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Study on the Dynamics and Ecological Impact of Lamprey Populations Based on Growth Indicators and Mathematical Models

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Abstract. This study explores the growth and population dynamics of lampreys, focusing on the relationship between growth rate, sex ratio, and interspecific interactions. Utilizing the Von Bertalanffy growth equation, we establish a non-monotonic growth rate function and an asymmetrical S-shaped weight gain curve for lampreys. Population distribution analysis reveals a normal distribution for mass and age, with a joint probability density function for coupled distributions. The study delves into the impact of sex ratio on growth rate and reproductive strategies, highlighting that slow-growing individuals are more likely to become male in resource-limited environments. The Lotka-Volterra Equation is employed to examine interspecific relationships, revealing distinct roles for males and females in predation, competition, and mutualistic symbiosis. Our findings suggest that sex ratios significantly influence population dynamics, with implications for understanding lamprey ecology and managing their populations in aquatic ecosystems.

Keywords: Lamprey growth models; Population dynamics; Sexual differentiation; Mathematical modeling; Reproductive strategies.

1. Introduction

The study of lampreys, a group of ancient jawless fish, has garnered significant interest due to their unique biological characteristics and ecological roles in aquatic ecosystems [1]. Lampreys are known for their distinct life cycle, which includes a prolonged larval stage, a parasitic juvenile stage, and a non-feeding adult stage [2, 3]. Understanding the growth and population dynamics of lampreys is crucial for ecological research and the management of their populations [4], especially in regions where they are considered invasive species or are of conservation concern.

Recent studies have highlighted the importance of examining the growth rate and sex ratio of lampreys, as these factors are closely linked to their reproductive strategies and population structure [5]. The Von Bertalanffy growth equation has been widely used to describe the growth patterns of various aquatic organisms, including lampreys [6]. This mathematical model provides insights into the growth rate and age-weight relationship of lampreys, which are essential for understanding their development and maturity.

Furthermore, the sex ratio of lampreys plays a pivotal role in their population dynamics [7]. Factors such as environmental conditions, resource availability, and predation pressure can influence the sex ratio, which in turn affects the reproductive success and sustainability of lamprey populations [8]. Investigating the interplay between growth rate, sex ratio, and interspecific interactions is crucial for developing effective conservation and management strategies for lampreys [9].

This study aims to explore the growth and population dynamics of lampreys, with a focus on the relationship between growth rate, sex ratio, and interspecific interactions. By employing mathematical models such as the Von Bertalanffy growth equation and the Lotka-Volterra Equation, we seek to provide a comprehensive understanding of lamprey biology and ecology. The findings of this research will contribute to the knowledge base on lampreys and inform management practices for their populations in aquatic ecosystems.

2. Analysis of Lamprey Growth Dynamics Using the Von Bertalanffy Model

To understand the relationship between the growth rate of lampreys and their sex ratio, this study initially attempted to establish the Von Bertalanffy equation. The Von Bertalanffy growth equation can be used to quantitatively describe the growth of lampreys [10]. In this paper, the growth equation has been constructed based on this equation for lampreys. Its expression is shown in the equation below.

$$m = m_{\infty}\left[1 - e^{-k(t-t_0)}\right]^{b} \tag{1}$$

$$L = L_{\infty}\left[1 - e^{-k(t-t_0)}\right] \tag{2}$$

Where m denotes the age weight. m_{∞} the asymptotic weight. t is the age. t_0 is the theoretical age when the weight is 0. k the Ford growth coefficient, i.e. the average curvature of the growth curve. b the growth index.

A derivation of Eq.(1) gives the equation for the growth rate as shown in Eq.(3).

$$\frac{dm}{dt} = abm_{\infty}ke^{-k(t-t_0)}\left[1 - e^{-k(t-t_0)}\right]^{b-1} \tag{3}$$

The function of the growth rate of body weight A is a non-monotonic function whose monotonicity shows an increasing and then a decreasing trend. This suggests that there is a positive correlation between body weight growth rate and time during the juvenile period, but a negative correlation later in life.

The Von Bertalanffy growth equation is derived in quadratic form, which gives the equation for the acceleration of growth as shown in Eq.(4).

$$\frac{d^2m}{dt^2} = abm_{\infty}k^2e^{-k(t-t_0)}\left[be^{-k(t-t_0)} - 1\right]\left[1 - e^{-k(t-t_0)}\right]^{b-2} \tag{4}$$

Solving for the zero roots of this growth acceleration equation yields its solution as $t=lnb/k+t_0$. Furthermore, when the relevant parameters for both males and females are incorporated into the results of this calculation, it is straightforward to see that the zero solution of the growth acceleration equation for females is 1.8, whereas the zero solution of the growth acceleration equation for males is 2.4.

Applying Eq.(1), it can be concluded that the weight gain curve of lampreys is an asymmetrical S-shaped curve. The age of the inflection point in the growth curve for the females was 1.8 years while for the males it was 2.4 years.

3. Characteristics of Population Distribution in Lampreys

From the information on the lampreys, it is easy to see that their populations in each of the masses have a normal distribution concerning mass. In other words $n(m) \sim N(\mu 1, \sigma_1^2)$. Its mathematical expression is shown in Eq.(5).

$$n(m) = \frac{1}{\sqrt{2\pi}\,\sigma_1}e^{-\frac{(m-\mu_1)^2}{2\sigma_1^2}} \tag{5}$$

Where σ^2 represents the degree of concentration of values taken. μ_1 is the most concentrated mass of lampreys.

The passage conveys that for lampreys within each mass category, the distribution of fish of a particular age can also be described using a normal distribution. Essentially, this means that the majority of the lampreys' masses are concentrated around a certain value, with fewer lampreys having significantly higher or lower masses. This specific value where the mass is most densely concentrated is denoted by μ2, and σ2 represents the spread of the masses around this value. In simple terms, the mass distribution of lampreys across different ages follows a normal distribution, with its mean and variance characterizing the central tendency and the variability of the masses, respectively. As shown in Figure 1.

$$n(m)\,|_{t=constant} = \frac{1}{\sqrt{2\pi}\,\sigma_2} e^{-\frac{(m-\mu_2|_{t=constant})^2}{2\sigma_2^{\,2}}} \tag{6}$$

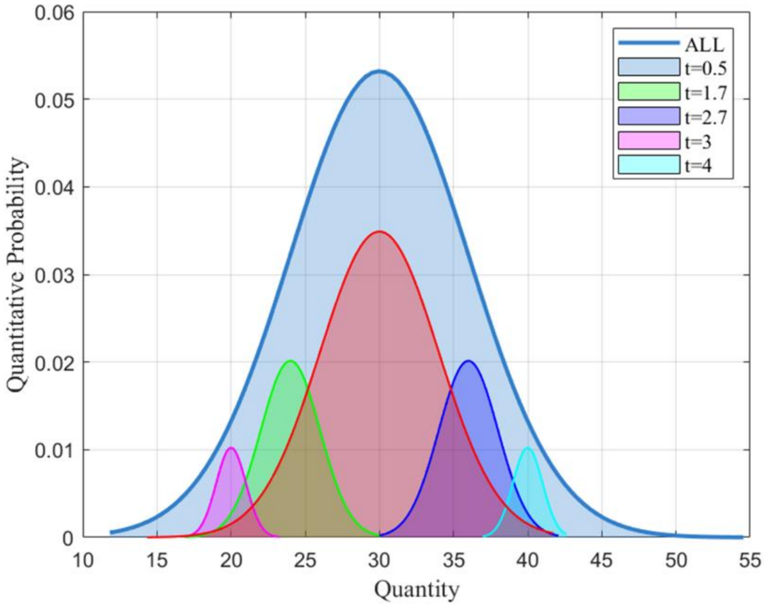


Figure 1. Probability distribution of weight frequencies

For a random variable m, if $n(m) \sim N(\mu 1,\sigma^2)$, the standardized variable can be expressed in the form shown in Eq7.

$$\bar{m} = \frac{m-\mu_1}{\sigma_1} \tag{7}$$

Likewise, the two standardized variables below are easy to derive.

$$\hat{m} = \frac{m-\mu_2}{\sigma_2} \tag{8}$$

After standardization, there is the fact that $\hat{m} \sim N(0,1)$. Same to $\bar{m}$.

Based on the equation for the positron distribution in the previous section, the total number of lampreys can be calculated from Eq.(9) by algorithmic derivation

$$N=\int_{t_{\min}}^{t_{\max}} n(t)\,dt=\int_{t_{\min}}^{t_{\max}}\left(\int_{m_{\min}}^{m_{\max}} n(m)\,|_{t=constant}\,dm\right)dt \tag{9}$$

By inserting the average mass at that age calculated into Eq.(9), the equation can be reformulated as Eq.(10).

$$N=\int_{t_{\min}}^{t_{\max}}\left(\int_{\hat{m}_{\min}}^{\hat{m}_{\max}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\hat{m}^2}{2}}\,d\hat{m}\right)dt \tag{10}$$

Based on the two normal distributions mentioned beforehand, the two are coupled by the correlation coefficient ρ to obtain the joint probability density function of the binary normal distribution as in Eq.(11).

$$n(m,t)=\frac{1}{\sqrt{2\pi}}\exp\left\{\frac{-1}{2(1-\rho^2)}\left[(\hat{m}-1)^2-2\rho(\hat{m}-1)(t-1)+(t-1)^2\right]\right\} \tag{11}$$

In an attempt to combine the two normal distributions mentioned above into a single formula, the shared probability density function is used in this thesis. The joint likelihood density function expression defined in this paper is shown in Eq.(12).

$$n(m,t)=n_t(m)\cdot n_m(t) \tag{12}$$

Where n_t (m) and n_m (t) are the probability density functions of the mass and age of the lampreys, respectively. This assumption implies that the mass and age of lampreys are independent of one another.

For these two probability density functions, the probability density function for age shown in Eq. (13) is used as an example of marginalization, through which a marginal probability density function, a probability density function obtained by integral over one or more of the variables in a multivariate probability distribution, is obtained

$$n_m(t)=\int_{-\infty}^{\infty} n(m,t)\,dm \tag{13}$$

Similarly, it is relatively trivial to write the expression for a, as seen in Eq.(14).

$$n_t(m)=\int_{-\infty}^{\infty} n(m,t)\,dt \tag{14}$$

Eq.(1) can be further modified to Eq.(11) by incorporating it into the joint distribution probability density function of the binary normal distribution obtained in the previous section, which is shown in Eq.(15).

$$n(m,t)=n(m(t),t)=n(t) \tag{15}$$

Depending on the external environment, the sex ratio of marine seven-gill eels varies. Whether an eel becomes a male or a female depends on its growth rate during its juvenile stage. To explain this phenomenon using mathematical tools, this paper counts the weight of several adult lampreys at the age of 3 years. It was found that female lampreys weighed a significantly lower proportion of their body weight than male lampreys, which means that females grow much slower than males.

Scientists have shown through extensive research that there are minimally sexually mature individuals in lampreys and that the average growth rate of sexually mature individuals can determine the sex at which they eventually mature. The steps for gender determination are as follows.

STEP 1: Minimally sexually mature individuals satisfy the following basic relationships of ages.

$$t \geqslant t_{adult} \tag{16}$$

STEP 2: Gender judgment. The sex of an individual satisfying Eq.(16) can be determined by the following relationship.

$$S = sign\left(\frac{m_{adult} - m_0}{t_{adult} - t_0} - \overline{v}\right) \tag{17}$$

When S = 1, the gender is male otherwise it is female.

The model used a 10-day time limit, 500 g mass limit, and 0.18 g per day growth rate limit. As shown in Figure 2, when resources are limited, slow-growing individuals are more likely to become male, i.e. more adapted to survival in a competitive environment, while fast-growing individuals are more likely to become female in an environment where resources are sufficient. This is because females tend to need more resources to develop eggs and become more reproductive. It can be seen that the point of equilibrium for the growth rate of lampreys is around 0.4 grams per day, at which point the ratio of females to males tends to be equal. This phenomenon allows a fish to adjust its sex throughout its life cycle in response to environmental factors to maintain relatively balanced sex ratios.

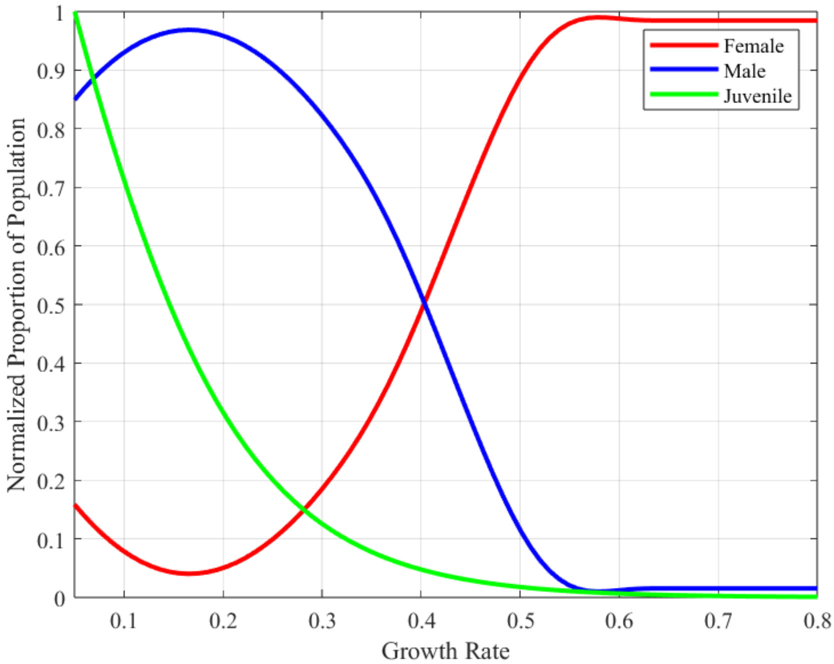


Figure 2. Sexual differentiation of populations.

Furthermore, this study attempts to explore the impact of changes in sex ratio on reproduction. Sex ratios are inextricably linked to population propagation. Many biologists and mathematicians have conducted numerous studies on the reproduction of different types of organisms, which have led to the conclusion that a mathematical relationship exists between the number of newborn offspring in a population and the sex ratio.

$$\Omega_{new} = MX\Omega \min\{\beta_{male}, \beta_{female}\} \tag{18}$$

In the equation, Ω_{new} stands for the number of newborn offspring. M is the mating coefficient. X is the proportion of adults at that midpoint. β is the proportion of the total number of adults in the population that are of a particular sex. Otherwise, $\beta_{\text{male}}+\beta_{\text{female}} = 1$.

The body length and weight of lampreys increase with age, and the absolute fecundity of lampreys also increases. It has been found that there is a mathematical relationship between absolute fecundity and body weight of lampreys as shown in Eq.(19). In summary, the lamprey is a fish with low fecundity and a short reproductive period. Lampreys also have a late sexual maturity, usually around 3 to 4 years.

$$F = am^2 + bm + c \tag{19}$$

Logistic differential equations are mathematical models for the description of population growth patterns in environments where resources are limited. The functional expression can be written as Eq.(20) when applying this model to the lamprey population

$$\begin{cases} \dfrac{dN}{dt} = rn\left(1 - \dfrac{N}{K}\right) \\ n(0) = n_0 \end{cases} \tag{20}$$

Where N is the population size of lampreys at time t. r is the population growth rate. K is the maximum environmental carrying capacity of lampreys. The solution of Eq.(20) is shown in Eq.(21).

$$N(t) = \frac{K}{\left[1 + \left(\frac{K}{n_0} - 1\right)e^{-rt}\right]} \tag{20}$$

The number of female and male individuals can thus be expressed as in Eq. (22) based on the above calculations.

$$\begin{cases} N_m = N \cdot \beta_{male} \\ N_w = N \cdot (1 - \beta_{male}) \end{cases} \tag{22}$$

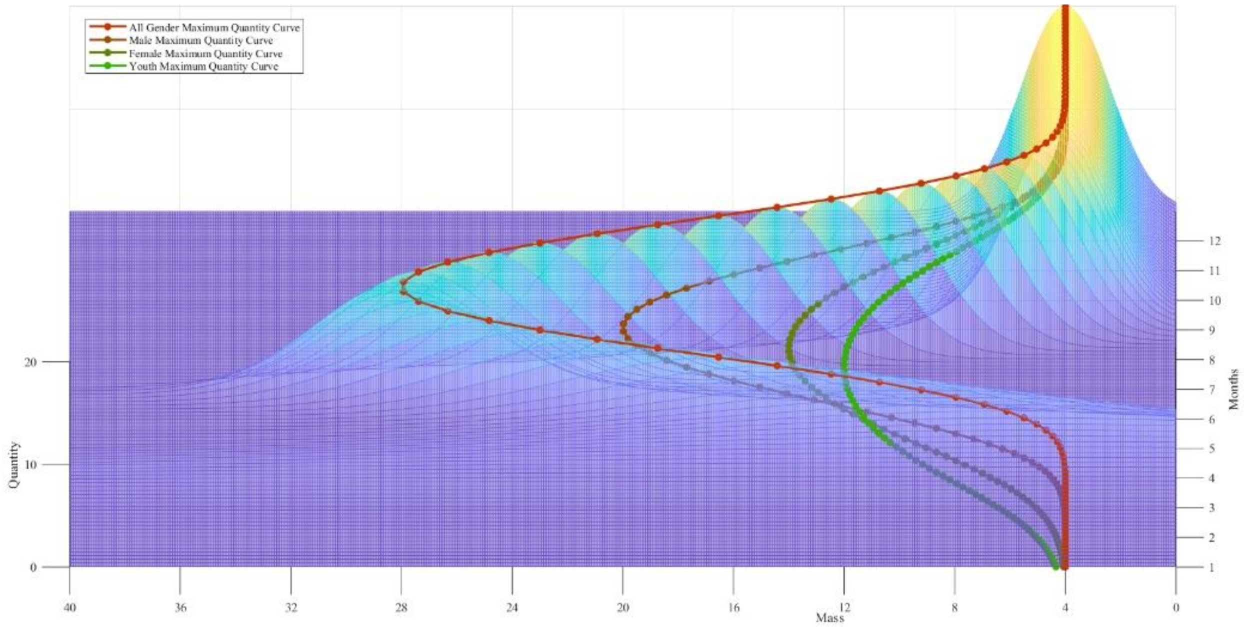


Figure 3. Visualisation of Logistic Functions

Where N_m represents males and N_w represents females. In addition, α is the percentage of males in the total population The specific results are shown in Figure 3.

To quantitatively describe the growth rate of lampreys at different ages, this dissertation employs four growth indicators: mass relative growth rate, growth specific velocity, growth constant, and growth index.

$$C_v m = \frac{\lg m_2 - \lg m_1}{\lg e \cdot (t_2 - t_1)} \tag{23}$$

$$C_i m = C_v m \cdot m_1 \tag{24}$$

Similarly, a similar indicator exists for body length.

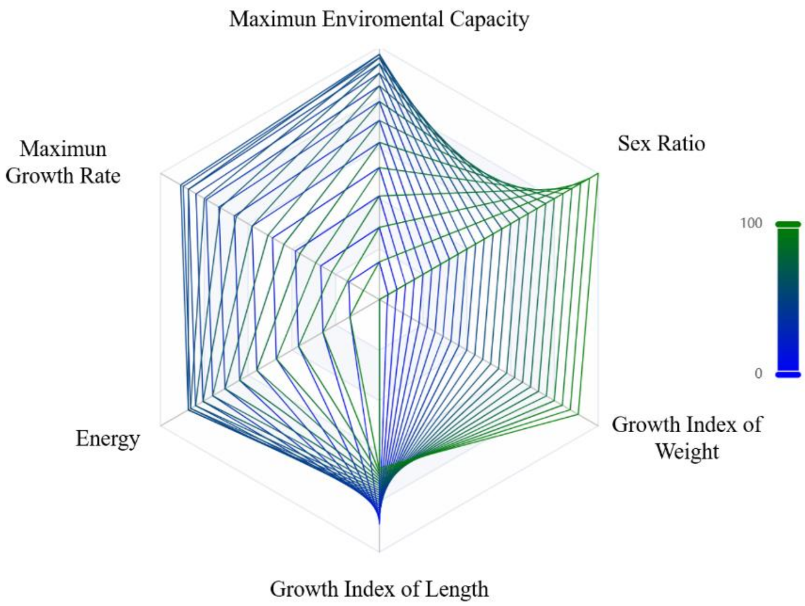


Figure 4. Analysis of sex ratio of population parameters

The situation shown in Figure 4 was obtained by processing the variation of the characteristic parameters of the lamprey population with sex ratio. Briefly, the maximum ecological capacity and the maximum population growth rate were maximal at 50% of the total number of males and increased with increasing proportion of males below 50%, while above 50% showed a decreasing trend. Body length growth index was negatively correlated with male ratio and body mass growth index was positively correlated. Otherwise, as the males increase in number, the energy of the ecosystem increases.

4. Analysis of Interspecific Relationships

The Lotka-Volterra Equation is a mathematical formula that explains how changes in population sizes of predators and prey depend on the amount and interactions between them. The changes in the population size of the predators and the population size of the prey stock over time are governed by the Lotka-Volterra Equation. It is well known that eq25 can be used to calculate the energy of a biological population according to the energy theory of ecology.

$$Q_i(t) = \sum\left(\omega_{ij} \cdot \left(\xi_{ij}^{(-1)^{\psi_{ij}}} \gamma_{ij}\right) \cdot A_{ij}(t)\right) \tag{25}$$

where Q_i is the total energy required by the current population *i* from producers. ω_{ij} is the proportion of energy sources in the food chain *j* for population *I* again. ξ_{ij} is the energy transfer efficiency of population *i* in food chain *j*. γ_{ij} is the trophic level of population *i* in food chain *j*. A_{ij} denotes the time-varying energy of population *i* in food chain *j*.ψ_{ij} is the energy flow factor of population *i* in food chain *j*.

The next step is to focus on the mass distribution of the lampreys. The marginalization is performed as shown below. The marginalization results are incorporated into the joint probability density function to obtain the following equation.

$$n_{ij}(m) = \int_{-\infty}^{\infty} n(m(t), t) \cdot n_m(t) dt \tag{26}$$

In which

$$A_{ij}(t) = \int_{-\infty}^{\infty} n_{ij}(m) \cdot m dt \tag{27}$$

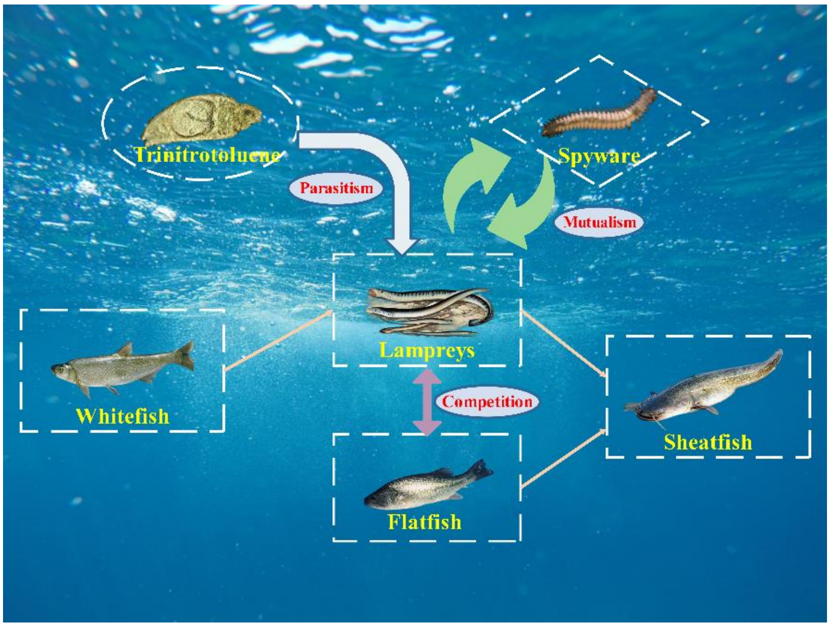
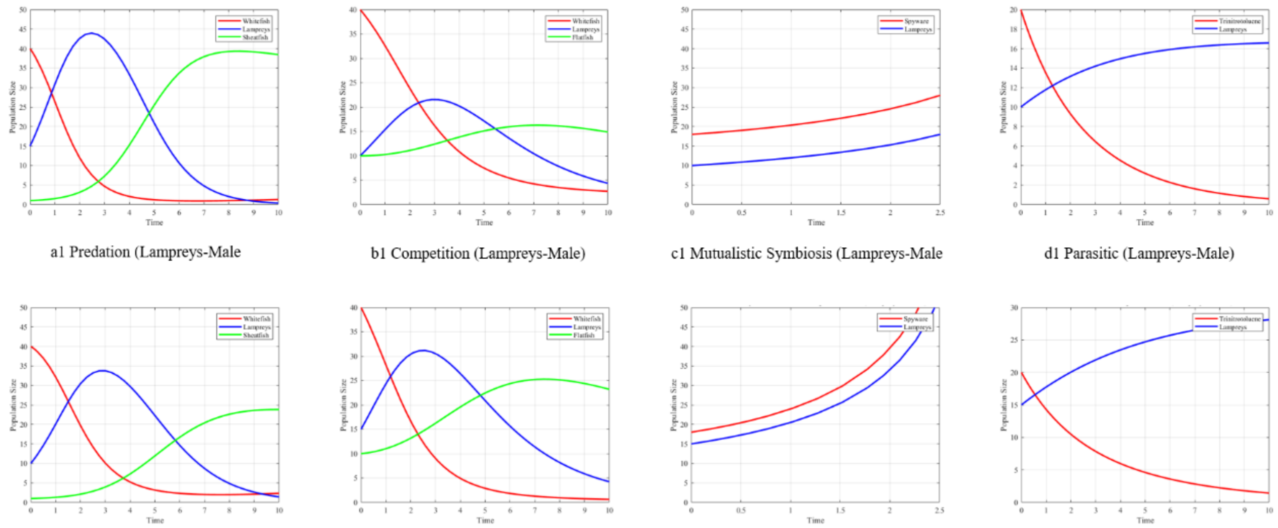


Figure 5. Food chains

It is well known that when there are two or more populations in an ecosystem, they either compete, interdepend, or prey on each other. In the river or marine environment where lampreys live, many, many organisms survive. Mr. Lamprey's natural habitat is home not only to his favorite food, the whitefish, but also to his nemesis, the flatfish, and the most feared of all, the sheatfish. It is straightforward to understand the connection between the creatures mentioned above in Figure 5.

Using the Volterra composite model for small-scale biological networks developed in this thesis, it was analyzed for each of the four interspecific relationships (predation, competition, mutualistic symbiosis, and parasitism) between the male and female populations of the lampreys, obtaining Figure 6. It can be seen that males are more capable of predation and competition than females, females are more prominent in mutualistic symbiosis, and there is no significant difference between the two in the case of parasitism.



a1 Predation (Lampreys-Male
b1 Competition (Lampreys-Male)
c1 Mutualistic Symbiosis (Lampreys-Male
d1 Parasitic (Lampreys-Male)
a2 Predation (Lampreys-Female
b2 Competition (Lampreys-Female)
c2 Mutualistic Symbiosis (Lampreys-Female
d2 Parasitic (Lampreys-Female)

Figure 6. Interspecies relationship role curves

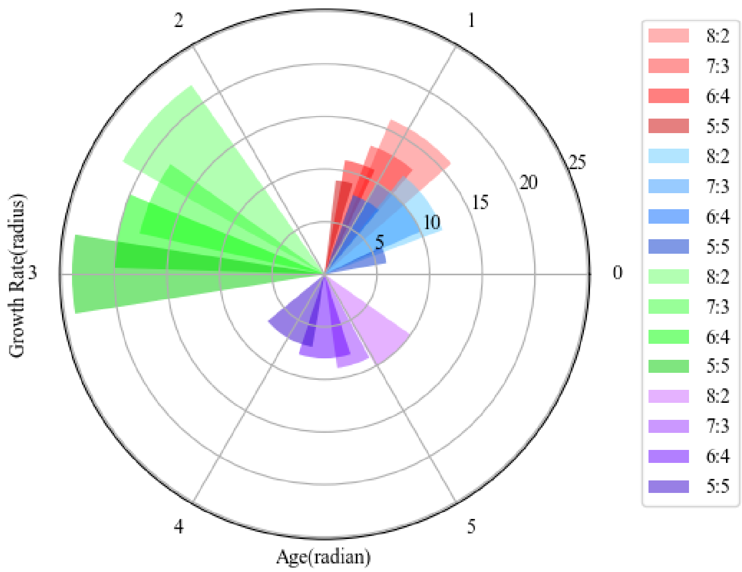


Figure 7. Analysis of lampreys populations

Figure 7 shows the effect of changing sex ratios on lamprey age composition distribution and maximum population growth rate. The lamprey age composition distribution narrowed, resulting in a decrease in the maximum population growth rate. With the decline in male abundance and the rise in female abundance, the distribution in ages under predation and competition became narrower, resulting in a decline in the maximum growth rate. The average age of the population increased, and no new content was added. It uses clear, unbiased language, with a formal register and precise vocabulary.

The study found that as the proportion of females in the parasitic relationship decreased, the mean age and maximum growth rate of the population also decreased, suggesting that female lampreys are the primary parasites in this relationship.

An important indicator of the ability of species and populations to use resources is

ecological niche width, which is the sum of the different resources used by a species in a community. The larger the value, the greater the capacity for resource use. Eq.(30) is its calculation formula.

$$EW_i = -\sum_{j=1}^{k}(P_{ij}\ln P_{ij}) \tag{28}$$

Where P_{ij} is the proportion of the biomass of population i in the total biomass of station j. k is the total number of stations.

Ecological niche overlap can be used to indicate the degree of co-utilization of resources of a particular dimension between species and can reflect potential competitive relationships between species. The formula of it is given in Eq29.

$$EO_i = \frac{\sum_{j=1}^{k}(P_{ij}P_{kj})}{\sqrt{\sum_{j=1}^{k}(P_{ij})^2 \sum_{j=1}^{k}(P_{kj})^2}} \tag{29}$$

The Ecotope Overlap Index takes values between 0 and 1, with larger values indicating a higher degree of overlap. It makes sense to consider the overlap when EO_i > 0.3. Additionally, it is considered a significant overlap when EO_i > 0.6.

The ecological niche is an organism in the process of evolution, through the long-term adaptation to its environment, can be in a certain handle resting place to obtain survival resources, thereby forming the greatest survival advantage. The formula is given in Eq.(30).

$$EN_i = \frac{1}{\sum_{i=1}^{k} {P_{ij}}^2} \tag{30}$$

According to the Shannon-Wiener species diversity index formula with Eq.(37), the RSR improved species diversity index can be expressed as below:

$$EH_i = -\sum \frac{WRSR_i}{\sum WRSR_i} \log_2 \frac{WRSR_i}{\sum WRSR_i} \tag{31}$$

The functional diversity index is also an important parameter for describing ecosystems. It is calculated as Eq.32

$$ED_i = \sum_{i=1}^{k} \left(\sum_{i=1}^{k} \left| F_{ij} - \overline{F}_i \right| \right) \tag{32}$$

5. Conclusion

In this study, we established the Von Bertalanffy growth equation for lampreys to understand the relationship between their growth rate and sex ratio. Our findings indicate that the growth rate of lampreys is non-monotonic, with a positive correlation during juvenility and a negative correlation later in life. The weight gain curve exhibits an asymmetrical S-shape, with the inflection point occurring at different ages for males and females. Population distribution analysis revealed a normal distribution for mass and age, with a joint probability density function for coupled distributions. The sex ratio was found to be a crucial factor in lamprey growth, with slow-growing individuals more likely to become male in resource-limited environments. Interspecific relationships were examined using the Lotka-Volterra Equation, highlighting distinct roles for males and females in predation, competition, and mutualistic symbiosis. Additionally, the study explored the impact of sex ratio changes on reproduction and population dynamics.

This study enhances our understanding of lamprey growth and population dynamics, offering insights into ecological management and conservation efforts in aquatic ecosystems.

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