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A Fourier series model and parametric study for single person three-way continuous walking load

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Abstract. The characteristics and mathematical model of the walking load is the basis for studying human-induced vibration in large-span flexible structures. In this paper, 2842 sets of effective walking load datum were collected by three-way force plate test. The correlation analysis shows that the correlations between the vertical component, the horizontal component and the longitudinal component of the walking load are weak, the Fourier series models of the three directional components of the walking load are established. The model parameters of the dynamic loading factor, the phase angle and the walking frequency function with respect to walking frequency are analyzed statistically, and the function of dynamic load factor and phase angle with respect to walking frequency are obtained respectively. Compared with the existing walking load model, it shows that there are some differences from the former models.

Keywords: three-way continuous walking load; walking load characteristics; Fourier series model; dynamic load factor; phase angle.

More and more large span footbridges have broken through the requirement of the vibration frequency of the bridge in the design specification.[1] Under the action of crowd loading, the human induced vibration of large-span footbridges has gradually increased and aroused the interest of researchers from all over the world.[2,3] Research on human induced vibration mainly focuses on the three directions of load modelling, structural analysis and assessment criteria, and the basic content of the dynamic design of pedestrian bridges[4-9]. The excitation source of human induced vibration is the dynamic load of pedestrians, so the study of human induced load becomes the first task to solve the problem of pedestrian bridge human induced vibration, which includes the study of the characteristics of human induced load, the measurement of human induced load and the establishment of mathematical models[11,14-15,18-19].

At present, the research on anthropogenic loads is not sufficient, and there are large differences between researchers, without forming a more unified anthropogenic load model. Anthropogenic load models can be categorized into time domain models and frequency domain models. The available data show that there are some differences in the data observed by different scholars and their statistical results[4,5]. The available data show that there are some differences in the observed data and statistical results of different scholars, and some of the differences deviate greatly. A large number of studies have shown that there is a corresponding functional relationship between the dynamic load factor and the step frequency, so the dynamic load factor is widely expressed as a function of frequency. Currently, Young[6] The idea of the dynamic load factor of the vertical component of walking load is accepted by many researchers. Chen[7] has done quite sufficient research work on the vibration and dynamic design of pedestrian bridges, especially in the theory of transverse vibration and vibration damping design of pedestrian bridges; Chen Jun[8,9] proposed a single person walking load model and a single person jumping load model in the human-induced loading on the basis of sufficient experiments; Nie Jianguo[10] based on Young[6] and Ebrahimpour[11] and Ebrahimpour, the standard load excitation for pedestrians in normal walking condition was constructed by using the Fourier series model for the vertical load excitation characteristics of pedestrian loads. However, for the most common walking loads, many scholars have studied the characteristics of walking loads and proposed their own load models at different times, and the experimental and statistical data are still few.

In this paper, a three-way force plate load test is carried out and 2842 sets of effective data of walking load are collected, and based on the correlation analysis of variables, the appropriate walking load time history expansion method is adopted and the functional relationship equation between the model parameters dynamic load factor and phase angle and the frequency are statistically studied an compared.

1. Walking load test

In this paper, a walking load test was conducted using a Kistler force plate, and the test site map and schematic diagram are shown in Figures 1 and 2. A total of 100 healthy adult volunteers, including 71 males and 29 females, participated in the walking load test, and their characteristic statistics are shown in Table 1. The range and variation of the test samples were significant and representative, and a total of 2842 sets of valid walking load time history curves were obtained from the walking load test.

The walking load frequency (steps per second) ranged from 1.0 to 2.5 Hz, and each tester was guided by an electronic metronome to perform the walking load test, and each tester independently performed 20-30 tests to obtain the walking load time curve of each test and construct a database of the walking load time curve. The steps for the testers are as follows:

(1) Inform the tester of the test needs and requirements;

(2) Registration of tester information;

(3) The tester performs a pre-test;

(4) The tester performs a formal trial, which is guided by an electronic metronome;

(5) Collect data and construct a database.

Table 1. Statistical data on the characteristics of testers in the walking load test

distinguishing between the sexes	number	Weight (kg)			Height (cm)		
		average value	variance (statistics)	range	average value	variance (statistics)	range
male	71	67.9	111.7	46.3~99.0	174.0	36.9	160.8 to 190.3
women	29	48.3	2.1	46.7~50.8	160.4	10.7	155.8 to 164.4

2. Walking load analysis

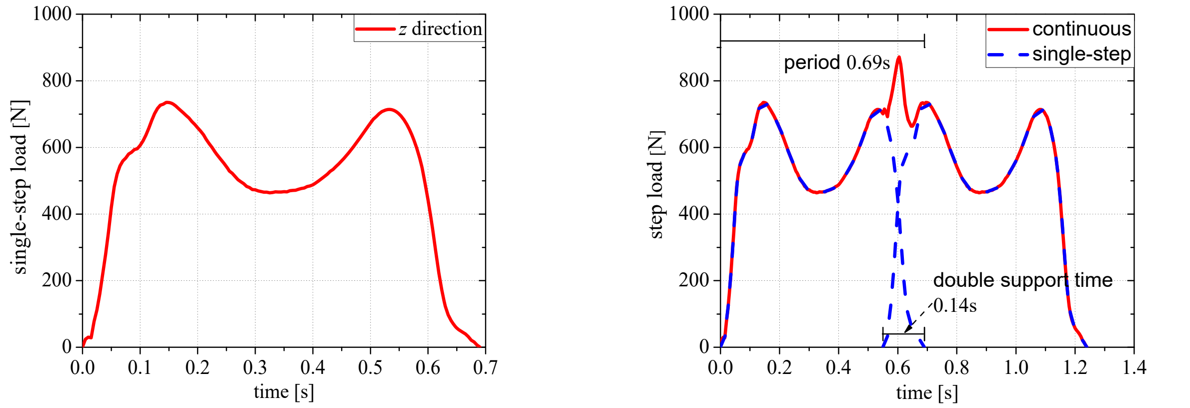
The walking load measured in the test is a single-step time history, and the analysis of the walking load test results is mainly the walking load through the time-expansion method to obtain the continuous walking load time history, the continuous walking load time history through the data quality detection and data filtering, the fast Fourier transform, the spectral characteristics of the continuous walking load. Assuming that the fundamental frequency of walking load is f_p , the Fourier expressions of the longitudinal component F_x (t), the transverse component F_y (t) and the vertical component F_z (t) of the continuous walking load in the time domain are:

$$F_x(t) = G \cdot \left[a_{x0} + \sum_{i=1}^{n} \alpha_{xi} \sin(i\pi ft + \varphi_{xi}) \right] \quad (1)$$

$$F_y(t) = G \cdot \left[a_{y0} + \sum_{i=1}^{n} \alpha_{yi} \sin(2i\pi ft + \varphi_{yi}) \right] \quad (2)$$

$$F_z(t) = G \cdot \left[a_{z0} + \sum_{i=1}^{n} \alpha_{zi} \sin(2i\pi ft + \varphi_{zi}) \right] \quad (3)$$

Assuming that the left and right feet produce the same load, the continuous walking load duration was obtained by the time-expansion method based on the single-step walking load duration and the double-support time (the time at which the left and right feet landed at the same time). Figure 1 shows the typical measured records of walking loads in *x*, *y* and *z* directions for a male tester (height 178.4cm, weight 64.3kg) at the walking frequency f_p =1.449Hz. The single-step walking load timescale and the continuous walking load timescale obtained by expanding the single-step walking load timescale are shown in Figures 1(a) and (b), and the human-caused load is obtained by performing the Fourier transform on the continuous walking load timescale. The parameters dynamic load factor (DLF) and phase angle in the load Fourier series model are shown in Fig. 1.



a) Single-step load timescale (z-direction)   b) Continuous load timescale (z-direction)

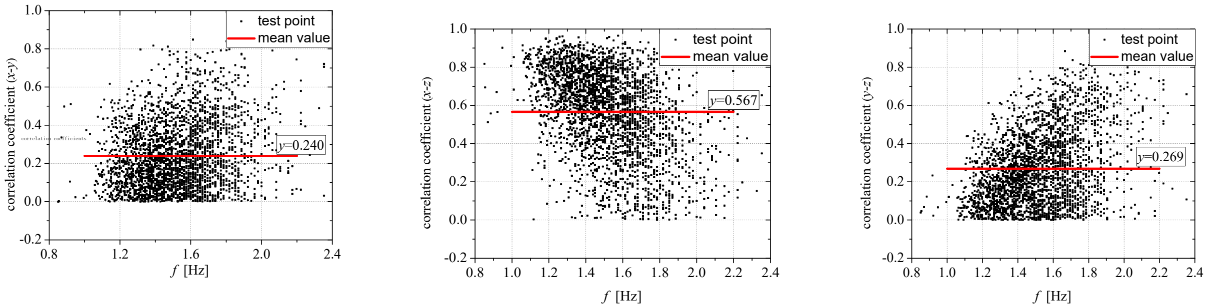
Fig. 1 Walking load analysis diagram (z-direction)

2.1. Walking load characterisation

(1) Correlation analysis

Before the three-way components of walking load are analysed as independent variables, their correlation must be[12] The correlation coefficients of the variables are calculated as shown in Eq. 4. The degree of linear correlation between the variables are analyzed and listed in Fig. 2.

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{4}$$



a) correlation between transverse and longitudinal loads   b) correlation between transverse and vertical loads   c) correlation between longitudinal and vertical loads

Fig. 2 Plot of three-way walking load correlation coefficients

As can be seen from Fig. 2, the horizontal component of walking load is very weakly linearly correlated with the longitudinal and vertical components, and the longitudinal and vertical components show moderate linear correlation. Accordingly, the three-way components of walking load can be modelled separately as independent variables.

(2) Peak load factor

The peak load factor f_m is the ratio of the peak value of each of the three-way components of the walking load $F_{x,y,z}$ to the pedestrian's body weight G, see Eq. 5, which reflects the dynamic characteristics of the walking load and is used to estimate the dynamic effect of the walking load.

$$f_m = \frac{F_m}{G} \qquad (m = x, y, z) \tag{5}$$

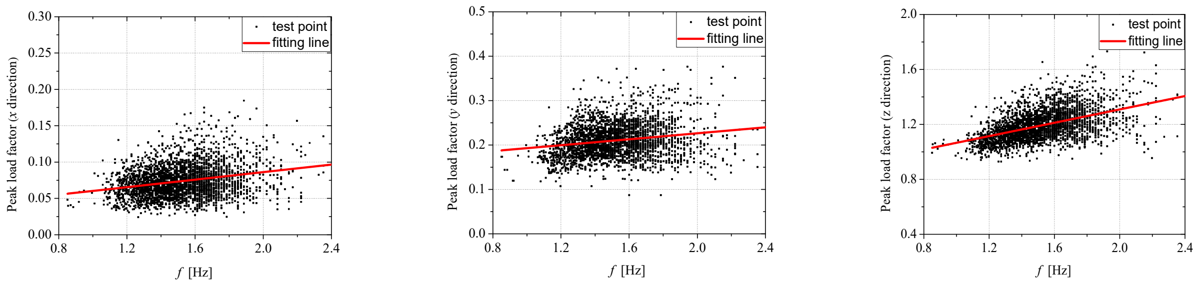
Table 2 shows the peak load factor f_m statistical structure of 2842 sets of valid data of walking load, the peak load factor f_m of lateral walking load is about 0.074 times of body weight, the peak load factor f_m of longitudinal walking load is about 0.210 times of body weight, and the peak load factor f_m of vertical walking load is about 1.193 times of body weight. The dynamic effect of the vertical load is tens of times that of the lateral and longitudinal directions, but since the influence of the lateral walking load component on the structural vibration is not negligible, the three-way component of the walking load must be reasonably modelled to be applied in the structural analysis and calculation.

Table 2. Walking Load Peak Factor Statistics

orientations	x-direction	y-direction	z-direction
average value	0.074	0.210	1.193
variance (statistics)	0.00050	0.00144	0.01261
range	0.024 to 0.185	0.087 to 0.376	0.930 to 1.811

Through the walking load test data, the fitting relationship between the three-way walking load peak factor and the step frequency is statistically presented in Fig. 3, from which it can be seen that the three-way walking load peak factor shows the trend of increasing with the step frequency, among which the vertical direction is the most significant. Assuming that the peak walking load factor varies linearly with the walking frequency, the expression for the linear relationship between the peak walking load factor and the step frequency can be obtained through linear fitting as shown in Eq. 6. Where f_{mx} , f_{my} , f_{mz} are the peak load factors in the horizontal, vertical and longitudinal directions, respectively; f_p is the walking step frequency.

$$\begin{aligned} f_{mx} &= 0.02588 f_p + 0.03442 \\ f_{my} &= 0.03353 f_p + 0.15926 \qquad 0.8 \le f_p \le 2.4 \\ f_{mz} &= 0.24299 f_p + 0.82298 \end{aligned} \tag{6}$$



a) lateral load b) longitudinal c) vertical

Fig. 3 Three-way walking load peak load factor map

(3) Double support time

In normal symmetrical walking, it accounts for about 40% and 60% of the walking cycle[8], respectively, and the simultaneous landing of the right and left feet is called the double-support phase, which is known as the double-support phase. In this paper, the double-support stage time duration expansion method is adopted, and the relationship between the double-support time and the step frequency obtained from the test is shown in Fig. 4 and the following approximate relationship can be fitted in Eq. 7, t_{on} is the double-support time, and f_p is the walking frequency.

$$t_{on} = -0.037 f_p^{\,3} + 0.275 f_p^{\,2} - 0.697 f_p + 0.700 \qquad 0.8 \le f_p \le 2.4 \tag{7}$$

2.2. Walking load modelling and parameter analysis

(1) Model constant term coefficients

The three-direction continuous walking load data are taken to the sixth order of the model order after the quality inspection and filtering process. The statistics of the constant term coefficients of the valid 2842 data sets are shown in Table 3, and the fitting result is shown in Eq. 8.

Table 3. Statistics of constant term coefficients of walking load model

orientations	x-direction	y-direction	z-direction
average value	0.023	0.017	0.729
variance (statistics)	0.00019	0.00022	0.00113
range	0.000 to 0.075	0.000 to 0.106	0.580 to 0.874

$$\begin{aligned} a_{0x} &= -0.011 f_p + 0.040 \\ a_{0y} &= 0.018 f_p - 0.011 \qquad 0.8 \le f_p \le 2.4 \\ a_{0z} &= -0.034 f_p + 0.781 \end{aligned} \tag{8}$$

Through the walking load test, the constant term in the three-way continuous walking load model in this paper takes the values of $a_{0x}=0$, $a_{0y}=0$, $a_{0z}=1$, and the expression of the three-way continuous walking load model is:

$$\begin{aligned} F_x(t) &= G \cdot \sum_{i=1}^{n} \alpha_{xi} \sin(i\pi f_p t + \varphi_{xi}) \\ F_y(t) &= G \cdot \sum_{i=1}^{n} \alpha_{yi} \sin(2i\pi f_p t + \varphi_{yi}) \qquad 0.8 \le f_p \le 2.4 \\ F_z(t) &= G + G \cdot \sum_{i=1}^{n} \alpha_{zi} \sin(2i\pi f_p t + \varphi_{zi}) \end{aligned} \tag{9}$$

(2) Dynamic load factor

The parameter α_i Dynamic Load Factor (DLF) in the three-direction continuous walking load model are taken to the 6th order, Table 4 summarizes the regression functions of each order of the dynamic load factor of the three-way continuous walking load model and Table 5 shows the standard deviation of each order of the dynamic load factor of the three-way continuous walking load model. Further, the regression function of each order of the dynamic load factor of the three-way continuous walking load model is added with 1.645 times of the standard deviation to obtain the design values with 95% guarantee as shown in Table 6.

Table 4. Dynamic load factor regression function for three-way continuous walking load ($0.8 \le f_p \le 2.4$)

ordinal number	Frequency f_h (Hz)	x-direction	y-direction	z-direction
1	0.8 to 2.4	$0.00412f_h+0.00517$	$-0.00727f_h+0.07178$	$0.00289f_h+0.12999$
2	1.6 to 4.8	$-0.00030f_h+0.01077$	$0.00173f_h+0.01515$	$0.02401f_h+0.10589$
3	2.4 to 7.2	$0.00116f_h-0.00154$	$0.00241f_h-0.00059$	$-0.00314f_h+0.07637$
4	3.2 to 9.6	$0.00054f_h+0.00059$	$0.00063f_h+0.00887$	$0.00139f_h+0.01388$

5	4.0 to 12.0	-0.00014f_h +0.00432	-0.00042f_h +0.01153	0.00050f_h +0.00925
6	4.8 to 14.4	-0.00031f_h +0.00488	-0.00049f_h +0.00848	-0.00002f_h +0.00826

Table 5. Standard deviation of dynamic load factors for three-way continuous walking load ($0.8 \le f_p \le 2.4$)

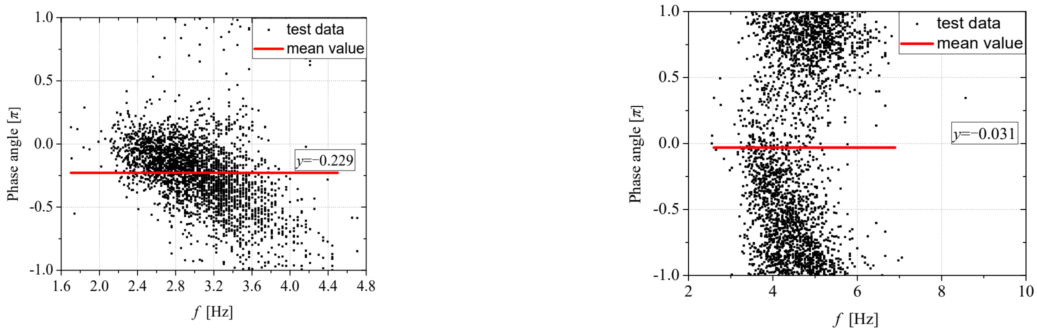
ordinal number	x-direction	y-direction	z-direction
1	0.00383	0.01219	0.03732
2	0.00416	0.00728	0.02901
3	0.00219	0.00431	0.01812
4	0.00194	0.00325	0.00965
5	0.00157	0.00266	0.00647
6	0.00115	0.00206	0.00441

Table 6. Design values of dynamic load factors for three-way continuous pedestrian loading ($0.8 \le f_p \le 2.4$)

ordinal number	Frequency f_h (Hz)	x-direction	y-direction	z-direction
1	0.8 to 2.4	0.00412f_h +0.01147	-0.00727f_h +0.09183	0.00289f_h +0.19138
2	1.6 to 4.8	-0.00030f_h +0.01761	0.00173f_h +0.02713	0.02401f_h +0.15361
3	2.4 to 7.2	0.00116f_h +0.00206	0.00241f_h +0.00650	-0.00314f_h +0.10618
4	3.2 to 9.6	0.00054f_h +0.00378	0.00063f_h +0.01422	0.00139f_h +0.02975
5	4.0 to 12.0	-0.00014f_h +0.00690	-0.00042f_h +0.01591	0.00050f_h +0.01989
6	4.8 to 14.4	-0.00031f_h +0.00677	-0.00049f_h +0.01187	-0.00002f_h +0.01551

(3) Phase angle value

The parameter φ_i phase angle in the three-way continuous walking load model are taken to the 6th order, and φ_1 =0 is usually taken, so φ_i , i=2,3,... is the phase difference of the *i*-order dynamic load relative to the first order dynamic load. Since the phase angle results obtained from the experimental data are more discrete, as can be seen in Fig. 4, the average value of the harmonic phase angle of each order is used in the calculation of this paper, and its mean line is shown as the red solid line in the figure, and the statistical results are shown in Table 7 and the phase angle of the second and third order of the walking load is shown in Fig. 4.



a) 2nd order phase angle (x-direction) b) 3rd order phase angle (x-direction)

Fig. 4 Phase angle distribution of three-way continuous walking load model

Table 7. Values of phase angle for three-way continuous walking load model ($0.8 \le f_p \le 2.4$)

ordinal number	Phase angle (π)		
	x-direction	y-direction	z-direction
1	0	0	0
2	-0.229	-0.128	-0.195
3	-0.031	0.511	-0.295
4	0.403	0.568	-0.189
5	0.458	0.438	-0.139

6	0.245	0.323	0.030

3. Comparative analysis of walking load models

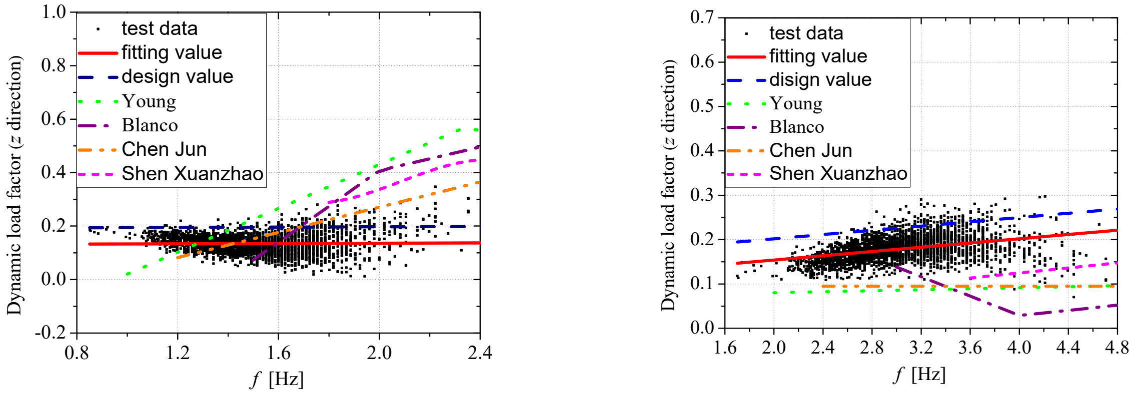
The vertical and lateral components of walking loads are statistically derived and the relationship between the dynamic load factor and the step frequency are summarized in Table 8 and Table 9,respectively. While some comparative results are shown in Fig. 5 and Fig.6. From the analysis of Fig. 5, it can be seen that the first two orders of the dynamic load factor of the vertical model of walking load have a large difference between different researchers, and the last two orders of the dynamic load factor have a better agreement between different researchers. And based on Fig. 6 (a), it can be seen that, compared with other researchers, the first-order dynamic loading factor takes a smaller value, the design value takes a value that is less different from that of the domestic scholar Chen Jun, with a higher degree of agreement, Ingolfsson's value is on the large side, and Bachmann's value involves only the fixed-frequency dynamic loading factor; from the analysis of the above figure (b), the value of the second-order dynamic loading factor takes a value that is in between that of Chen Jun and that of Ingolfsson In the above figure (b), it can be seen that the values of the second-order dynamic load factor are between Chen Jun and Ingolfsson, the fitted value of the second-order dynamic load factor matches well with Ingolfsson's, and the designed value of the second-order dynamic load factor matches better with Chen Jun's, and the values of the second-order dynamic load factor and the fourth-order dynamic load factor are not involved in the study of Bachmann.

Table 8. Dynamic load factors for some of the vertical pedestrian load models

author	Step frequency (Hz)	dynamic load factor	annotation
Young[6]	1.0 to 2.8	$\alpha_1 = 0.41f_p - 0.39 \leq 0.56$ $\alpha_2 = 0.0056f_p + 0.069$ $\alpha_3 = 0.0064f_p + 0.033$ $\alpha_4 = 0.0065f_p + 0.013$	Design value with 75 per cent guarantee
Blanco C.M.[13]	1.5 2.0 2.5	$\alpha_1 = 0.073$, $\alpha_2 = 0.138$, $\alpha_3 = 0.018$ $\alpha_1 = 0.408$, $\alpha_2 = 0.079$, $\alpha_3 = 0.018$ $\alpha_1 = 0.518$, $\alpha_2 = 0.058$, $\alpha_3 = 0.041$	interpolation
Chen Jun[14-16]	1.2~3.0	$\alpha_1 = 0.2358f_p - 0.201$ $\alpha_2 = 0.0949$ $\alpha_3 = 0.0523$ $\alpha_4 = 0.0461$ $\alpha_5 = 0.0339$	Design value with 95 per cent guarantee Sample size: 73
Shen [17]	1.8~2.6	$\alpha_1 = -1.0168f_p^3 + 6.3646f_p^2 - 12.9233f_p + 8.8597$ $\alpha_2 = 0.0285f_p + 0.0107$ $\alpha_3 = 0.0102f_p - 0.0105$ $\alpha_4 = 0.0084f_p - 0.0232$	a (close) fit Sample size: 8

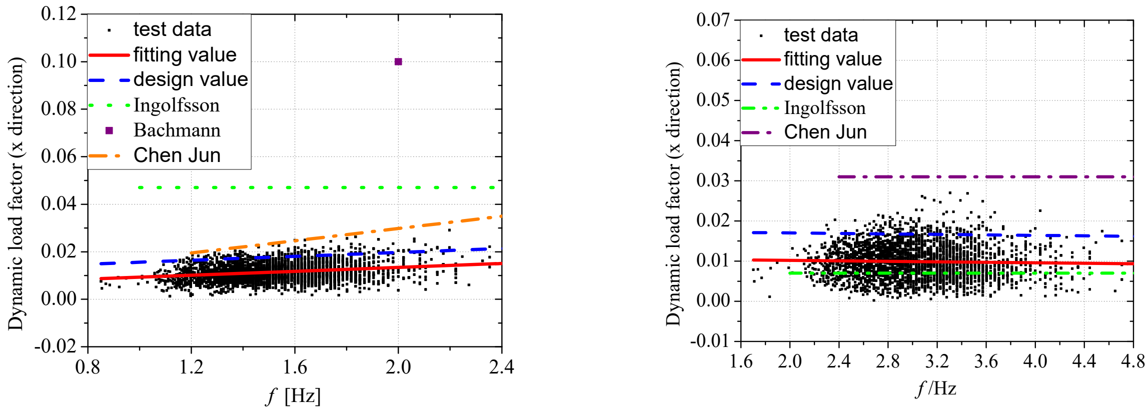
Table 9. Lateral walking load model dynamic load factors

author	Step frequency (Hz)	dynamic load factor	annotation
Ingolfsson[18]	1.0 to 2.8	$\alpha_1 = 0.047$ $\alpha_2 = 0.007$ $\alpha_3 = 0.025$ $\alpha_4 = 0.005$	average value Sample size: 71
Bachmann[19]	2	$\alpha_1 = 0.1$ $\alpha_3 = 0.1$	specific step frequency
Chen Jun[8]	1.2~3.0	$\alpha_1 = 0.0129f_p + 0.0040$ $\alpha_2 = 0.0310$ $\alpha_3 = 0.0083f_p + 0.0001$ $\alpha_4 = 0.0099f_p - 0.0036$	Design value with 95 per cent guarantee Sample size: 73



a) 1st order DLF b) 2nd order DLF

Fig. 5 Comparison of dynamic load factors of different vertical walking load models (Z direction)



a) 1st order DLF b) 2nd order DLF

Fig. 6 Comparison of dynamic load factors of different lateral walking load models

4. Conclusion

The article investigates the three-way continuous walking load characteristics and model establishment and parameter analysis through the three-dimensional force plate walking load test, and analyses and compares with the existing model, and the main conclusions are as follows:

(1) The correlation between the three components of the walking load is weak, so the three components of the single three-way continuous walking load can be analyzed separately as independent variables.

(2) The double-support phase time duration expansion method is used to expand the single-step footfall time history curve into a continuous walking load time history, and then carry out walking load modelling. The trend of double-support time roughly showed a decrease with the increase of step frequency, and the relationship expression was obtained by fitting.

(3) The fitting relationship between the three-directional continuous walking load peak factor and the walking step frequency is statistically presented, and the three-directional walking load peak factors all show the trend of increasing with the step frequency, with the z-direction being the most significant. The peak factor of the vertical load is tens of times that of the lateral and longitudinal directions, but since the influence of the lateral walking load component on the structural vibration is not negligible, the three-way component of the walking load must be reasonably modelled to be applied in the structural analysis and calculation.

(4) The sixth-order Fourier series model of the three-way component is established respectively, and the fitting relationship and standard deviation of the first six orders of dynamic load factor with the walking step frequency are given, and then the design value of the dynamic load factor is proposed; due to the discrete nature of the phase angle, the mean value of the first six orders of the phase angle is statistically obtained and used as the basis for its value.

(5) The differences between the dynamic load factors of existing walking load models were analyzed and compared, and there is a large variability among different researchers, and walking load models should be selected based on different objectives.

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