

# Research on the Impact of Sex Ratio of Lamprey on Ecosystem Stability Based on Dynamic System Model

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**Abstract.** In the Great Lakes of the United States, lampreys, as an invasive species, have seriously affected the survival of native fish and destroyed the ecological environment. To find natural ways to control its population, it is necessary to study the relationship between lamprey and other species. The sex ratio of lampreys has an enormous impact on their population size. To study the impact of lampreys on ecosystem, it's important to investigate the relationship between the sex ratio of lampreys and their population size. Based on Logistic model, Lotka-Volterra model, Competition model and Independence model, a Dynamic System model of simplified ecosystem is established. The effects of lampreys on other species under different interspecific relationships were explored. A population quality assessment model was established, and the population of lampreys was quantitatively assessed from eight aspects including Predation Control, Food Chain Role, Ecosystem Provider, et al.

**Keywords:** Logistic model; Lotka-Volterra model; Dynamic System model; Population Quality Evaluation model.

## 1. Introduction

In aquatic ecosystems, every species plays a unique role, contributing to the overall ecological balance through intricate interactions within their specialized niches. Among them, lampreys have a great impact on their ecological environment due to their adaptive sex ratio variation and special status in the ecosystem [1]. It is of great significance in finding methods to control its damage to the ecological environment to investigate their gender ratio, population size, and impact on the ecological environment. Heather A. Dawson studied the lamprey was a significant factor in the collapse of fish stocks in the Great Lakes [2]. Docker studied the lampreys' population structure [3]. Johnson studied that slower growth in unproductive environments contributed to the sex ratio differences by directly influencing sex determination [4]. Ferreira Martins studied the impact of lamprey's population on the ecological environment of the Five Great Lakes [1].

But they did not directly link the gender ratio with the impact of the lamprey population on the ecological environment, nor did they use mathematical models to describe it. Based on the traditional Logistic model, this article considered various interspecies relationships. Combined with Lotka-Volterra model, Competition model and Independence model to construct a Dynamic system model of the lamprey ecosystem [5]. It helped study the impact of lamprey sex ratio on the ecological environment.

## 2. Model established

### 2.1. Logistic model

Logistic model is a fundamental model for studying changes in the population size of species. At the beginning, the exponential growth model was used to study the population size of species, but it did not consider the blocking effect of natural resources, environmental conditions, and other factors on population growth, and the blocking effect became increasingly significant with the increase of

population. Logistic model is derived by considering these factors and modifying the basic assumptions of the exponential growth model. Its basic form is as follows:

$$\begin{cases} \frac{dx}{dt} = rx(1 - \frac{x}{x_m}) \\ x(0) = x_0 \end{cases} \quad (1)$$

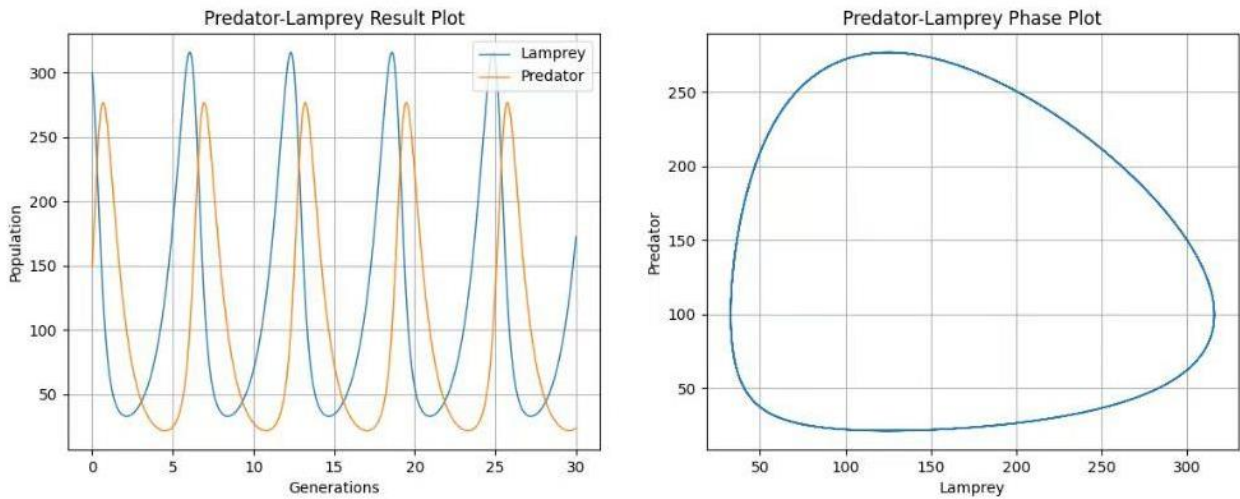
Where  $r$  donates the increase rate of the population,  $x$  donates the population,  $t$  donates time,  $x_m$  donates the maximum population that resources and environment can accommodate while  $x_0$  represents initial population.

## 2.2. Lotka-Volterra predator-prey model

To study the relationship between predators and prey, this article introduced the Lotka Volterra model. The amount of prey and predator at time  $t$  is denoted as  $x_1(t)$  and  $x_2(t)$ , respectively. Considering that in the case of abundant resources, it can be assumed that if prey survive independently, the growth rate of  $r_1$  will increase exponentially. The presence of predators reduces the growth rate of prey, and if the reduction is proportional to the number of predators,  $x_1(t)$ ,  $x_2(t)$  satisfies the equation

$$\begin{cases} \dot{x}_1(t) = x_1(r_1 - \lambda_1 x_2) \\ \dot{x}_2(t) = x_2(r_2 - \lambda_2 x_1) \end{cases} \quad (2)$$

The scale factor  $\lambda_1$  reflects the predator's ability to take prey while the scale coefficient  $\lambda_2$  reflects the ability of prey to provide for predators. The relationship between the number of predators and Lamprey over time and their numbers in the model is shown in Figure 1.



**Figure 1.** Lotka-Vilterra model

## 2.3. Competition model

The lampreys can affect the number of native fish in the five major lakes, indicating the existence of competitors for the lampreys in the ecological environment. [6] Therefore, this article introduces the competitor model which examines the effects of two populations of the same trophic level on each other when they live in the same natural environment [7]:

$$\begin{cases} \dot{x}_1(t) = r_1 x_1 (1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}) \\ \dot{x}_2(t) = r_2 x_2 (1 - \sigma_2 \frac{x_1}{N_1} - \frac{x_2}{N_2}) \end{cases} \quad (3)$$

Where  $x_1(t)$  denotes the population size of population A and accordingly,  $x_2(t)$  denotes the population size of population B. The meaning of  $\sigma_1$  is that the amount of food consumed by unit quantity B (relative to  $N_2$ ) to feed A is  $\sigma_1$  times the amount of food consumed by unit quantity A (relative to  $N_1$ ) to feed A. The same as  $\sigma_2$ .

## 2.4. Independence model

In nature, it's common for two populations to coexist dependently and engage in symbiotic or parasitic relationships within the same environment. Considering this situation, this article utilizes a variant of the logistics model to examine this phenomenon.

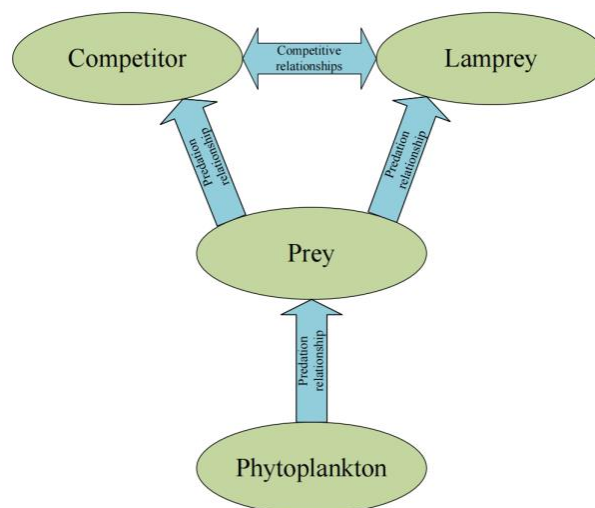
For population A, which can exist independently and grows according to the logistic rule, population B provides food and contributes to the growth of A. Without the presence of A, population B would perish. Thus, the evolution law of population A's number can be formulated as follows.

$$\begin{cases} \dot{x}_1(t) = r_1 x_1 \left(1 - \frac{x_1}{N_1} + \sigma_1 \frac{x_2}{N_2}\right) \\ \dot{x}_2(t) = r_2 x_2 \left(-1 + \sigma_2 \frac{x_1}{N_1} - \frac{x_2}{N_2}\right) \end{cases} \quad (4)$$

Where  $x_1(t)$  denotes the population size of population A and accordingly,  $x_2(t)$  denotes the population size of population B. The meaning of  $\sigma_1$  is that the amount of food provided by unit quantity B (relative to  $N_2$ ) to feed A is  $\sigma_1$  times the amount of food consumed by unit quantity A (relative to  $N_1$ ), and the meaning of  $\sigma_2$  can be defined in the same way.

## 2.5. Dynamic system model of ecosystem

To construct a dynamic system model of the lamprey ecosystem, the food chain in the ecosystem was first analyzed, as shown in Figure 2.



**Figure 2.** Food chain in the ecosystem

In the food chain, phytoplankton are the bottom producers and the cornerstone of the lamprey ecosystem. On the one hand, lamprey mainly invades the Five Great Lakes where the environment is good and resources are abundant [8]. On the other hand, what this article concerned is not the inorganic environment of lamprey ecosystem but the impact between species. Therefore, it is assumed that the inorganic environment of the ecosystem, such as water quality, sunlight, etc., is abundant and good. Under this assumption, the factors that affect the population size of phytoplankton are only the population size of prey which eats phytoplankton and the environmental capacity. Based on the Logistic model and Lotka Volterra model, the following equation is obtained:

$$\frac{dP}{dt} = g_P P \left(1 - \frac{P}{K_P} - \sigma_1 H\right) \quad (5)$$

Where  $P$  donates the population size of phytoplankton,  $t$  donates time,  $g_P$  is the natural growth rates of phytoplankton,  $K_P$  is the environmental capacity of phytoplankton,  $\sigma_1$  donates the predatory ability of prey to plankton and  $H$  donates the population size of prey.

Next, consider the population changes of Prey. Prey is a predator of phytoplankton and is also preyed upon by lampreys and their competitors. Based on the logistic model, Lotka-Volterra model, the following equations can be obtained:

$$\frac{dH}{dt} = g_H H \left(1 - \frac{H}{K_H} - \sigma_2 L - \sigma_3 C + \lambda_1 P\right) \quad (6)$$

Where  $g_H$  is the natural growth rates of prey,  $K_H$  is the environmental capacity of prey,  $\sigma_2$  donates the predatory ability of lamprey to prey,  $\sigma_3$  donates the predatory ability of competitor to prey,  $L$  donates the population size of lamprey,  $C$  donates the population size of competitor and  $\lambda_1$  represent the supporting ability of plankton to prey.

The lamprey eats prey and has a competition with its competitor. Based on the logistic model, Lotka-Volterra model and competition model, the following equations can be obtained:

$$\frac{dL}{dt} = g_L L \left(1 - \frac{L}{K_L} + \lambda_2 H - \sigma_4 \frac{C}{K_C}\right) \quad (7)$$

Where  $g_L$  is the natural growth rates of lamprey,  $K_L$  is the environmental capacity of lamprey,  $K_C$  is the environmental capacity of competitor,  $\lambda_2$  represents the supporting ability of prey to lamprey. The amount of food consumed per unit quantity  $C$  (relative to  $K_C$ ) is  $\sigma_4$  times the amount of food consumed per unit quantity  $L$  (relative to  $K_L$ ).

Compared to lampreys, their competitors have an additional symbiotic population in the ecosystem. Therefore, by adding the independence model to the equation of lampreys, the following equation is obtained:

$$\frac{dC}{dt} = g_C C \left(1 - \frac{C}{K_C} + \lambda_3 H - \lambda_4 \frac{L}{K_L} + \sigma_5 \frac{S}{K_S}\right) \quad (8)$$

Where  $g_C$  is the natural growth rates of competitor,  $K_S$  is the environmental capacity of symbiosis population,  $S$  donates the population size of symbiosis population,  $\lambda_3$  represents the supporting ability of prey to competitor. The amount of food consumed per unit quantity  $L$  (relative to  $K_L$ ) is  $\lambda_4$  times the amount of food consumed per unit quantity  $C$  (relative to  $K_C$ ) and the amount of food consumed per unit quantity of symbiosis (relative to  $K_S$ ) is  $\sigma_5$  times the amount of food consumed per unit quantity competitor (relative to  $K_C$ ).

The symbiosis population has only independence relationship with competitor. Based on the Logistic model and Independence model, the following equation is obtained:

$$\frac{dS}{dt} = d_S \left(-1 - \frac{S}{K_S} + \lambda_5 \frac{C}{K_C}\right) \quad (9)$$

Where  $d_S$  denote the death rate of symbiosis and  $\lambda_5$  denotes the promoting effect of competitor on the growth of symbiosis.

Last but not the least, the sex ratio of lampreys has an impact on its population size [9]. The closer the gender ratio of the population approaches 1:1, the higher the reproduction rate of the population.

After trying, simulate the effect of gender ratio on the population size of lampreys using the following equation:

$$K_L = K(1 - R)R \quad (10)$$

Where  $K$  donates the environmental capacity of lamprey when the male proportion is 50% and  $R$  represents the male proportion.

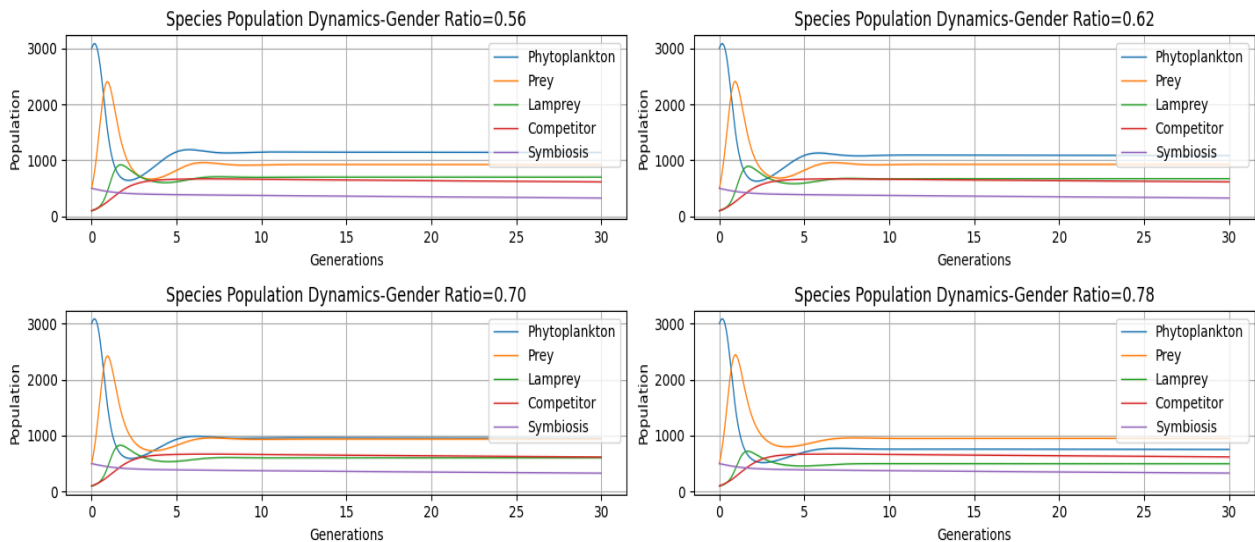
The fundamental dynamic system model of the lamprey ecosystem was established as followed:

$$\left\{ \begin{array}{l} \frac{dP}{dt} = g_P P \left(1 - \frac{P}{K_P} - \sigma_1 H\right) \\ \frac{dH}{dt} = g_H H \left(1 - \frac{H}{K_H} - \sigma_2 L - \sigma_3 C + \lambda_1 P\right) \\ \frac{dL}{dt} = g_L L \left(1 - \frac{L}{K_L} + \lambda_2 H - \sigma_4 \frac{C}{K_C}\right) \\ \frac{dC}{dt} = g_C C \left(1 - \frac{C}{K_C} + \lambda_3 H - \lambda_4 \frac{L}{K_L} + \sigma_5 \frac{S}{K_S}\right) \\ \frac{dS}{dt} = d_s \left(-1 - \frac{S}{K_S} + \lambda_5 \frac{C}{K_C}\right) \\ K_L = K(1 - R)R \end{array} \right. \quad (11)$$

### 3. Result

#### 3.1. Dynamic system model analysis

Simulate the model parameters above and take different gender ratios. Visualize the changes of the five populations in the system using the matplotlib library in Python, as shown in Figure 3:



**Figure 3.** specis' population viration under different lamprey's sex ratio

The Figure 3 illustrates the population dynamics within an ecological system model with different sex ratio. As primary producers, phytoplankton maintain substantial populations, which is crucial as they form the foundation of the aquatic food web. Their stable numbers indicate a continuous supply of energy and nutrients to higher trophic levels.

In the initial iterations, prey populations experienced a sharp decline due to overconsumption by predators or competitors, subsequently stabilizing, suggesting a balance between reproductive rates and predation pressure. The lamprey population initially increased and then stabilized at a certain level. Competitors sharing ecological niches with lampreys, whether as predators or in resource

competition, display relatively stable population numbers, indicating a sustainable population size that neither overexploits their prey nor is significantly suppressed by lampreys.

Regarding the overall stability of the ecosystem, populations stabilized after initial fluctuations, suggesting the system reached a steady state where producers, consumers, and competitors could sustain their populations without drastic changes. This indicates that the ecosystem’s adaptability to sex ratio variations contributes to enhanced stability, demonstrating resilience—a positive sign of ecosystem stability. However, it is noteworthy that this is a simplified model, and real-world ecosystems are subject to many additional factors and potential pressures.

### 3.2. Population Quality Evaluation Model

To quantitatively study the impact of lampreys on the ecological environment, a population quality evaluation model was established [10]. Scores the lamprey population gained when the sex ratio of lampreys is equal (50% male), male\_dominant (more than 50% male) and female\_dominant (less than 50% male) under this system is shown in Table 1:

**Table 1.** Summary of Ecological Indices across Different Gender Ratios

Ecological Indices	Equal	Male_Dominant	Female_Dominant
Predation Control	0.88	0.41	0.14
Food Chain Role	2.45	0.90	0.62
Ecosystem Provider	11.00	5.17	1.70
Adaptability	0.57	-0.05	-0.33
Predation Threat	-0.41	-0.38	-0.41
Reproduction Threat	-0.20	-0.49	-0.15
Overfishing Risk	-0.41	-0.31	-0.20
Environmental Pressure	-0.34	-0.28	-0.43
Overall Score	2.71	1.10	0.33

In a scenario where the sex ratio is balanced, the lamprey exhibits the most positive impact on the ecosystem, with an overall score of 2.71. In contrast, when the sex ratio is skewed towards males or females, the overall score significantly decreases to 1.10 and 0.33, respectively. This variation is primarily attributed to a decline in adaptability and an increase in environmental stress. Particularly under female-dominated conditions, the contribution of ecosystem providers is markedly reduced, highlighting the potential ecological service degradation due to sex ratio imbalance.

## 4. Conclusion

Based on traditional population models, a Dynamic System model of the lamprey ecosystem was established. Under the prediction of this model, the impact of the gender ratio of the lamprey population on the ecological environment was explored. The population quality evaluation model was constructed to quantitatively compare the impact of lamprey populations on various aspects of the ecological environment under different gender ratios.

Compared to traditional population models, the dynamic system model of lamprey ecosystem exhibits good scalability. Within this model, various interspecies relationships such as competition, predation, symbiosis, and parasitism are considered. The same rationale of this model can be applied to model complex relationships between different species based on classic models like the Lotka-Volterra model. Therefore, it can simulate the evolution of various population dynamics under natural conditions. In the meantime, this article discretizes the sex ratio to simplify the computational complexity. However, the actual adaptive changes in the sex ratio should be considered a continuous dynamic relationship, and constructing such a complex dynamic system is not feasible for quantitative analysis given our current practical constraints. Moreover, with sufficient computational power and

time, stability analyses of the ecological dynamic system at any given moment through stability analysis could be provided.

## References

- [1] Ferreira-Martins, Diogo, Champer, Jackson, Mccauley, David W., et al. Genetic control of invasive sea lamprey in the Great Lakes[J]. *Journal of great lakes research*, 2021, 47 (S1): S764 - S775.
- [2] Heather A. Dawson, Courtney E. Higgins-Weier, Todd B. Steeves, Nicholas S. Johnson. Estimating age and growth of invasive sea lamprey: A review of approaches and investigation of a new method [J]. *Journal of Great Lakes Research*, 2020.
- [3] Docker, Margaret F., Bravener, Gale A., Garroway, Colin J., et al. A review of sea lamprey dispersal and population structure in the Great Lakes and the implications for control [J]. *Journal of great lakes research*, 2021, 47 (S1): S549 - S569.
- [4] Johnson, Nicholas S., Swink, William D., Brenden, Travis O. Field study suggests that sex determination in sea lamprey is directly influenced by larval growth rate[J]. 2017, 284 (1851).
- [5] Melanie Gasser, Jean Keller, Pascale Fournier, et al. Identification and evolution of nsLTPs in the root nodule nitrogen fixation clade and molecular response of Frankia to AgLTP24 [J]. *Scientific Reports*, 2023, 13 (1).
- [6] Lauri Leach, Mitch Simpson, Justin R. Stevens, et al. Examining the impacts of pinnipeds on Atlantic salmon: The effects of river restoration on predator–prey interactions [J]. 2022, 32 (4): 645 - 657.
- [7] Carson L. Arends, Hannah B. Vander Zanden, Margaret M. Lamont. Isotopic niche partitioning in a multi-species assemblage [J]. *Marine Biology*, 2023, 171 (1).
- [8] Ellen M. Weise, Kim T. Sribner, Olivia Boeberitz, et al. Evaluating the utility of effective breeding size estimates for monitoring sea lamprey spawning abundance [J]. *Ecology and Evolution*, 2023, 13 (9): n/a-n/a.
- [9] Dhamelincourt, Marius, Buoro, Mathieu, Rives, Jacques, et al. Individual and group characteristics affecting nest building in sea lamprey (*Petromyzon marinus* L. 1758) [J]. *Journal of Fish Biology*, 2021, 98 (2): 557 - 565.
- [10] Adams, Jean, V, Birceanu, Oana, Chadderton, W. Lindsay, et al. Trade-offs between suppression and eradication of sea lampreys from the Great Lakes [J]. *Journal of great lakes research*, 2021, 47 (S1): S782 - S795.