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A study of the competitive relationship between trout and bass based on differential equations

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Abstract. In this paper, the competitive relationship between trout and perch populations was analyzed by model differential equations, and the stability and final stabilization points under different parameters were explored. It is found that under the premise that global stability is better than local stability, trout and perch can coexist and reach the final stable state when the competition factors satisfy specific conditions; while under other competition conditions, it may lead to the number of one species tends to zero or the number of the other species tends to the maximum capacity. This has important implications for the understanding of biological population competition and the construction of competition models for software products.

Keywords: Competition modeling; Biological populations; Differential equations.

1. Introduction

The study of the stability of biological populations has always been an important and complex topic in ecology and competition theory. Competitive relationships among biological populations do not only affect their respective growth rates, but also have a profound impact on the balance and dynamics of the entire ecosystem. Competition modeling provides us with a theoretical framework to help us understand and predict the interactions among different biological populations [1].

In this paper, we will focus on the stability analysis under the competition model for biological populations [2]. We use trout and perch as representative competing populations, and model and analyze their population changes in the form of model differential equations. In this competition model, we considered many factors, including the growth rate of the population itself, the effect of competing populations on its growth rate, and the efficiency of resource utilization.

The stability of biological populations under the competition model is not only related to the local equilibrium problem, but also to the global stability and the long-term evolution of the ecosystem. Through the adjustment of model parameters and mathematical derivation, we can reveal the stability characteristics of biological populations under different competition conditions, thus providing important theoretical guidance and decision support for ecology and resource management [3-4].

Through the research in this paper, we hope to gain a deeper understanding of the mechanisms and laws of biological population stability under the competition model, to provide a scientific basis for ecosystem protection and management, and to promote the harmonious development of mankind and nature.

2. Basic mode

2.1. Terms, definitions and symbols

The curve of trout number (I) and bass number (II) are analyzed in the form of model differential equation. The growth rate of an application is affected not only by its own growth rate, but also by its competitive population.

Terms:

Global stability: it refers to the process that the system will be able to return to equilibrium if these external forces are eliminated, no matter what kind of external forces or disturbances the system is subjected to.

Local stability: the system is stable under the action of external force or disturbing force [5]. If the system moves in a certain range, it will be unstable beyond that range. This characteristic is called local stability. The description of variable is shown in table 1.

Table 1. The description of variable

Variable	Description
dx/dt	The growth rate of trout
$(1 - x / k_1)$	With the increase of trout number, the blocking effect on its own growth rate
bxy	Effect of perch number on trout growth
dy/dt	Growth rate of perch users
$(1 - y / k_2)$	With the increase of perch population, the blocking effect on its own growth rate
nxy	Effect of trout on the growth of perch

2.2. Hypothesis

The model parameters are shown in table 2:

Table 2. The description of variable

Variable	Description
a	1
m	1
k1	100
k2	100

b: The limited living resources consumed by a unit number of bass (relative to k2) for trout are b times of the limited living resources consumed by a unit number of trout (relative to k1).

n: The limited living resources consumed by a unit number of trout (relative to k1) are in times of the limited living resources consumed by a unit number of bass (relative to k2).

Assume that a, m, k1, k2, b, b are all fixed values, which do not change with time and a=m=1, k1=k2=100.

For model (2), b and n are independent of each other. In some special cases, bn=1. When bn is not equal to 1, we solve (2) to get four equilibrium points $\mathrm{P1}(\mathrm{k}_1, 0), \mathrm{P2}(0, \mathrm{k}_2), \mathrm{P3}\left(\frac{\mathrm{k}_1(1-\mathrm{b})}{1-\mathrm{bn}}, \frac{\mathrm{k}_2(1-\mathrm{n})}{1-\mathrm{bn}}\right), P4(0,0)$

In this topic, we extend the competition among biological populations to the competition of software products, so we prefer the global stability to the local stability. That is, regardless of the initial value, the global stable equilibrium point is stable [6-7].

Combined with this topic, we first study the local stability of the equilibrium point. Let's look at the matrix first according to the judgment of the stability of the equilibrium band.

$$\mathrm{A} = \begin{bmatrix} f_{x_1} & f_{x_2} \\ g_{x_1} & g_{x_2} \end{bmatrix} \begin{bmatrix} a\left(1 - \frac{2x}{k_1}\right) - b\frac{y}{k_2} & -ab\frac{x}{k_2} \\ -mn\frac{y}{k_1} & m\left(1 - n\frac{x}{k_1} - \frac{2y}{k_2}\right) \end{bmatrix} \quad (1)$$

Therefore, P=-(+)$\mathrm{f}_{\mathrm{x}_1}\mathrm{g}_{\mathrm{x}_2}|\mathrm{p}_\mathrm{i}, \mathrm{q} = \det\mathrm{A}|\mathrm{p}_\mathrm{i}, \mathrm{i} = 1,2,3,4.$

When q>0, it is locally stable at this point; When q<0, it is unstable.

When it is preliminarily determined that the point is locally stable, we use the phase trajectory to determine whether it is globally stable.

List algebraic equations:

$$\varphi(x,y) = 1 - \frac{x}{k_1} - b\frac{y}{k_2}, \psi(x,y) = 1 - n\frac{x}{k_1} - \frac{y}{k_2} \tag{2}$$

At this point, we see

$$\begin{cases} x(t) = ax\left(1 - \frac{x}{k_1} - b\frac{y}{k_2}\right) = ax\varphi(x_1, x_2) \\ y(t) = my\left(1 - n\frac{x}{k_1} - \frac{y}{k_2}\right) = my\psi(x_1, x_2) \end{cases} \tag{3}$$

Therefore, when determining whether the point is globally stable, we need to observe the positional relationship between straight line =0 and straight line =0 and the sign of derivative in each region.φω

(1) When b < 1, n < 1

The differential equation can be solved when t tends to positive infinity, when trout and perch can coexist, and the number of coexistences can be determined:

(Number of trout, number of bass) is stable at P3.$\left(\frac{k_1(1-b)}{1-bn}, \frac{k_2(1-n)}{1-bn}\right)$

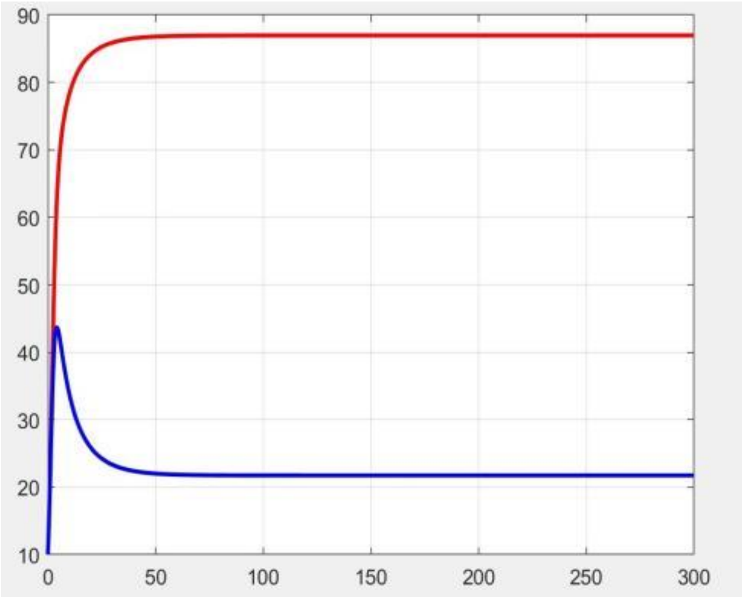

Figure 1. Graphs Called Population Size Changes

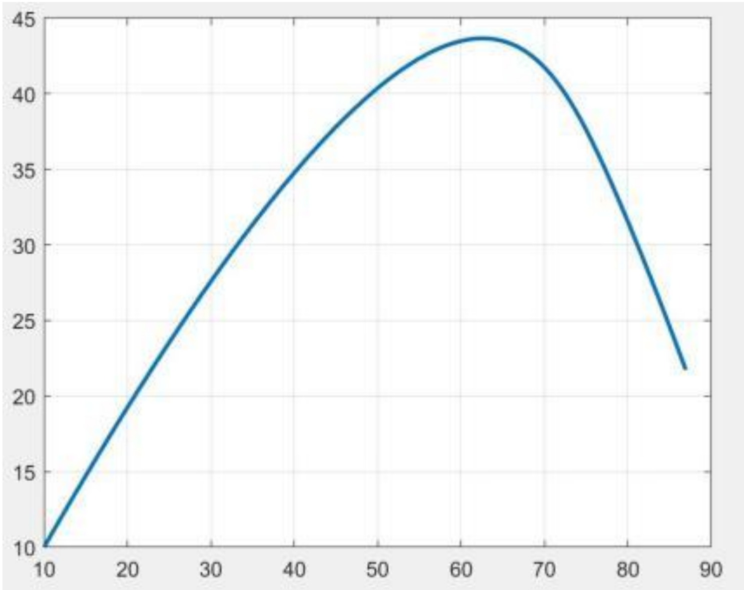

Figure 2. Analysis of population size track lines

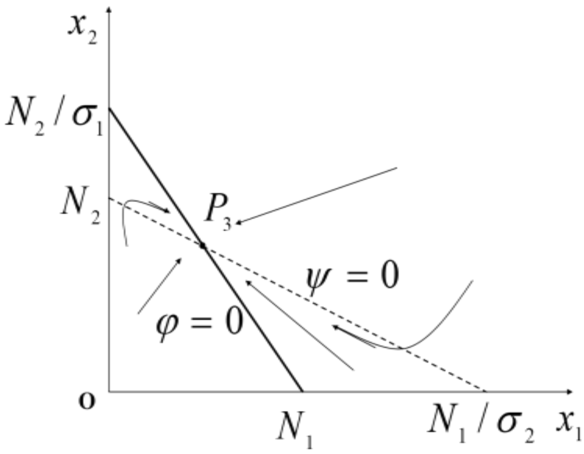

Figure 3. Solving for Stability Points

According to figure 1, figure 2 and figure 3, the stability of the solution is verified by phase trajectory analysis, and the streamline in the first quadrant (corresponding to the practical solution in the first quadrant) converges to the equilibrium point P3, which shows that P3 has very strong stability when $b < 1$, $n < 1$, and this equilibrium point is the final stable point of any initial condition, that is, the global stable point we are looking for [8-9].

(2) when $b > 1$, $n > 1$

Solving the differential equation can make the number of trout and perch unstable when T approaches positive infinity.

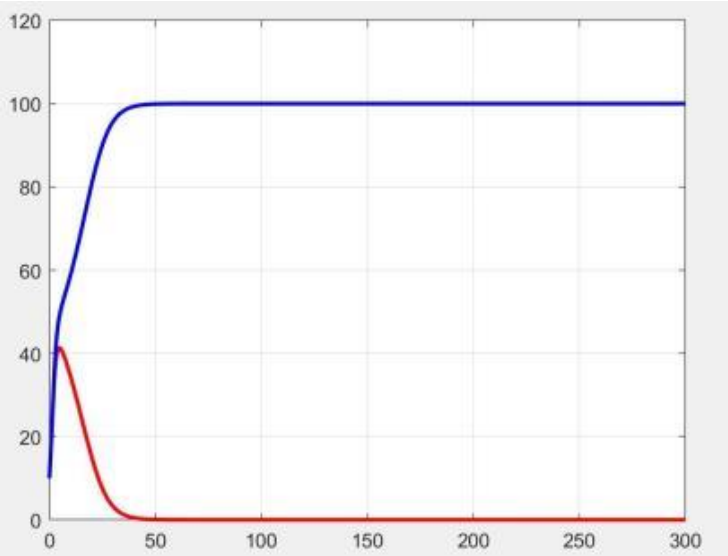

Figure 4. Graphs Called Population Size Changes

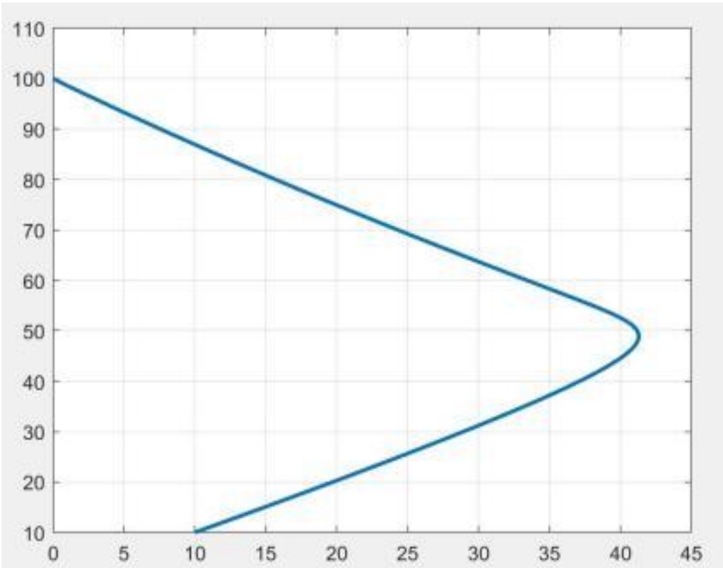

Figure 5. Analysis of population size track lines

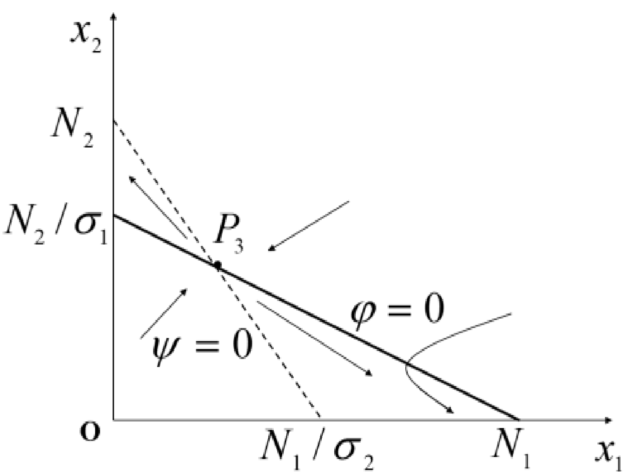

Figure 6. Solving for Stability Points

According to figure 4, figure 5 and figure 6, the stability of the solution is verified by phase trajectory analysis. The streamline in the first quadrant (corresponding to the practical solution in the first quadrant) converges to the equilibrium point P3, while the streamline converges to other points, which shows that P3 is unstable when b > 1, n > 1.

(3) when b > 1, n < 1

When t approaches positive infinity, the number of trout and perch is stable, the number of trout tends to zero, and the number of perch tends to the maximum of its environmental capacity [10].

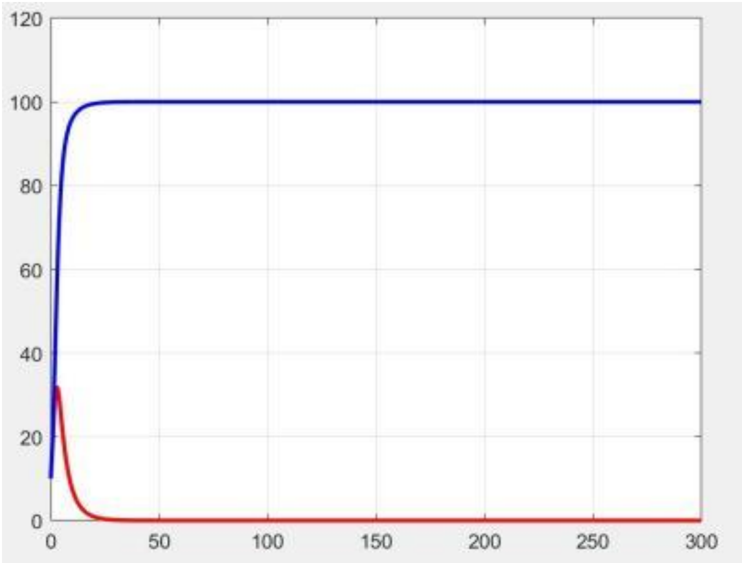

Figure 7. Graphs Called Population Size Changes

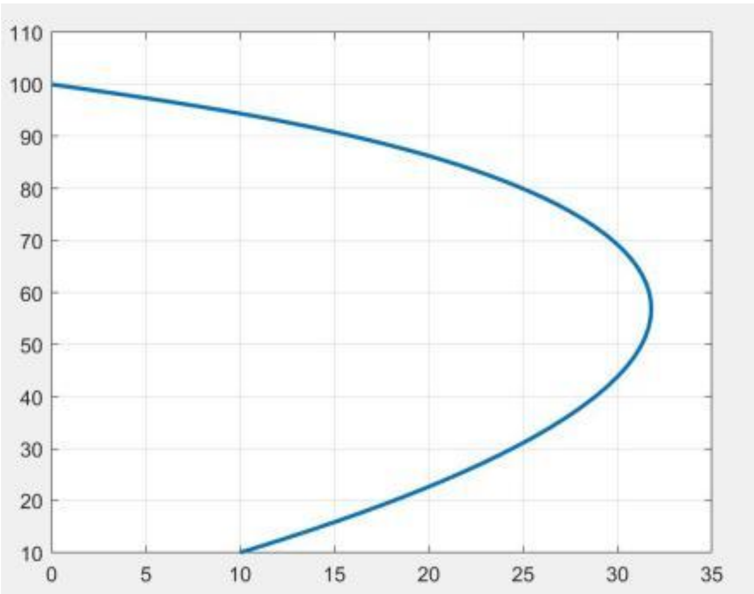

Figure 8. Analysis of population size track lines

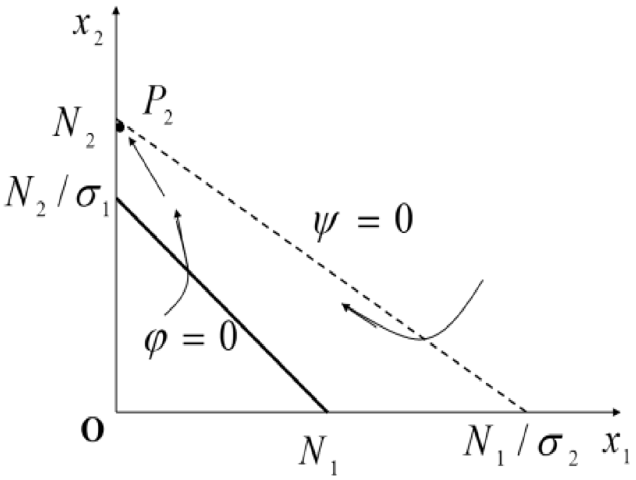

Figure 9. Solving for Stability Points

According to figure 7, figure 8 and figure 9, the stability of the solution is verified by the phase trajectory analysis, and the streamline in the first quadrant (corresponding to the practical solution in the first quadrant) converges to the equilibrium point P2, which shows that when $b > 1$ and $n < 1$, P2 has very strong stability, and this equilibrium point is the final stable point of any initial condition, that is, the global stable point we are looking for.

(4) when $b < 1$, $n > 1$

By solving the differential equation, it can be found that when t tends to positive infinity, the number of trout tends to the maximum of environmental capacity, and the number of perch tends to zero.

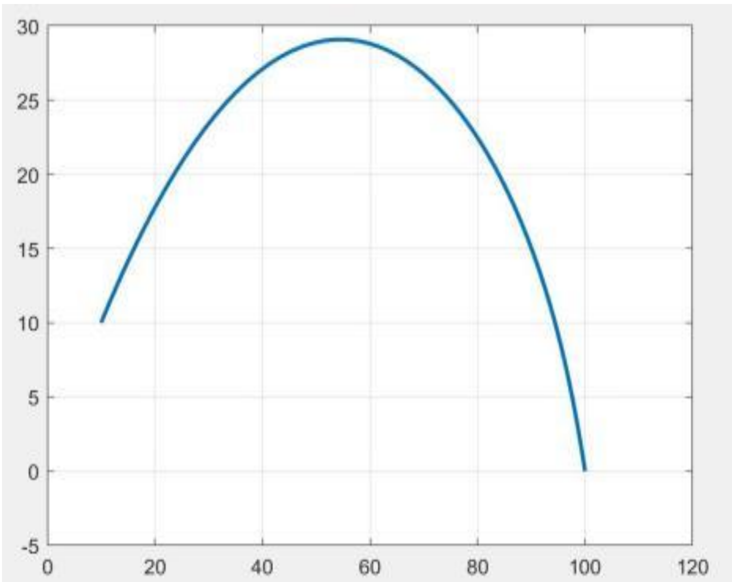

Figure 10. Graphs Called Population Size Changes

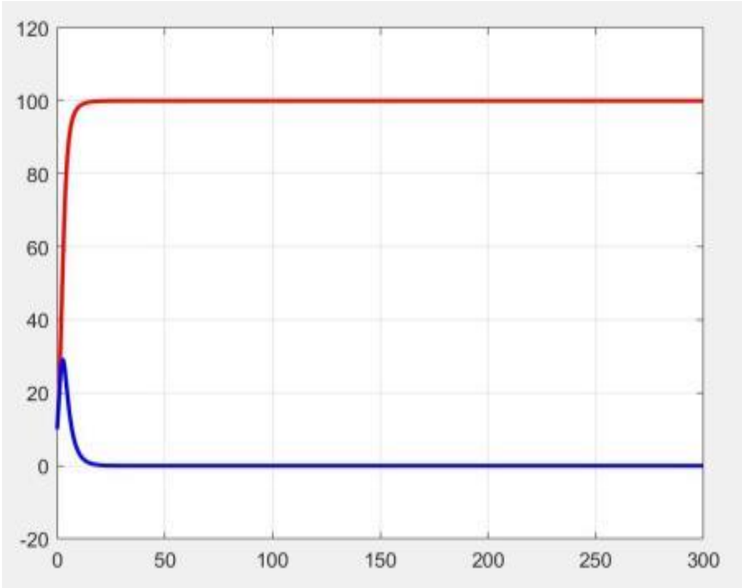

Figure 11. Analysis of population size track lines

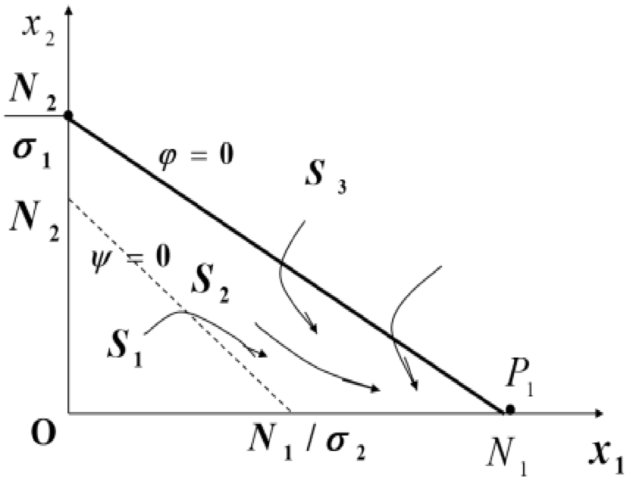

Figure 12. Solving for Stability Points

According to figure 10, figure 11 and figure 12,The stability of the solution is verified by phase trajectory analysis, and the streamline in the first quadrant (corresponding to the practical solution in the first quadrant) converges to the equilibrium point P1, which shows that when $b < 1$, $n > 1$, P1 has very strong stability, and this equilibrium point is the final stable point of any initial condition, that is, the global stable point we are looking for.

According to the above classification discussion:

When $b<1$, $n>1$, $b<1$ means that the trout is weaker than the trout in the competition for trout resources, and $n>1$ means that the trout is stronger than the perch in the competition for trout resources, so the perch will eventually become extinct, and the trout population will tend to the maximum capacity, thus tending to the equilibrium point P1.

When $b>1$ and n $b> 1$, $n> 1$ means that trout is stronger than trout in the competition for resources supporting trout, and $n>1$ means trout is stronger than bass in the competition for resources supporting trout, which is unstable due to the strong competition between trout and trout.

When $b>1$, $n<1$, $b>1$ means that the trout is stronger than the trout in the competition for trout resources, and $n<1$ means that the trout is weaker than the trout in the competition for trout resources, so the trout will eventually become extinct, and the perch population will tend to the maximum capacity, so it will tend to the equilibrium point P2.

When b<1 and n<1, b < 1 means that trout is weaker than trout in the competition for trout resources, and n < 1 means that trout is weaker than trout in the competition for trout resources, and they can coexist in a certain range, so they approach the equilibrium point p3.

3. Conclusion

This paper focuses on the stability of biological populations under the competition model, and through modeling and analysis of model differential equations, the population change law of trout and perch as competing populations is discussed in depth. Through the adjustment of model parameters and mathematical derivation, conclusions on the stability under different competition conditions were drawn. These research results provide important theoretical guidance for ecology and resource management, and help to promote the harmonious development of human and nature.

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