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Dynamic food web model based on Lotka Volterra model

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Abstract. Lampreys are regarded as parasites with significant impacts on the ecosystem in some lake habitats, and analyzing the relationship between the population characteristics of lampreys and the ecosystem is of great significance for improving the ecological environment and maintaining the ecological balance. The sex ratio of lampreys varies depending on the external environment. To gain insight into the issue of sex ratios of lampreys in ecosystems and their dependence on local conditions, this paper firstly develops a Lotka-Volterra model to characterize the effects of changes in the sex ratio of lampreys on predators. The dynamics of male and female lampreys were modeled separately. A reproduction rate impact factor k was introduced to describe the effect of the poison TFM on lamprey populations, considering human intervention. A pattern of change in predator population size was derived: the proportion of males was negatively correlated with the number of predators. Considering that the larvae and adults of the lamprey are in different ecological niches, the model was further optimized to establish a dynamic food web model containing multiple trophic levels, and to derive the changes in the number of groups in the ecosystem over time. These data were used to construct a comprehensive evaluation index of ecological stability, establish an AHP-PCA integrated model, calculate the weights of each index, and analyze the comprehensive evaluation index. It was concluded that the proportion of males in the lamprey population fluctuated regularly over time, and when the proportion of males in the lamprey population increased, the comprehensive evaluation index declined, forming a fluctuating trend like that of the lamprey population.

Keywords: Lamprey; Lotka-Volterra Model; Dynamic Food Web Model; AHP-PCA Integrated Model.

1. Introduction

In nature, most species exhibit dioecious sexual differentiation into females and males. However, not all species have an even sex ratio, and the sex ratio of some species can change under certain external environmental conditions, as in this paper, the lamprey is a species whose sex ratio changes with the environment. The lamprey is a semi-parasitic organism [1], which is able to securely attach itself to a fish with its sucker-like mouth, then use its teeth to bite through the host's body and burrow into the host to feed on body fluids and flesh. Lampreys have a huge impact on the ecological environment of some lakes and habitats. For example, the invasion of lampreys in the Great Lakes region of North America in the 20th century caused great damage to the ecology of the Great Lakes of North America, and most of the fish in the Great Lakes, including whitefish, salmon, and catfish, were parasitized by the lamprey, which made the fish caught by fishermen scarred and unsaleable. The incalculable damage caused by the proliferation of lampreys to the Great Lakes fisheries has severely impacted the fisheries production in the Great Lakes region [2]. Studies have been conducted to quantify the sex ratios produced in unproductive lake and productive stream environments by tagging and releasing juvenile sea lampreys into these environments, recapturing tagged adult individuals, and concluding that sex determination of lampreys is directly influenced by larval growth rate resulting in sex ratio differences [3]. The purpose of this paper is to examine how changes in the sex ratio of the lamprey may in turn differentially affect local ecosystems.

Just as invasive species such as the Fusiliers and the Canadian Monarch have caused serious harm to biodiversity, threatening and striking a blow to agricultural security and ecosystems [4], as early as 1932 the journalist Lillian introduced readers to the lampreys in the middle of the sea and pointed out that "because the lampreys have been hunting fish, they have actually become an economic problem ". And because the impact of the sex ratio of lampreys on the surrounding ecological environment has rarely been explored, this paper investigates the impact of the change of the sex ratio of the lamprey on the ecosystem to control the threat to the ecological environment, to maintain the balance of the ecosystem and the protection of biodiversity has extraordinary practical significance and ecological value, but also to reduce the related economic losses.

Any biological populations are in a certain community, and with other populations occur a certain connection, so there are both interdependence and mutual constraints between different populations, ecology will be the interaction between different species is called interspecific relationships. Among them, predation is the most common interspecific relationship, and many scholars have established models based on the interaction relationship between two species and their own characteristics, and studied the dynamics of the models. Regarding the dynamics of predation model Lotka and Volterra proposed the famous Lotka-Volterra population model [5], which is often used in the study of ecosystems.

In this paper, we used the predator-feeder model in the Lotka-Volterra model to describe the interactions of the lamprey population with the ecosystem when the sex ratio changed, and analyzed the effects of the lamprey on the ecosystem under the change of sex ratio. The Lotka-Volterra model was also improved and combined with the sex ratio adjustment function to construct a dynamic food web model including producers, primary consumers and secondary consumers. A combined AHP-PCA model was developed to further analyze the stability of the ecosystem by constructing indicators to assess the stability of the ecosystem.

2. Effects of Lamprey Populations on the Ecosystem

This paper uses data related to the number of lamprey populations in the Great Lakes, trout populations, and survival of lamprey larvae in response to human use of drugs such as TFM [6]. The data sources are summarized in Table 1.

Table 1. Data source collation

Database Name	Database Websites	Data Type
GLFC	http://www.glfc.org:3838/slcp/	Geography
ScienceDirect	https://www.sciencedirect.com/	Academic journal

2.1. Lotka-Volterra Model Introduction

The Lotka-Volterra model is a set of differential equations that describe the dynamics of predator-prey interactions in ecological systems. It predicts oscillations in the populations of prey and predators, indicating a dynamic balance between the two populations [7-8]. The basic model consists of two equations, one for the prey population and one for the predator population. The equations are as follows:

$$\begin{cases} \frac{dx}{dt} = x[\alpha_1 + \beta_1 x + \gamma_1 y] \\ \frac{dy}{dt} = y[\alpha_2 + \beta_2 x + \gamma_2 y] \end{cases} \tag{1}$$

The equations describe changes in predator and prey populations respectively. Where $\alpha_i, \beta_i, \gamma_i (i = 1,2)$ are constants, β_1, γ_2 reflect the density action coefficients of the two populations, called intraspecific action coefficients, respectively, γ_1, β_2 reflect the factors that the two populations interact with each other, called interspecies action coefficients, and α_1, α_2 denote the in-growth rates of the two populations, respectively.

For the parasitism of the lampreys studied in this question, the predator-feeder model in the Lotka-Volterra model applies, i.e., one species acts as a feeder for the other species. The parasite is the predator and the host is the predator food bait, at this time the equation (1), $\gamma_1 \times \beta_2 < 0$, and $\gamma_1 > 0, \beta_2 < 0$, then the calculation formula for the model is:

$$\begin{cases} \frac{dx}{dt} = x[\alpha_1 - b_1 x + c_1 y] \\ \frac{dy}{dt} = y[\alpha_2 - b_2 x - c_2 y] \end{cases} \tag{2}$$

y is the number of the predator.

2.2. Model Building

This paper uses the Lotka-Volterra model to describe the interactions between the lampreys and their ecosystem. Based on the fact that humans use relevant means to control the population size of lampreys and to study the effects of different sex ratios, this paper models the dynamics of male and female lampreys separately:

$$\begin{cases} \frac{dy_1}{dt} = r_1 y_1 k - d_1 y_1 + b_1 \alpha x y_1 \\ \frac{dy_2}{dt} = r_2 y_2 k - d_2 y_2 + b_2 \alpha x y_2 \\ \frac{dx}{dt} = r_3 x - \alpha x (y_1 + y_2) \end{cases} \tag{3}$$

Where y_1, y_2 is the population size of male and female lampreys, respectively, r_1, r_2 is is the reproduction rate of male and female lampreys, respectively, d_1, d_2 is the mortality rate of male and female lampreys, respectively, b_1, b_2 is the initial feeding efficiency (the efficiency of converting food into population growth) of male and female lampreys, respectively, α is the probability of the prey being preyed upon, x is the population size of the prey, r_3 is the natural rate of growth of the prey, and k is the survival rate of lamprey larvae under the effects of the TFM and Bayluscide drugs.

In this model, the lamprey as the predator, its impact on the ecosystem is mainly reflected in the changes in the number of its own population and the number of its prey population. Since the curves of the population numbers of male and female lampreys respectively over time are not convenient to analyze the impact on the ecosystem when the population can change the sex ratio, this paper analyzes the change rule as a curve of the sex ratio over time and draw conclusions.

2.3. Model Results

In equation (4), this paper sets the values of each parameter. Where male and female lamprey reproduction rate $r_1 = r_2 = 0.2$, survival rate of lamprey larvae under the effect of TFM and Bayluscide drugs $k = 0.1$, and the natural mortality rate of female lampreys is relatively high compared to that of males because they die after spawning and reproduction. Some of the data used in the modeling calculations came from the data this paper collected on lampreys in the Great Lakes, and calculate the initial predation efficiencies of male and female lampreys accordingly.

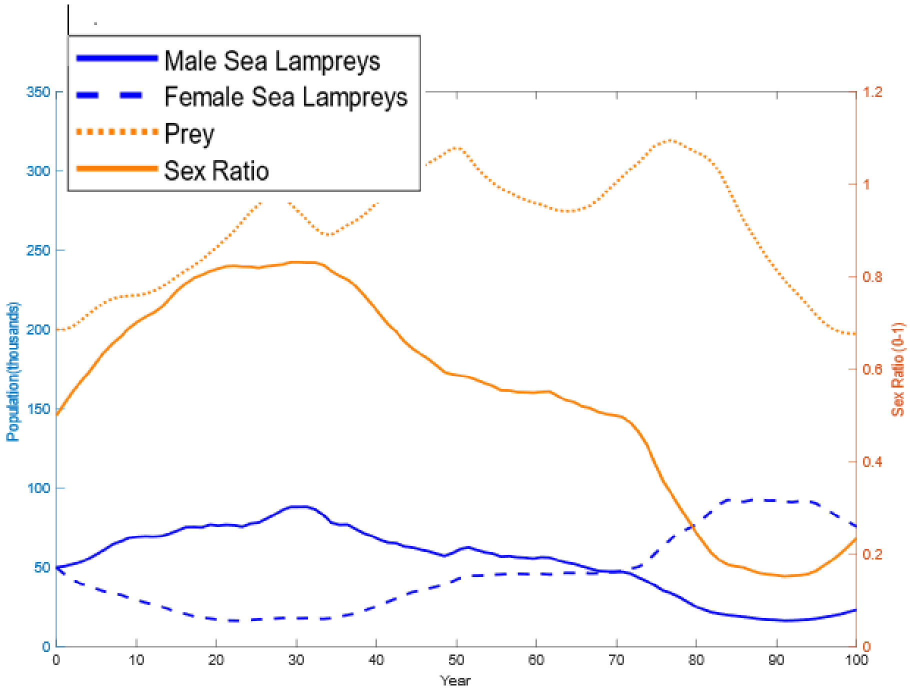

Figure 1. When the initial ratio of the number of male lampreys is 1, male and female lamprey population change chart

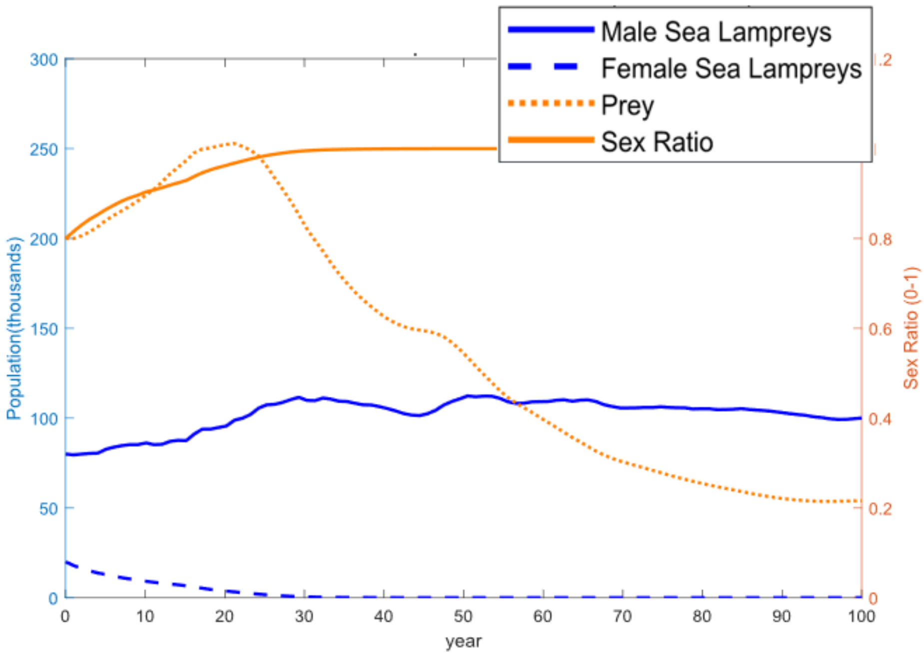

Figure 2. When the initial ratio of the number of male lampreys is 4, male and female lamprey population change chart

Through simulation, this paper obtains a chart of the variation in the effect of sex ratio of male and female lampreys on the number of the prey, and analyzing the above Figure 1and Figure 2, it can be concluded that when the number of male lampreys is higher, it leads to a decrease in the number of the prey, and it can be found that the sex ratio of the lampreys increases and the number of the prey decreases as the process progresses.

3. Result

3.1. Dynamic Food Web Model

In this part, a dynamic food web model including the lamprey as well as other related species is first constructed. Consider the interactions between different species in the ecosystem, including food chains, predation and competition relationship [9]. For the food web in the ecosystem analyzed in this paper, the producers are plankton and other organisms in the water, the primary consumers are the larvae of the lamprey and the prey, and the secondary consumers are the lamprey and other competitors that also feed on the prey.

3.1.1. Sex Ratio Adjustment Function.

First, this part will consider the following food availability to lampreys as the number of the preys. For the analysis of the relationship between sex ratio and food availability, this paper will model this relationship using the Sigmoid function, which is a nonlinear threshold function for neurons with outputs ranging from 0 to 1, which makes it particularly suitable for outputting probability values or for generating activation values in neural networks. Its mathematical expression:

$$\varphi(x) = \frac{1}{1+e^{-x}} \tag{4}$$

For the problem of this research, this paper defines a Sigmoid function - sex ratio adjustment function:

$$\varphi(z) = \frac{1}{1+e^{-k(z-z_0)}} \tag{5}$$

Where $\varphi(z)$ is the proportion of the total population size accounted for by male lampreys, z is the level of food availability, i.e., the number of the prey, z_0 is the food availability threshold that affects the sex ratio, and k is a parameter that affects the steepness of the curve.

3.1.2. Model Building.

This paper optimizes and improve the Lotka-Volterra model built in 2.1 and incorporate the sex ratio adjustment function $\varphi(z)$ in 3.1.2 to build a dynamic food web model by incorporating the relationship between producers, primary consumers and secondary consumers into the model. The improved Lotka-Volterra model is as follows (6) ~ (8):

The producer:

$$\frac{dz}{dt} = r_z z - \alpha_1 zP - \alpha_2 zy' \tag{6}$$

The primary consumer:

$$\begin{cases} \frac{dP}{dt} = r_P P - \alpha_3(y_1 + y_2)P - \alpha_4 y_3 P - d_P P + b_4 \alpha_1 Pz \\ \frac{dy'}{dt} = r y_2 - (m + d)y' \end{cases} \tag{7}$$

The secondary consumer:

$$\begin{cases} \frac{dy_1}{dt} = y'\varphi(z) - d_1y_1 + b_1\alpha_3Py_1 \\ \frac{dy_2}{dt} = y'(1-\varphi(z)) - d_2y_2 + b_2\alpha_3Py_2 \\ \frac{dy_3}{dt} = r_3y_3 - d_3y_3 + b_3\alpha_4Py_3 \end{cases} \quad (8)$$

A brief explanation of each of the new parameters in the above equations is given next. z, y', y_3 are the number of producers (i.e., plankton in the water, etc.), lamprey larvae, and other competitors of the lamprey, respectively. It is known that humans can use the drugs TFM and Bayluscide to kill the larvae of the lamprey, thereby controlling the lamprey population size, nd thus this paper introduces the parameters m, denotes the mortality rate of lamprey larvae under the effect of drugs, and d denotes the natural mortality rate of lamprey larvae.

3.1.3. Model Results.

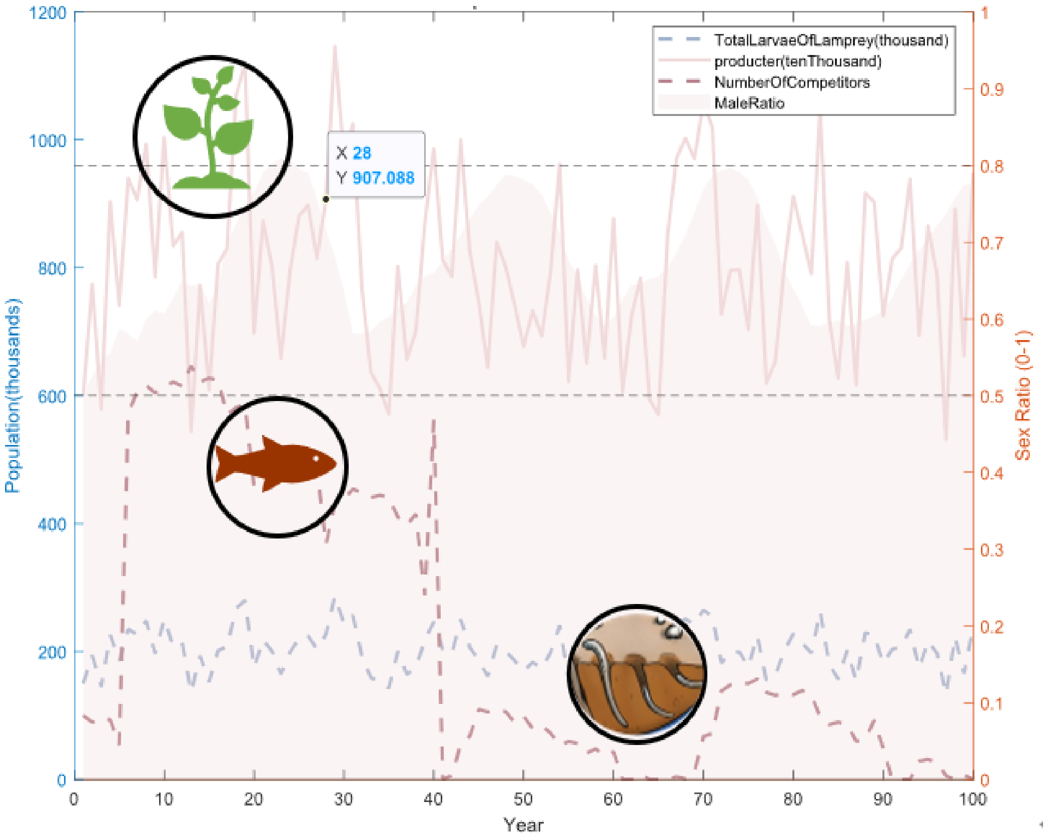

Figure 3. Plot of the producer, juvenile lampreys, and other competitors against sex ratios

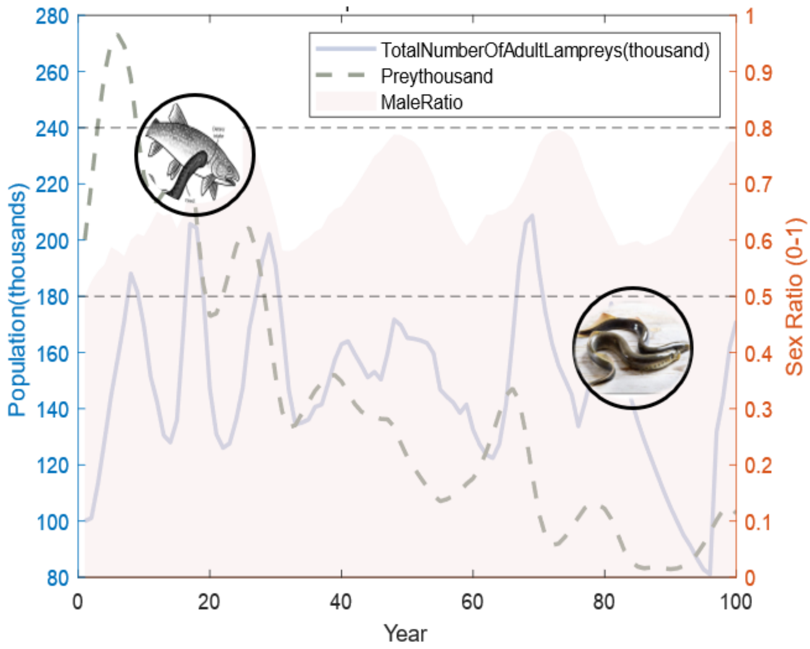

Figure 4. Plot of adult lampreys and predators versus sex ratio

This paper used the method of simulated ecosystems, and the analysis of Figure 3 shows that the sex ratio is maintained between the double dashed lines, and the proportion of males varies dynamically between 50% - 80%. At this point, the curve of the number of the producer maintains dynamic fluctuation at a certain level height, and the number of competitors increases at the beginning, and due to the increase of the number of lampreys, the number of competitors decreases to a lower level. Lamprey larvae, as primary consumers, fluctuate at a more stable level similar to the changes in the number of producers. Plot of adult lampreys and predators versus sex ratio is shown in Figure 4. Over the 100 years of the simulation, the populations of all species are in dynamic fluctuation, reflecting the sustainability of the ecosystem.

3.2. Ecosystem Stability Indicator System Based on AHP-PCA Combined Model

3.2.1. Model Introduction.

This paper adopts the combined entropy weight model of AHP-PCA to determine the optimal weight value of each evaluation index.

AHP determines the subjective weights: AHP constructs the judgment matrix by determining the target layer, criterion layer, element layer and indicator layer, and annotating the importance of each indicator using Satty's 1~9 scale method. The weight value of each indicator is obtained through eigenvector normalization, and the consistency test is carried out by using the consistency ratio (C_R) with the following formula.

$$C_R = \frac{C_I}{R_I} = \frac{\lambda_{max}-n}{R_I \times (n-1)} \tag{9}$$

Where C_I,R_I is the stochastic consistency index, C_R is the average stochastic consistency ratio, n is the number of elements, and λ_{max} is the maximum characteristic root of the judgment matrix. When $C_R < 0.1$, the judgment matrix is considered to have satisfactory consistency, and the weights of the model evaluation indicators are calculated reasonably.

PCA determines the objective weights: firstly, this paper needs to construct the evaluation index matrix $A = (x_{ij})_{m\times n}$, then calculate the index characteristic ratio r_{ij}, then calculate the information entropy S_j of the jth index according to the characteristic ratio r_{ij}, and finally calculate the corresponding index weight v_j, whose formulas are as follows (10) ~ (12):

$$r_{ij} = \frac{b_{ij}}{\sum_{i=1}^{m} b_{ij}} \tag{10}$$

$$S_j = -(lnm)^{-1} \sum_{i=1}^{m} r_{ij} lnr_{ij} \tag{11}$$

$$v_j = \frac{1-S_j}{\sum_{j=1}^{n} (1-S_j)} \tag{12}$$

Where r_{ij} is the characteristic ratio of the evaluation index matrix, S_j is the information entropy of the evaluation index, and v_j is the objective weight of the evaluation index.

AHP-PCA combination model to determine the comprehensive weights: after determining the subjective weights and objective weights of the evaluation indexes, the multiplier normalization method is used to calculate the comprehensive weights, and the formula is:

$$a_j = \frac{v_j P_j}{\sum_{j=1}^{n} (v_j P_j)} \tag{13}$$

Where a_j is the comprehensive weight determined by AHP-PCA combination model, v_j is the objective weight of evaluation indexes, and P_j is the subjective weight of evaluation indexes.

3.2.2. Model Building.

According to the characteristics of the lampreys ' own population and the characteristics of the ecological environment in which it survives, the ecosystem stability evaluation system is more suitable for the multi-level and multi-indicator indicator framework system. The study shows that the method of decomposing the weight structure of each indicator into subjective weights reflecting the degree of influence of different indicator attributes on vulnerability and objective weights characterizing the degree of influence of the amount of information of diffe- rent indicators on them, solving the problem using hierarchical analysis (AHP) and principal component analysis (PCA) respectively, and introducing the minimum relative entropy model to complete the computation of their combined weights has a high degree of accuracy[10]. Based on the AHP-PCA combination model mentioned above, the ecosystem stability indicator system model is composed by adopting the relationship framework of target layer-criteria layer-indicator layer, and providing indicators that can comprehensively reflect the evaluation of ecosystem stability based on the principles of assessment indicator selection, and the model

structure is shown in the Figure 5 below:

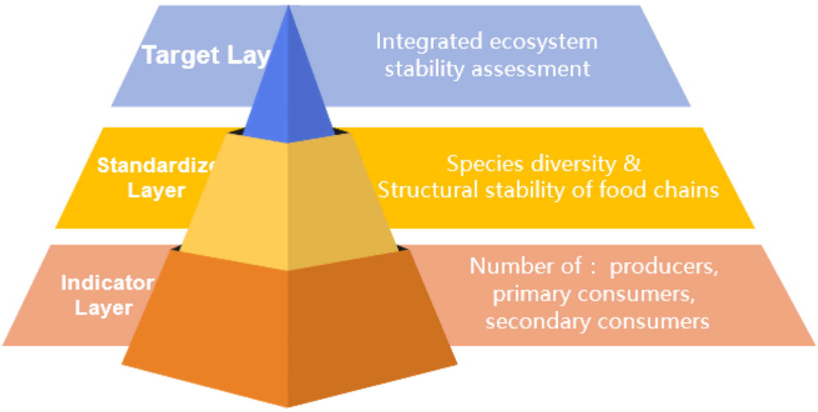

Figure 5. Chart of the model structure

In order to verify whether the selected indicators are suitable for principal component analysis, i.e., to verify whether the values of the indicators are independent of each other, the indicators are examined with the help of Bartlett's sphericity test.

Table 2. Bartlett's Spherical Test Results

KMO	Approximate chi-square	P
0.619	322.835	0.000

The results in Table 2 show that it is seen that KMO=0.619 and Bartlett's test of sphericity P=0.000<0.05, therefore the selected indicators are suitable for principal component analysis. The criterion for judgment here is KMO>0.5 and P<0.05, then it is suitable for principal component analysis.

The model calculation introduces the variables E for species evenness, D for species richness, C_1 for the total number of individuals of primary consumers, and C_2 for the total number of individuals of secondary consumers. If each variable contains information of 1, the extraction of principal components is carried out here, and the more information extracted from each variable, the better the information is condensed, which can be seen from the table that the extraction of each common factor is greater than 0.5, so the information is condensed better. The results are shown in the Table 3 below:

Table 3. Information on the Extraction of the Variance of the Common Factor

E	D	C_1	C_2
0.926	0.572	0.563	0.871

This paper also extracted one public factor, ecosystem stability, using principal component analysis. Using the maximum variance method, the eigenvalue of component 1 is 2.857, and the variance contribution rate of these two is 71.432%. In general, the original indexes have less information loss, and the effect of principal component analysis is more desirable, which is of significance to the study.

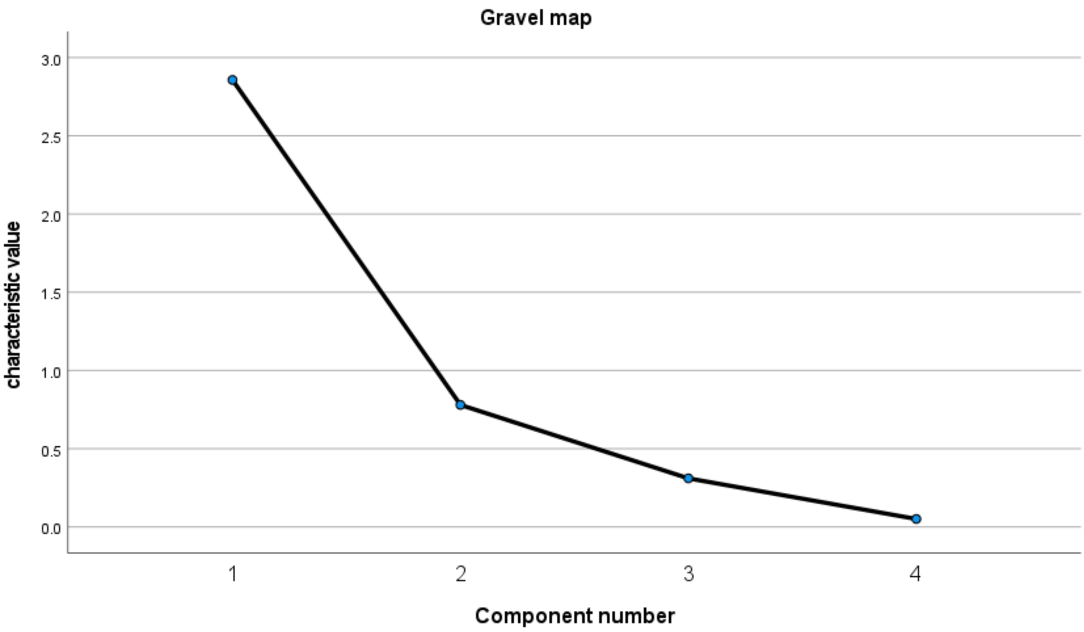

Figure 6. Gravel map

From the above Figure 6, it can be seen that the higher the point, the higher the potential energy, the more important it is for the study, the first factor has the highest eigenvalue and contributes the most to explaining the original question items, the eigenvalues from the 2nd onwards are smaller and contribute little to explaining the original question items and can be ignored, so extracting 1 factor is more appropriate.

3.2.3. Model Results.

Table 4. Results of weighting of evaluation indicators

Target Layer	Standardized Layer	Indicator Layer
Ecosystem stability	Species Homogeneity	0.337
	Species Richness	-0.265
	Ratio of number of producers to number of primary consumers	0.245
	Ratio of secondary to primary consumers	0.327

From the weights in the above Table 4, the formula for the comprehensive evaluation indicator is obtained:

$$Comprehensive\ indicators\ = 0.337E - 0.265D + 0.245C_1 + 0.327C_2 \tag{14}$$

Substituting the simulated data to plot the curve of the comprehensive evaluation indicator, its relationship with the change in the sex ratio is shown in the Figure 7 below:

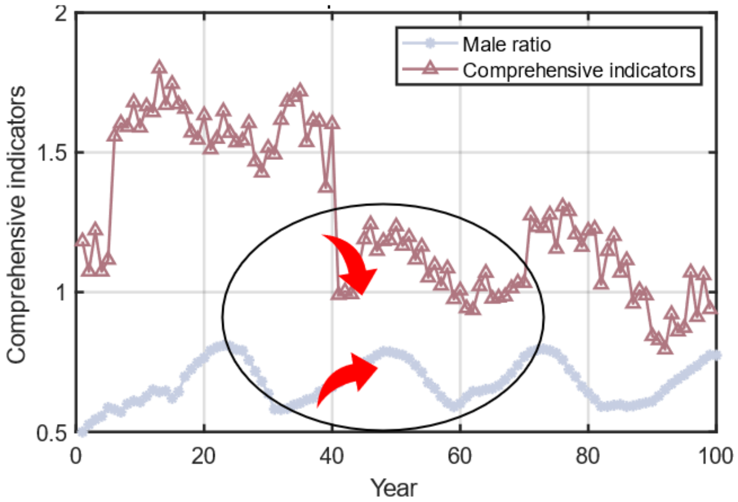

Figure 7. Comprehensive indicators chart

Parameters in the ecosystem are extracted as indicators, the weights are analyzed, and the relationship between ecosystem stability and the proportion of males reflected by the comprehensive indicators is analyzed. It can be seen from the above figure that, over time, when the comprehensive indicators for evaluating the ecosystem fluctuate, the proportion of males in the population of the lamprey rises in order to adapt to changes in the environment and make full use of the environmental resources.

Analyzing the relationship between the sex ratio and the comprehensive evaluation index in the black box, the comprehensive evaluation index decreases when the proportion of males increases, which corresponds to a poor ecological environment and is consistent with the actual situation.

4. Conclusion

In this paper, to address the ecological impacts of the sex ratio of the lamprey, a Lotka-Volterra model was constructed to describe the role of changes in the sex ratio of the lamprey on predators, modeling male and female lampreys separately, and considering anthropogenic interventions, the effect of the toxicant TFM on the population of the lamprey was introduced by introducing the reproduction rate impact look-alike k. In addition, a dynamic food web model containing multiple trophic levels was constructed in this paper. The changes in the number of various groups in the ecosystem over time were deduced by building the food web model of producers, primary consumers and secondary consumers, and these data were used to construct comprehensive evaluation indexes of ecological stability, and the AHP-PCA comprehensive model was established, and the weights of each index were calculated.

The strength of the interaction in this paper's model was constant, but there was a complex interaction between changes in the sex ratio of lampreys and ecological stability. Therefore, the model could not capture the effects of more complex dynamic changes, which sometimes led to unstable results. In fact, the effects of lamprey sex ratio on the ecosystem are related to too many factors, and more factors should be further introduced to improve the model in the following, so as to conduct a more in-depth study on the effects of lamprey sex ratio on the ecosystem.

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