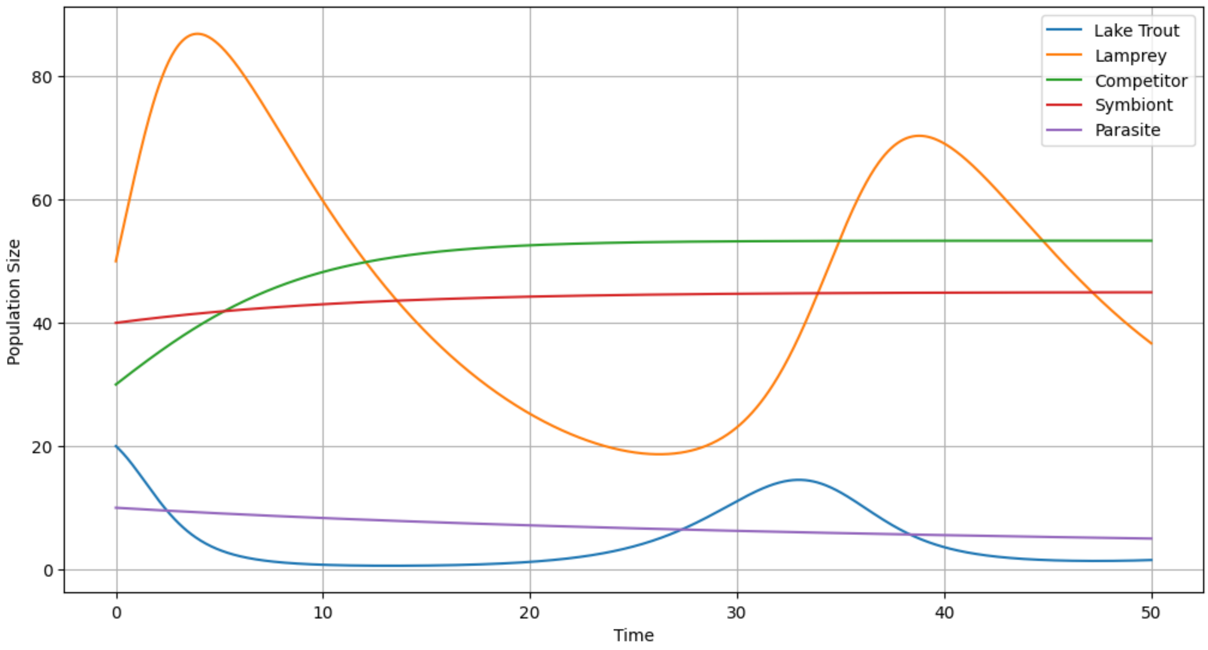

Figure 7. Trends in population size over time within ecosystems considering population growth

As can be seen from the figure, lampreys and lake trout are in a cyclical fluctuation state, similar to the model above; while the number of competitors first rises and then stabilizes; the number of symbionts grows slowly over time and tends to a certain lower, but stable, value; and the number of parasites, however, has been in a slow process of decreasing.

Based on these observations, this paper preliminarily estimates that lake trout are highly influenced by lampreys, while competitor populations are stable and may have occupied a stable position in the ecological niche, creating an equilibrium with lake trout and lampreys. Meanwhile the low and steady increase in the number of symbionts may reflect their interdependence with other populations, and in particular may be dependent on the lake trout population. In contrast, parasites are strongly influenced by other populations and cannot effectively adapt to fluctuations in the host population or to defense mechanisms developed by the host.

5. Conclusion

In this paper, a mathematical model based on the logistic model and the Lotka-Volterra model was developed to study the population's numerical changes and its effect on the stability of the ecosystem. When the sex ratio varies within a certain range, the number of lampreys populations decreases with the increase of the number of males and increases with the increase of the number of females; the number of competitors in the ecosystem firstly rises and then tends to be stabilized; and the number of various populations in the ecosystem remains in a stable state within a reasonable fluctuation range.

Based on the above predictions, the researchers can reasonably control the population size of lampreys and better study the interactions between the populations in the food web, and further regulate the number of related species based on the food web, which is of guiding significance to maintain the stability of the local ecosystem.

In the face of actual different survival environments, the model can be applied to predict the number of other exotic species and regulate the stability of the ecosystem, but with the change of the external environment and the change of the number of different populations, the applicability of the system may be reduced, so it is necessary that the relevant researchers need to strengthen the collection of data, fieldwork the ecosystems within the new scenarios, and simulation of the survival environment within the scenarios. This will help researchers to get high-precision prediction data and make reasonable and scientific decisions.

References

[1] Wilkie M P, Johnson N S, Docker M F. Invasive species control and management: The sea lamprey story [J]. Fish Physiology, 2022 (39): 489 - 579.

[2] S N J, D W S, O T B. Field study suggests that sex determination in sea lamprey is directly influenced by larval growth rate. [J]. Proceedings. Biological sciences, 2017, 284 (1851): 20170262 - 20170262.

[3] G. B, F. M D, S. N J, et al. Exploiting the physiology of lampreys to refine methods of control and conservation [J]. Journal of Great Lakes Research, 2021, 47: 723 - S741.

[4] F. M D, A. G B, J. C G, et al. A review of sea lamprey dispersal and population structure in the Great Lakes and the implications for control [J]. Journal of Great Lakes Research, 2021, 47 (S1): S549 - S569.

[5] Diogo F, Jackson C, W. D M, et al. Genetic control of invasive sea lamprey in the Great Lakes [J]. Journal of Great Lakes Research, 2021, 47 (S1): S764 - S775.

[6] ThresherE R, JonesM, DrakeR A D. Evaluating active genetic options for the control of sea lamprey (Petromyzon marinus) in the Laurentian Great Lakes [J]. Canadian Journal of Fisheries and Aquatic Sciences, 2018, 76 (7): 1186 - 1202.

[7] A H M, E E C. The evolutionary impact of invasive species. [J]. Proceedings of the National Academy of Sciences of the United States of America, 2001, 98 (10): 5446 - 51.

[8] B. J H, A. G B, Shane F, et al. What can commercial fishery data in the Great Lakes reveal about juvenile sea lamprey (Petromyzon marinus) ecology and management? [J]. Journal of Great Lakes Research, 2021, 47 (S1): S590 - S603.

[9] Haifang N, Gerrit K, Irene K, et al. Evaluation of Bayesian Hui-Walter and logistic regression latent class models to estimate diagnostic test characteristics with simulated data [J]. Preventive Veterinary Medicine, 2023, 217105972 - 105972.

[10] József D, Tamás J. Generalizing the sigmoid function using continuous-valued logic [J]. Fuzzy Sets and Systems, 2022, 44979 - 99.

[11] Aksoy Y, G?kta?, ünal, Pakdemirli M ,et al. Application of perturbation–iteration method to Lotka–Volterra equations [J]. Alexandria Engineering Journal, 2016: 1661 - 1666.