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Optimization Study on Swimmers' Velocity Distribution Based on Euler-Lagrange Method

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ABSTRACT

In addressing the limitations of conventional experience-based speed allocation in competitive swimming, the objective of this study is to develop optimal strategies from a biomechanical and metabolic perspective, leveraging advanced tools such as mathematical modeling. Specifically, the forces on swimmers during freestyle at different stages were analyzed, and energy consumption was assessed with the ATP-CP model. The analysis included energy expenditure, speed limits, and collision losses. Subsequently, an optimization model for speed allocation was developed using the Euler-Lagrange method and Particle Swarm Optimization (PSO). The results show that in the 50-meter race, swimmers achieve a 0.3-second improvement when they rapidly attain and maintain their maximum sprint speed. For the 100-meter event, the initiation of a rapid increase in speed during the first half, followed by the maintenance of a steady pace, results in a 0.7-second enhancement. In the 200-meter race, a more uniform distribution of speed, coupled with a slight acceleration towards the finish line, results in a performance enhancement of 1.2 seconds. Furthermore, numerical simulations demonstrate that optimal strategies can reduce average power consumption by approximately 15% in comparison to conventional methods. These findings provide scientific support for the effectiveness of optimized speed distribution in improving athlete performance.

KEYWORDS

Swimming Force Analysis, Adenosine Triphosphate - Creatine Phosphate System, Euler-Lagrange Equation, Particle Swarm Optimization, Speed Distribution.

1. INTRODUCTION

In the domain of competitive swimming, scientific speed allocation strategies have emerged as a pivotal factor in enhancing athletes' performance [1]. Conventional research methodologies predominantly employ empirical summarization and statistical analysis of data, exhibiting a dearth of systematic theoretical models to support these analyses [2]. From the perspective of sports biomechanics, athletes must overcome the resistance of water throughout the race and adjust their movement state through various technical maneuvers. From the perspective of energy metabolism, the ATP-CP system, lactate system, and aerobic system play different roles in different stages of the race [3]. Such complex system characteristics make it difficult for the traditional empirical speed allocation strategy to achieve optimal results. However, recent advancements in mathematical modeling and computer technology have paved the way for novel scientific approaches to the study of optimal speed allocation strategies. The integration of mathematical tools such as the Euler-Lagrange method and the particle swarm optimization algorithm has not only enhanced the theoretical underpinnings of this field but also provided tangible benefits to athletes' training and competitive strategies.

Herein, the present study integrates the principles of sports biomechanics and energy metabolism, employing the Euler-Lagrange method to formulate a kinetic model that incorporates the resistance of water flow, the physical exertion of the athlete, and the energy supply mechanism. Subsequently, the particle swarm optimization (PSO) algorithm was employed to solve the model, thereby identifying a speed allocation scheme that can minimize energy consumption and maximize race performance [4]. The experimental results demonstrate that the proposed speed optimization strategy not only effectively reduces the energy expenditure of the athletes, but also significantly improves their race performance [5]. This finding provides a scientific and effective method for athletes and their coaching teams to formulate training and competition strategies, which can help them achieve better performance in real competitions. In addition, this study lays the foundation for subsequent research in related fields and promotes the cross-fertilization and development of sports science and engineering.

2. ANALYSIS OF SPORTS MECHANICS CHARACTERISTICS IN COMPETITIVE FREESTYLE SWIMMING

2.1. Whole process analysis

The data for this study were obtained from https://www.saikr.com. In order to better understand the movement state and energy consumption of the swimmers in the freestyle competition and thus facilitate the subsequent establishment of our speed model, we selected the performance of the swimmers in the 100-meter freestyle for the entire force analysis, as shown in Figure 1.

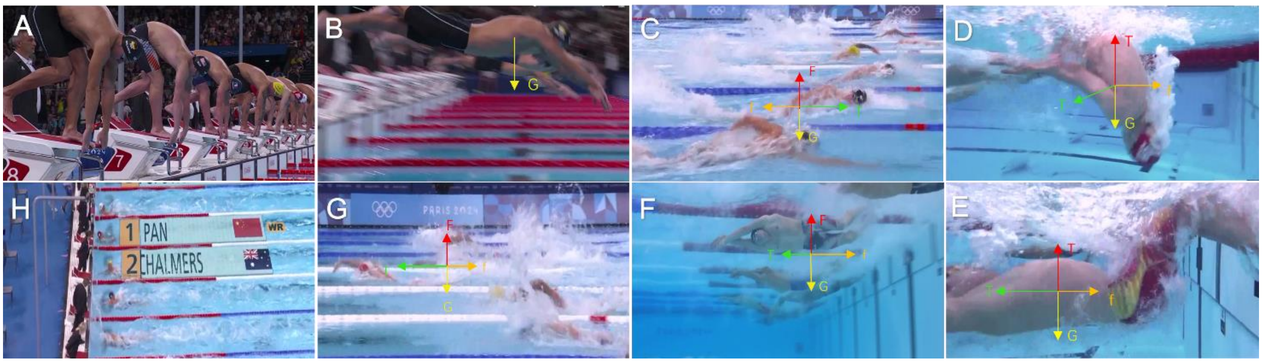

Figure 1. Force analysis for a freestyle swimming race athlete throughout the race

During the takeoff phase, competitors attain substantial initial velocity due to the reaction force of the ground, subsequently traversing a considerable distance through the air, unencumbered by the effects of gravity [6]. Upon entering the water and advancing, competitors are confronted with the interplay of their gravity, buoyancy, and the resistance and thrust from the water. When the thrust surpasses the resistance offered by the water, the competitors undergo acceleration. At the 50-metre mark, the competitor executes a tumbling turn by propelling the water with rapid kicking and hand movements, thereby generating a downward thrust. During this rotation, the competitor adjusts their posture through hand and foot movements to maintain the necessary thrust for the tumble. It is at this juncture that the player becomes subject to the resistance of the water, which acts in opposition to the tumbling action. In order to overcome this resistance and the inertial force, which results in a reduction in speed and an increase in energy consumption, the player must adjust their movement state. Subsequent to a successful turn, the competitor propels themselves against the wall in the direction of the finish line, thereby achieving significant propulsion for instant acceleration. This is followed by a slide that covers a certain distance, thus entering the next stroke cycle. In the sprint stage, the player will increase his thrust by increasing the stroke frequency and amplitude and accelerate to the endpoint until he reaches the endpoint.

2.2. Analysis of the rolling and turning process

In order to simplify the tumbling and turning process, a physical collision model was developed for research purposes [7]. Eight athletes of each gender were selected to fit the relationship between their velocity before and after the collision. The coefficient of velocity division before and after the collision was selected as the attenuation factor to obtain the respective fitting relationship map. The maximum correlation coefficient was recorded as 0.77 for males and 0.45 for females, as illustrated in Figure 2.

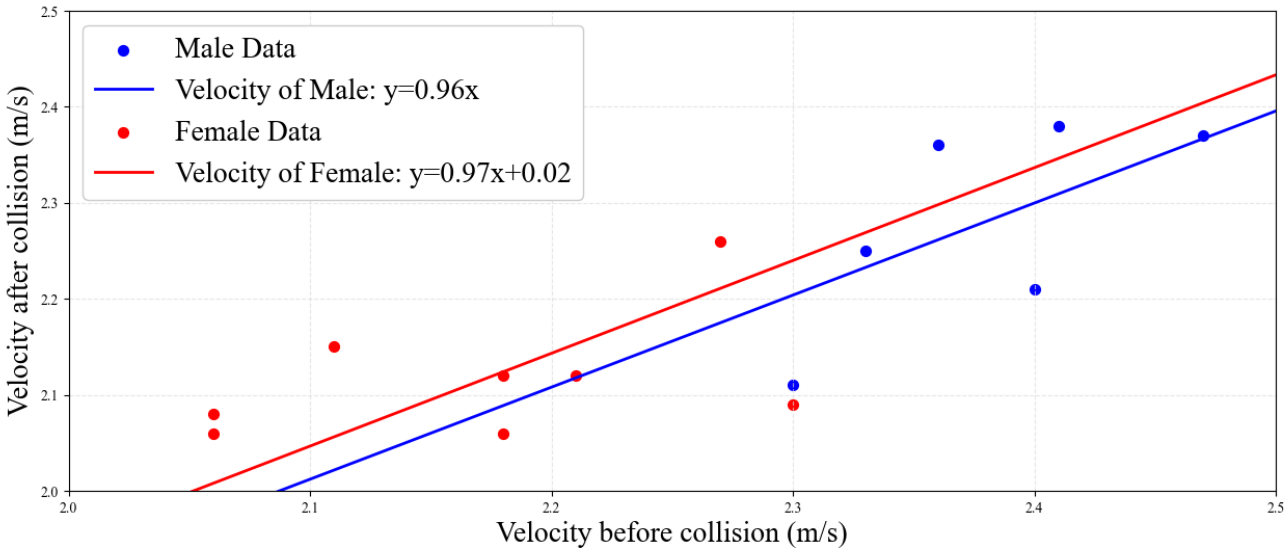

Figure 2. Velocity comparison between male and female

Following a thorough examination of the available data, the relationship between the velocities of male and female subjects was analyzed both prior to and following the collision. The resultant linear fitting map provided valuable insights into the dynamics of the collision. The velocities exhibited a strong positive correlation, with a collision loss of 0.95 before and after the collision, as illustrated in Figure 3.

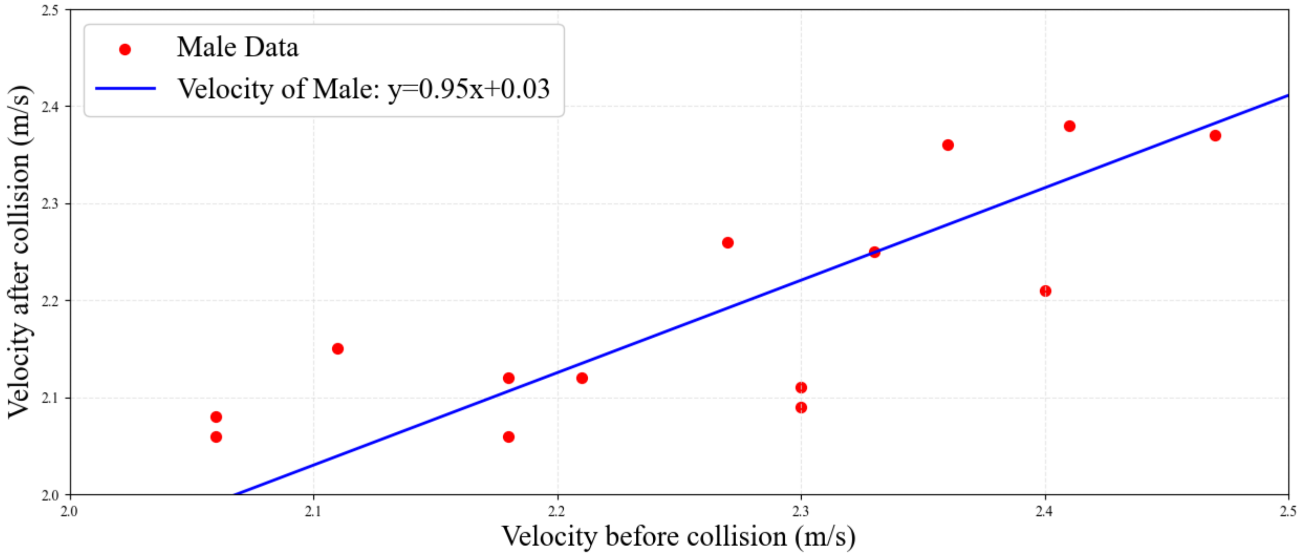

Figure 3. Velocity comparison of all swimmers

3. ENERGY SUPPLY SYSTEMS AND POWER ANALYSIS IN SWIMMING

3.1. Energy systems in swimming

The energy supply in swimming is dependent on three core energy systems: the ATP-CP system, the lactic acid system, and the aerobic system. The role of these systems varies in relation to the intensity and duration of the swim. As exercise time increases, the primary stage of the energy supply undergoes a gradual change, as illustrated in Figure 4.

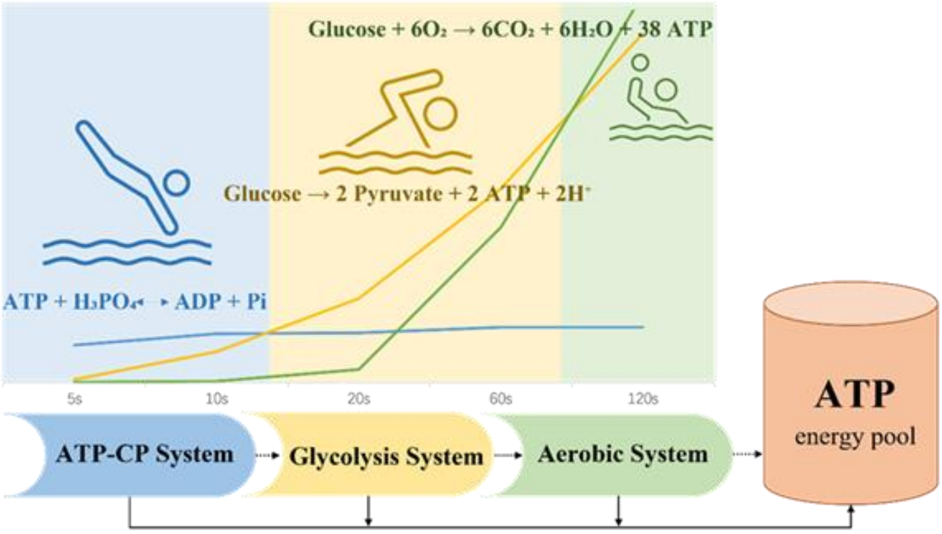

Figure 4. Diagram of the Three Major Energy Supply Systems

The energy expenditure associated with swimming is typically quantified in calories (kcal), and its calculation formula is as follows:

$$W(kcal) = METs \times G(kg) \times t(h) \tag{1}$$

The determination of METs (Metabolic Equivalents) is based on three factors: swimming speed, stroke type, and intensity. According to the formula $P = W/t$, if the work W and the duration t are known, the average power can be calculated.

3.2. Energy supply characteristics and power function

Assuming that power (P) is a function of time (t) and is primarily influenced by the energy supply rates of the phosphocreatine system and the glycolysis system; while also considering the gradual involvement of the aerobic system, a simplified power function can be constructed in Formula 2.

$$P(t) = P_{ATP} - CP(t) + P_{Glycolysis}(t) + k(t) \cdot P_{Aerobic}(t) \tag{2}$$

In the context of this study, $P_{ATP} - CP(t)$, $P_{Glycolysis}(t)$, $P_{Aerobic}(t)$ represent the power output of the phosphocreatine system, glycolysis system, and aerobic system, respectively [8]. The term k(t) denotes a time-varying coefficient, which indicates a gradual increase in the contribution of the aerobic system. The determination of "k" was conducted through the implementation of the least squares method, a statistical technique employed to find the best-fit line through a set of data points.

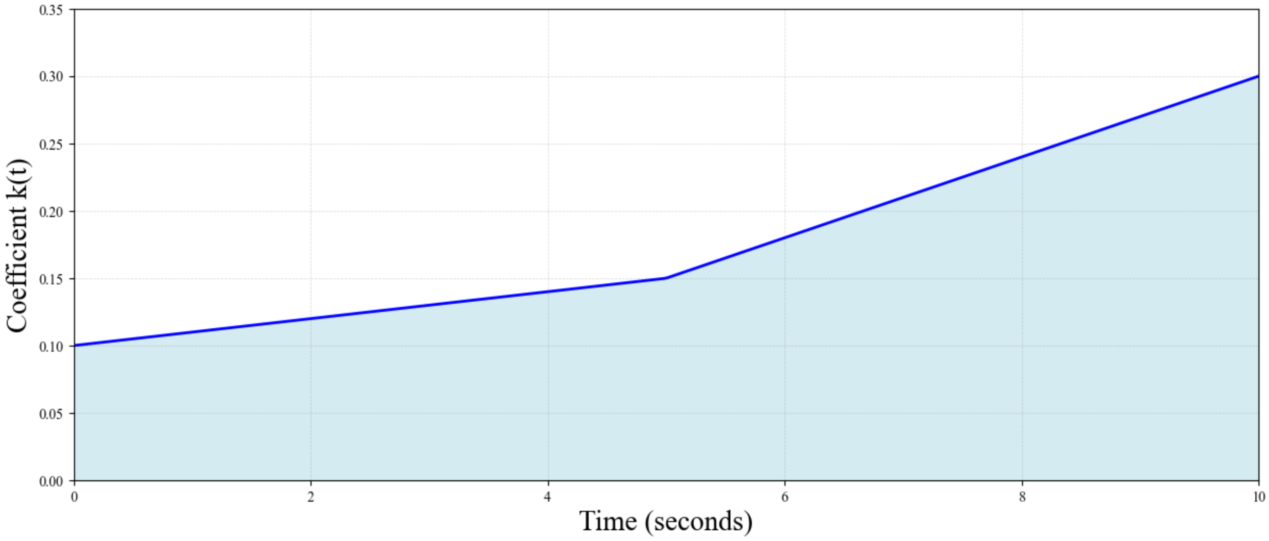

Figure 5. Variation of k(t) over Time in a 100 m Race

As illustrated in Figure 5, the parameter k exhibits a consistent and relatively low level throughout the energy supply process, ranging from 0.05 to 0.1 at the initial stage and increasing from 0.1 to 0.2 in the latter half. As time progresses and the swimming distance increases, the proportion of aerobic respiration gradually rises, and the parameter k gradually increases to 0.2-0.3, reflecting an increase in the proportion of energy supplied by the aerobic system as the race progresses.

4. SPEED OPTIMIZATION STRATEGY IN COMPETITIVE SWIMMING

In order to address the question of how competitors should arrange their swimming speed for optimal performance, a speed allocation model for freestyle swimming will be built. The model recognizes that different distances have distinct speed requirements. For short distances (50m), swimmers need explosive speed throughout, quickly reaching and maintaining maximum velocity. For middle distances (100m), the focus is on balancing initial sprint speed with a sustainable pace, starting fast, and maintaining an efficient speed in the second half. In the case of long distances (200m), the emphasis shifts to endurance and maintaining a steady speed to prevent early fatigue. The objective function is time, with the aim of optimizing speed distribution across stages to achieve the shortest race time and enhance competitive performance [9].

4.1. Minimization time solution model

Let T denote the total time taken by the athlete to complete the race, $v(t)$ represent the athlete's speed at time t, L be the total distance of the race, and E be the total energy expenditure. The objective of the model is to minimize the race completion time T, which can be formally expressed as:

$$\min_{v(t)} T = \int_0^L \frac{1}{v(t)} dx \tag{3}$$

To ensure the effectiveness and practicability of the model, this study considers the following constraints.

Firstly, it is imperative to consider the total energy expenditure E of the athlete, which must not exceed their maximum available energy $E_{\max}$. This can be expressed through the following formula.

$$E=\int_{0}^{T}P(v(t))dt\leqslant E_{\max} \tag{4}$$

Herein, the power consumption function $P(v(t))$ is employed to represent the energy consumption rate of athletes at speed $v(t)$. This function can be approximately expressed as follows.

$$P(v(t))=av(t)^{3}+bv(t)+c \tag{5}$$

In this expression, $av(t)^{3}$ represents power consumption due to fluid resistance, $bv(t)$ represents propulsion-related power consumption, and c represents baseline maintenance power.

Secondly, the athlete's speed $v(t)$ is constrained by the physiological limit $v_{\max}$, indicating that the range of speed variation is constrained by the following formula.

$$0\leqslant v(t)\leqslant v_{\max},\forall t\geqslant 0 \tag{6}$$

This constraint reflects the physical and biological boundaries of human athletic capability, ensuring the realism and feasibility of the simulation outcomes.

Lastly, taking into account the particular circumstances at the outset and conclusion of the race, it is imperative that the initial speed $v(0)$ and final speed $v(T)$ satisfy specific criteria. Specifically, the initial speed and the final speed should meet the following formula.

$$v(0)=v_{start},v(T)=v_{end}. \tag{7}$$

Where v_{start} is typically close to zero, signifying the start from a stationary position, while v_{end} may approach zero or another predetermined value, reflecting deceleration after a final sprint or other tactical maneuvers.

4.2. Lagrange multiplier method and Euler-Lagrange equation

In order to minimize the race time T while satisfying the aforementioned energy constraints, this study introduces the Lagrange multiplier λ to construct the Lagrange function.

$$L(v(t))=\int_{0}^{L}[v(t)+\lambda(av(t)^{3}+bv(t)+c)]dt \tag{8}$$

Given that the problem at hand is of a functional extreme value type, this study employs the variational method to solve for the extreme value of $L(v(t))$, thereby establishing the Euler-Lagrange equation and consequently determining the optimal velocity distribution $v(t)$.

In accordance with the fundamental principle of the variational method, the extreme value of the Lagrange function L is equivalent to the Euler-Lagrange equation. Given the independence of L with respect to v'(t) (the first derivative of velocity), the Euler-Lagrange equation can be simplified to:

$$-\frac{1}{v^{2}}+\lambda(3av^{2}+b)=0 \tag{9}$$

4.3. PSO Algorithm Process

After constructing an exact solution model to minimize the time through the Euler-Lagrange equation, the solution process is further optimized using the PSO algorithm. This optimization is based on the determination that the objective is to minimize the race time T while ensuring that all the constraints are satisfied.

In addressing this issue, it is imperative to note that each particle's position vector is equivalent to a set of discretized velocity values $x_i = \{v(t_i)\}$. This is because t_i is a discrete point in the time interval from start to finish. Consequently, the particle's velocity vector v_i is instrumental in determining how it updates its position at the subsequent iteration. The objective of particle swarm optimization is to identify a set of x_i that minimizes the objective function.

$$\min f(x_i) = T(x_i) \tag{10}$$

Where the objective function value of the i particle is denoted by $f(x_i)$, and the race time, calculated from the velocity distribution x_i, is denoted by $T(x_i)$.

The initialization of the particle swarm is achieved by employing the optimal velocity distribution, which was previously determined, as one of the initial guesses. This approach has been shown to expedite the convergence process and enhance the likelihood of identifying a globally optimal solution. Furthermore, the study incorporates the constraints derived from the aforementioned analysis to ensure that the solution represented by each particle aligns with the physical reality and the established rule requirements.

In order to achieve a more effective integration of the Lagrange multiplier method with the PSO algorithm, this paper proposes the incorporation of the Lagrange multiplier λ in the construction of the fitness function [10]. This function is employed to maintain a balanced ratio between the weights assigned to the minimization of the main objective and the constraints. The primary benefit of this approach is that it enables the direct utilization of the form of the Lagrange function to assess the quality of each particle. This, in turn, serves to guide the particles in adjusting their states to approximate the global optimal solution. The following formula presents the fitness function.

$$F(x_i) = T(x_i) + \lambda[E(x_i) - E_{\max}] + P_E(x_i) + P_v(x_i) \tag{11}$$

The study's findings reveal that the PSO algorithm is a viable solution for the aforementioned equations, leading to the optimization of the final speed allocation results.

4.4. Result Analysis and Evaluation

In accordance with the aforementioned optimization, the ensuing solution results have been obtained and subsequently compared with the extant data.

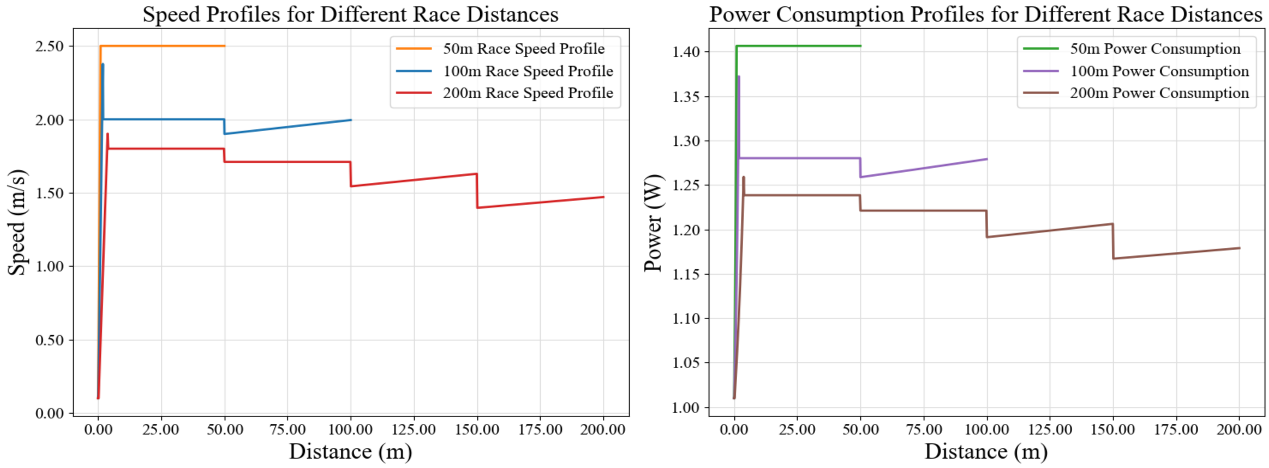

Figure 6. The fitted speed and power consumption profiles for different running distances

Figure 6 illustrates the optimal speed and power distribution curves for the 50-meter, 100-meter, and 200-meter freestyle races, respectively. The graph demonstrates that the optimal speed gradually decreases as the race distance increases, while the power allocation varies at different stages. For the 50m race, the optimal speed allocation peaks at 2.4 m/s initially and stabilizes around 2.1 m/s, suggesting a strategy of explosive start followed by maintaining high speed throughout to leverage the short distance. For the 100m race, it starts at 2.3 m/s and maintains an average of 1.9 m/s, indicating a balanced approach focusing on a strong start and conserving energy for a steady finish. For the 200m race, the speed begins at 2.2 m/s and averages around 1.6 m/s, recommending an initial burst with a focus on pacing and endurance to sustain performance over the longer distance.

Lastly, this study systematically collected detailed data on the speed distribution across various phases for actual swimmers in different events. The method of averaging was employed to compare these data with the fitted curves. It was found that the fitted curves closely matched the real-world conditions, thereby confirming the accuracy of the obtained results.

5. CONCLUSIONS

This study developed an optimal speed allocation model for competitive swimming using the Euler-Lagrange method, addressing the limitations of traditional strategies. The PSO algorithm was applied to the model to identify optimal velocity distributions for different race distances. The study's key findings include the following: a 0.3-second improvement in the 50-meter race by quickly reaching and maintaining maximum sprint speed; a 0.7-second enhancement in the 100-meter race through an initial rapid increase followed by steady pace; and a 1.2-second improvement in the 200-meter race with uniform speed distribution and slight acceleration towards the end. Furthermore, numerical simulations suggest that these optimized strategies result in a reduction of approximately 15% in average power consumption when compared to conventional methods, thereby providing scientific substantiation for their efficacy in competitive swimming. This study reveals that the proposed framework and methodology in sports science, specifically for optimal speed allocation in competitive swimming, show significant feasibility, which can help to solve challenges in capturing real-world swimming conditions and enhance the findings' generalizability.

The present study offers valuable insights into optimal speed allocation in competitive swimming. However, it is important to acknowledge the limitations of this study. While the kinetic model captures basic aspects, it may not fully reflect real-world swimming complexity and faces challenges with data and optimization methods. In light of these limitations, future research directions include refining the kinetic model by incorporating factors such as muscle fatigue, swimming technique, and environmental conditions, as well as exploring the potential of advanced optimization algorithms, particularly deep learning-based methods, to further optimize speed allocation strategies.

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