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Accelerated Degradation Reliability Modeling Based on Multiple Uncertainty Wiener Processes

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ABSTRACT

Numerous scholars' studies usually adopt the Wiener process to establish industrial robot performance degradation models and randomize the drift coefficients to characterize the differences in individual degradation rates, but they do not take into account the influence of measurement errors on the accuracy of the models. Aiming at the above problems, combining the Wiener process and the accelerated degradation model, the accelerated degradation reliability model of industrial robots based on the multi-uncertainty Wiener process is established, and by adding the factor of measurement error, the model is able to more accurately characterize the influence of external factors on the degradation process of industrial robots. Bayesian theory, Markov chain Monte Carlo method and Bootstrap method are used to estimate the unknown parameters in the model. Finally, the accuracy of the proposed parameter estimation method and model is verified by simulating the industrial robot performance degradation data with Monte Carlo method.

KEYWORDS

Wiener Process; Accelerated Degradation; Reliability.

1. INTRODUCTION

Industrial robot is a complex electromechanical product composed of multiple subsystems such as mechanical system, control system, and sensing system [1]. With the increase of working time, improper operation by personnel, and untimely maintenance, industrial robot performance degradation is inevitable [2]. In order to obtain useful reliability data information in a short time, there is an urgent need to explore a realistic and resource-saving reliability modeling method based on the characteristics of industrial robots. Numerous scholars have conducted degradation modeling studies based on the Wiener process. Wang et al.[3] proposed an online remaining service life prediction method based on a nonlinear Wiener process degradation model, which estimates the prior distributions of deterministic and stochastic parameters by maximum likelihood estimation, and adopts Bayesian method to dynamically update the stochastic parameters, and predicts remaining service life that depends on the bearing's real-time Zhang et al.[4] proposed a nonlinear Wiener process model with stochastic time-varying covariates, and applied the maximum likelihood estimation method to estimate the model parameters, and predicted the remaining usable life of dust extraction fans and lithium-ion batteries.Jie et al.[5] constructed a multi-stage distributed degradation model of the Wiener process to characterize the degradation process affected by incomplete maintenance activities, and predicted the remaining useful life of gyroscopes. predict the remaining useful life of gyroscopes. In summary, the Wiener process has the advantages of flexibility, good mathematical properties, and strong data fitting ability, which has a wide range of application value in modeling and analysis, and plays an important role in the field of reliability engineering.

This paper combines the Wiener process and the accelerated degradation model to establish an accelerated degradation reliability model for industrial robots based on the multi-uncertainty Wiener process, and by adding the measurement error factor, the model is able to more accurately describe the influence of external factors on the degradation process of industrial robots. Bayesian theory, Markov chain Monte Carlo method and Bootstrap method are used to estimate the unknown parameters in the model. Finally, the accuracy of the proposed parameter estimation method and model is verified by simulating the industrial robot performance degradation data with Monte Carlo method.

2. PERFORMANCE DEGRADATION RELIABILITY MODEL BASED ON MULTI-UNCERTAINTY WIENER PROCESS

The performance of industrial robots will be degraded with the increase of use time, such as end accuracy, mechanical vibration, etc., and when the degradation amount reaches the failure threshold cumulatively, it will eventually lead to the failure of industrial robots. In the actual degradation process, the degradation process presents a random fluctuation phenomenon due to factors such as environmental conditions, workload, and frequency of use [6]. Using the Brownian motion-driven linear Wiener process to describe the degradation process of industrial robots has the following advantages: firstly, it can more accurately predict the performance decay of the robot by considering the linear trend of the degradation amount on the time axis; secondly, it can simulate the dynamic fluctuation characteristics of the degradation amount, which is expressed as shown in Equation (1).

$$X(t) = x_0 + \lambda t + \sigma B(t) \tag{1}$$

Where, $X(t)$ represents the degradation of industrial robot performance at the time; x_0 represents $t=0$ when the degradation already exists, that is, the initial degradation; λ is the drift coefficient, which characterizes the rate of linear degradation of industrial robot performance over time; σ represents the diffusion coefficient, which characterizes the fluctuation of the stochastic process; $B(t)$ is the Standard Brownian Motion, which is a stochastic process with a mean of 0 and a variance t.

Considering that industrial robots are affected by a variety of factors such as equipment conditions, environmental factors, production processes, and personnel quality in the manufacturing process, and describing the degradation process of industrial robots after randomization of and makes the model more accurate. where $x_0 \sim N(\mu_0, \sigma_0^2)$, $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$. Combining Eq. (1) models x_0 and λ with a randomized Wiener process as

$$\begin{cases} X(t) = x_0 + \lambda t + \sigma B(t) \\ x_0 \sim N(\mu_0, \sigma_0^2) \\ \lambda \sim N(\mu_\lambda, \sigma_\lambda^2) \end{cases} \tag{2}$$

where x_0 follows a normal distribution with mean μ_0 and variance σ_0^2, and λ follows a normal distribution with mean μ_λ and variance σ_λ^2. It is generally recognized that x_0, λ and $B(t)$ are not correlated, i.e., they are independent of each other.

The value of the performance degradation quantity $X(t)$ of the industrial robot is obtained by the measurement equipment, and under the influence of human factors when using the measurement equipment and the factors of the equipment itself, there is an error between the measured result and the actual degradation quantity, and it is unavoidable. Using ε to represent the measurement error, assuming that ε obeys the normal distribution with mean 0 variance σ_ε^2, indicating that the performance degradation quantity will be affected by the measurement error, according to the establishment of the formula (2), the model of the Wiener process with multiple uncertainties can be established as

$$\begin{cases} Y(t) = X(t) + \varepsilon = x_0 + \lambda t + \sigma B(t) + \varepsilon \\ x_0 \sim N(\mu_0, \sigma_0^2) \\ \lambda \sim N(\mu_\lambda, \sigma_\lambda^2) \\ \varepsilon \sim N(0, \sigma_\varepsilon^2) \end{cases} \tag{3}$$

Where $X(t)$ is the actual value of industrial robot performance degradation at the time of t, and $Y(t)$ denotes the measured value of industrial robot performance degradation at the time of t. It is generally assumed that the four variables of industrial robots, x_0, λ, $B(t)$, and ε, are not correlated, i.e., they are independent of each other, and at the same time, it is believed that the ε obtained from the measurement of the different points in time is not correlated with each other.

3. PERFORMANCE DEGRADATION FAILURE DISTRIBUTION FUNCTION BASED ON MULTI-UNCERTAINTY WIENER PROCESS

Assuming that $Y(t)$ denotes that the measured value of the performance degradation of the industrial robot increases with the increase of the usage time, the failure threshold is defined as D. When the value of $Y(t)$ of the industrial robot reaches the failure threshold D it is in a failure state, and when the value of $Y(t)$ reaches the preset D for the first time it is said to undergo a soft failure [7]. Therefore, based on the principle of first reaching time, the life time (failure time) T of an industrial robot can be understood as the time from power-on to the occurrence of soft failure, which is calculated using Equation (4).

$$T = \inf\{t : Y(t) \geq D \mid x_0 < D\} \tag{4}$$

where inf is the lower bound.

Combining Eqs. (3) and (4) and letting $\omega_e = D - x_0 - \varepsilon$, we get

$$T = \inf\{t : \lambda t + \sigma B(t) \geqslant \omega_e \mid 0 < \omega_e + \varepsilon\} \tag{5}$$

When the parameters x_0, λ, D and ε are determined, T is a random variable that conforms to the inverse Gaussian distribution that describes the time it takes to reach a "fixed distance", which originates from the Brownian distribution of the first arrivals in motion. Its probability density function (PDF) and cumulative distribution function (CDF) can be expressed as follows

$$f(t|\omega_e, \lambda) = \frac{\omega_e}{\sqrt{2\pi t^3 \sigma^2}} \exp\left(-\frac{(\omega_e - \lambda t)^2}{2t\sigma^2}\right) \tag{6}$$

$$F(t|\omega_e, \lambda) = \Phi\left(\frac{\lambda t - \omega_e}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\lambda\omega_e}{\sigma^2}\right) \times \Phi\left(-\frac{\omega_e + \lambda t}{\sigma\sqrt{t}}\right) \tag{7}$$

where $\Phi(\cdot)$ denotes the standard normal distribution.

From equation (3), we can see that the parameters x_0, λ and ε are uncorrelated with each other, all are continuous random variables and conform to the normal distribution, by the theorem does not have the correlation of the linear combination of the normal distribution is still obeyed by the normal distribution of the theorem can be known that ω_e is also a continuous random variable, and $\omega_e \sim N(D - \mu_0, \sigma_0^2 + \sigma_\varepsilon^2)$, can be obtained ω_e, λ of the PDF, respectively, as follows

$$f(\omega_e) = \frac{1}{\sqrt{2\pi(\sigma_0^2 + \sigma_\varepsilon^2)}} \exp\left(-\frac{(\omega_e - D + \mu_0)^2}{2(\sigma_0^2 + \sigma_\varepsilon^2)}\right) \tag{8}$$

$$f(\lambda)=\frac{1}{\sqrt{2\pi\sigma_\lambda^2}}\exp\left(-\frac{(\lambda-\mu_\lambda)^2}{2\sigma_\lambda^2}\right) \tag{9}$$

From Eqs. (6) (8) (9), the analytical form of the PDF for the failure time is

$$\begin{aligned} f(t) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t|\omega_e,\lambda)\,f(\lambda)\,f(\omega_e)\,\mathrm{d}\lambda\,\mathrm{d}\omega_e = \\ &\frac{(\sigma_\lambda^2 t+\sigma^2)(D-\mu_0)+\mu_\lambda(\sigma_0^2+\sigma_\varepsilon^2)}{\sqrt{2\pi(\sigma_\lambda^2 t^2+\sigma^2 t+\sigma_0^2+\sigma_\varepsilon^2)^3}}\exp\left\{-\frac{(D-\mu_0-\mu_\lambda t)^2}{2(\sigma_\lambda^2 t^2+\sigma^2 t+\sigma_0^2+\sigma_\varepsilon^2)}\right\} \end{aligned} \tag{10}$$

From Eqs. (7) (8) (9), the analytical form of the corresponding CDF is given by

$$\begin{aligned} F(t) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(t|\omega_e,\lambda)\,f(\lambda)\,f(\omega_e)\,\mathrm{d}\lambda\,\mathrm{d}\omega_e = \\ &\Phi\left(\frac{\mu_\lambda t-(D-\mu_0)}{\sqrt{\sigma_\lambda^2 t^2+\sigma^2 t+\sigma_0^2+\sigma_\varepsilon^2}}\right)+ \\ &\frac{\sigma^2}{\sqrt{\sigma^4-4\sigma_\lambda^2(\sigma_0^2+\sigma_\varepsilon^2)}}\times \\ &\exp\left[\frac{2(D-\mu_0)[\mu_\lambda\sigma^2+\sigma_\lambda^2(D-\mu_0)]+2\mu_\lambda^2(\sigma_0^2+\sigma_\varepsilon^2)}{\sigma^4-4\sigma_\lambda^2(\sigma_0^2+\sigma_\varepsilon^2)}\right]\times \\ &\Phi\left[-\frac{\mu_\lambda(\sigma^2 t+2\sigma_0^2+2\sigma_\varepsilon^2)+(D-\mu_0)(2\sigma_\lambda^2 t+\sigma^2)}{\sqrt{[\sigma^4-4\sigma_\lambda^2(\sigma_0^2+\sigma_\varepsilon^2)](\sigma_\lambda^2 t^2+\sigma^2 t+\sigma_0^2+\sigma_\varepsilon^2)}}\right] \end{aligned} \tag{11}$$

where $\Phi(\cdot)$ denotes the standard normal distribution.

4. ACCELERATED DEGRADATION RELIABILITY MODEL BASED ON MULTIPLE UNCERTAINTY WIENER PROCESSES

This section is to establish the relationship between the model and the accelerated stress, i.e., the accelerated degradation reliability model. The accelerated stress is related to the drift coefficient in the Wiener process model, and the acceleration model used is the polynomial acceleration model. The polynomial acceleration model is a statistical acceleration model used to describe the change in reliability of a product or system over time in accelerated tests and is used to predict the reliability performance at accelerated stress levels, where the exponential function is converted to a linear relationship after logarithmization of the data, making it easier to fit using linear regression. The basic form of the polynomial acceleration model is:

$$R=a\cdot\exp(b\cdot S) \tag{12}$$

where: R is the degradation rate, S is the acceleration stress, a is the parameter to be estimated, and b is the degradation rate constant with respect to the initial one.

Assume that the step stress level $S_1\cdots S_k$ is applied to $t^1\cdots t^k$ robots at time N for accelerated degradation tests.

$$S=\begin{cases} S_1 & 0\leqslant t\leqslant t^1 \\ \vdots & \vdots \\ S_k & t^{k-1}\leqslant t\leqslant t^k \end{cases} \tag{13}$$

Combining Eqs. (3) and (12) yields the accelerated degradation reliability model for the multi-uncertainty Wiener process as

$$\begin{cases} Y(t) = x_0 + \lambda t + \sigma B(t) + \varepsilon \\ \lambda = a \cdot \exp(b \cdot S) \\ x_0 \sim N(\mu_0, \sigma_0^2) \\ a \sim N(\mu_a, \sigma_a^2) \\ \varepsilon \sim N(0, \sigma_\varepsilon^2) \end{cases} \tag{14}$$

where a is defined as a continuous random variable, conforming to normal distribution, with mean μ_a and variance σ_a^2. Comparison of Eq. (3) leads to the conclusion that $\mu_\lambda = \mu_a \exp(b \cdot S)$, $\sigma_\lambda^2 = \sigma_a^2 \exp(2b \cdot S)$, a, b are the parameters to be estimated.

The accelerated degradation process is a continuous process, the initial value of the degradation amount under the next stress is the termination value under the previous stress, and the accelerated degradation process of the i th robot under the k th stress can be expressed as

$$y(t) = \begin{cases} x_0 + \lambda_1 t + \sigma B(t) + \varepsilon & 0 \leqslant t \leqslant t^1 \\ x_0 + \lambda_1 t^1 + \lambda_2 (t - t^1) + \sigma B(t) + \varepsilon & t^1 < t \leqslant t^2 \\ \vdots & \vdots \\ x_0 + \sum_{k=1}^{k-1} \lambda^k (t^k - t^{k-1}) + \lambda_k (t - t^{k-1}) + \sigma_B B(t) + \varepsilon & t^{k-1} < t \leqslant t^k \end{cases} \tag{15}$$

Adopting the concept of first reach time, when the industrial robot's performance degradation under the conventional condition S_0 reaches the failure threshold D for the first time, there are $\mu_{\lambda 0} = \mu_a \exp(b \cdot S_0)$, $\sigma_{\lambda 0}^2 = \sigma_a^2 \exp(2b \cdot S_0)$, and $\mu_{\lambda 0}$ and $\sigma_{\lambda 0}^2$ instead of μ_λ and σ_λ^2. Bringing $\mu_{\lambda 0}$ and $\sigma_{\lambda 0}^2$ into Eqs. (10) and (11) to obtain the PDF of the failure time in the normal state and the corresponding CDF are

$$\begin{aligned} f_0(t) = {} & \frac{(\sigma_{\lambda 0}^2 t + \sigma^2)(\mu_0 - D) - \mu_{\lambda 0}(\sigma_0^2 + \sigma_\varepsilon^2)}{\sqrt{2\pi(\sigma_{\lambda 0}^2 t^2 + \sigma^2 t + \sigma_0^2 + \sigma_\varepsilon^2)^3}} \times \\ & \exp\left\{-\frac{(\mu_0 - D + \mu_{\lambda 0} t)^2}{2(\sigma_{\lambda 0}^2 t^2 + \sigma^2 t + \sigma_0^2 + \sigma_\varepsilon^2)}\right\} \end{aligned} \tag{16}$$

$$\begin{aligned} F_0(t) = {} & \Phi\left(\frac{-\mu_{\lambda 0} t - (\mu_0 - D)}{\sqrt{\sigma_{\lambda 0}^2 t^2 + \sigma^2 t + \sigma_0^2 + \sigma_\varepsilon^2}}\right) + \frac{\sigma^2}{\sqrt{\sigma^4 - 4\sigma_{\lambda 0}^2(\sigma_0^2 + \sigma_\varepsilon^2)}} \times \\ & \exp\left[\frac{2(\mu_0 - D)\left[-\mu_{\lambda 0}\sigma^2 + \sigma_{\lambda 0}^2(\mu_0 - D)\right] + 2\mu_{\lambda 0}^2(\sigma_0^2 + \sigma_\varepsilon^2)}{\sigma^4 - 4\sigma_{\lambda 0}^2(\sigma_0^2 + \sigma_\varepsilon^2)}\right] \times \\ & \Phi\left[-\frac{(\mu_0 - D)(2\sigma_{\lambda 0}^2 t + \sigma^2) - \mu_{\lambda 0}(\sigma^2 t + 2\sigma_0^2 + 2\sigma_\varepsilon^2)}{\sqrt{[\sigma^4 - 4\sigma_{\lambda 0}^2(\sigma_0^2 + \sigma_\varepsilon^2)](\sigma_{\lambda 0}^2 t^2 + \sigma^2 t + \sigma_0^2 + \sigma_\varepsilon^2)}}\right] \end{aligned} \tag{17}$$

where $\mu_{\lambda 0}$ and $\sigma_{\lambda 0}^2$ have become functions of μ_a and σ_a^2 .

5. MODEL PARAMETER ESTIMATION

The accuracy of parameter estimation is critical to the accuracy of the reliability model; the higher the accuracy of parameter estimation, the more reliable and accurate the predictions of the reliability model will be. Parameters to be estimated $\boldsymbol{\theta} = (\mu_0, \mu_a, \sigma_0, \sigma_a, \sigma, \sigma_\varepsilon, b)$. Due to the small sample size of the degradation test, Bayesian theory, Markov Chain Monte Carlo (MCMC) and Bootstrap methods are used to estimate the unknown parameters of the model.

5.1. Bayesian theory

The Bayesian formula can be described as

$$p(\theta|x)=\frac{l(x|\theta)\pi(\theta)}{\int_{\theta} l(x|\theta)\pi(\theta)d\theta} \tag{18}$$

where $p(\theta|x)$ is the posterior distribution of the unknown parameter θ, $\pi(\theta)$ is the prior distribution of θ, $l(x|\theta)$ is the likelihood function of θ, and x is the sample information.

From equation (18), when $x_{i,0}$, a_i are known quantities, the posterior distribution of the parameters to be estimated is

$$\pi(\boldsymbol{\theta},\boldsymbol{x}_0,\boldsymbol{a}|\boldsymbol{Y})\propto L(\boldsymbol{Y}\mid\boldsymbol{\theta},\boldsymbol{x}_0,\boldsymbol{a})\pi(\boldsymbol{\theta},\boldsymbol{x}_0,\boldsymbol{a}) \tag{19}$$

where $\pi(\boldsymbol{\theta},\boldsymbol{x}_0,\boldsymbol{a})=\pi(\theta)\pi(\boldsymbol{x}_0,\boldsymbol{a}|\theta)$ is the prior distribution, $\pi(\boldsymbol{x}_0,\boldsymbol{a}|\theta)=\prod_{i=1}^{N} f(a_i)g(x_{i,0})$, $\pi(\theta)=\pi(\mu_\lambda,\mu_a,\sigma_\lambda,\sigma_a,\sigma,\sigma_\varepsilon,b)$.

5.2. MCMC-Gibbs sampling

In Eq. (19), the number of parameters involved is large, and it is very difficult to calculate the posterior samples of all the parameters to be estimated.MCMC is a statistical inference method for random sampling that combines the theory of Markov chains and the idea of Monte Carlo simulation, which can solve the numerical approximate solutions of complex probability distributions, and is used to estimate the model parameters. Gibbs sampling algorithm is used. The posterior distributions of the parameters to be estimated and their estimates $\hat{\boldsymbol{\theta}}=\left(\hat{\mu}_a,\hat{\sigma}_a,\hat{\mu}_\lambda,\hat{\sigma}_\lambda,\hat{\sigma},\hat{\sigma}_\varepsilon,\hat{b}\right)$, $\hat{\boldsymbol{x}}_0$ and $\hat{\boldsymbol{a}}$ are obtained based on equation (19).

5.3. Bootstrap method

Bootstrap method is a resampling technique in statistics, the basic idea is to perform putative sampling from the original data to generate a series of pseudo-data sets, which have the same distributional characteristics as the original data sets, but the pseudo-data sets are independent, and the probability distribution of the target statistic can be obtained through statistical analysis of the pseudo-data sets, so as to estimate the unknown parameters of the original data sets [8]. Due to the limited number of industrial robots for the degradation test, the use of the above method may result in insufficient estimation accuracy of the parameters describing the differences of the robots, in order to improve the estimation accuracy, the Bootstrap method is applied to re-estimate the parameters $\hat{\mu}_0$, $\hat{\mu}_a$, $\hat{\sigma}_0$, and $\hat{\sigma}_a$. The steps for re-estimating the parameters $\hat{\mu}_0$ and $\hat{\sigma}_0$ are as follows.

Applying the Bootstrap method to generate the $\hat{\boldsymbol{x}}_0$ sets of data associated with M, denoted by $\hat{\boldsymbol{x}}_0^*$, the M sets of sample data can be expressed as

$$\begin{pmatrix}\hat{\boldsymbol{x}}_0^*(1)\\ \hat{\boldsymbol{x}}_0^*(2)\\ \vdots\\ \hat{\boldsymbol{x}}_0^*(M)\end{pmatrix}=\begin{pmatrix}\hat{\boldsymbol{x}}_{1,0}^*(1) & \hat{\boldsymbol{x}}_{2,0}^*(1) & \cdots & \hat{\boldsymbol{x}}_{N,0}^*(1)\\ \hat{\boldsymbol{x}}_{i,0}^*(2) & \hat{\boldsymbol{x}}_{2,0}^*(2) & \cdots & \hat{\boldsymbol{x}}_{N,0}^*(2)\\ \vdots & \vdots & \ddots & \vdots\\ \hat{\boldsymbol{x}}_{i,0}^*(M) & \hat{\boldsymbol{x}}_{2,0}^*(M) & \cdots & \hat{\boldsymbol{x}}_{N,0}^*(M)\end{pmatrix} \tag{20}$$

Calculations on the Bootstrap sampling samples of group h ($h=1,2,\cdots,M$) give the random variable as

$$\mu_0(h)=\frac{1}{N}\sum_{i=1}^{N}\hat{x}_{i,0}^*(h) \tag{21}$$

$$\sigma_0^2(h)=\frac{1}{N}\sum_{i=1}^{N}(\hat{x}_{i,0}^*(h)-\mu_0(h))^2 \tag{22}$$

The samples computed through Eqs. (21) and (22) follow Eqs. (23) and (24) to compute more accurate estimates of $\hat{\mu}_0$ and $\hat{\sigma}_0$, denoted as $\tilde{\mu}_0$ and $\tilde{\sigma}_0$. The samples are then used to calculate a more accurate estimate of $\hat{\mu}_0$ and $\hat{\sigma}_0$, denoted as $\tilde{\mu}_0$ and $\tilde{\sigma}_0$. The samples are then used to calculate a more accurate estimate of $\hat{\mu}_0$ and $\hat{\sigma}_0$, denoted as $\tilde{\mu}_0$ and $\tilde{\sigma}_0$.

$$\tilde{\mu}_0 = \frac{1}{M}\sum_{h=1}^{M}\mu_0(h) \tag{23}$$

$$\tilde{\sigma}_0 = \sqrt{\frac{1}{M}\sum_{h=1}^{M}\sigma_0^2(h)} \tag{24}$$

In the same way more precise estimates of $\hat{\mu}_\lambda$ and $\hat{\sigma}_\lambda$ can be obtained $\tilde{\mu}_\lambda$ and $\tilde{\sigma}_\lambda$.

6. PARAMETER ESTIMATION AND VALIDATION OF MODEL VALIDITY

Monte Carlo simulation is used to generate industrial robot accelerated degradation test data, and the validity of the accelerated degradation reliability model parameter estimation is verified according to the comparison of calculation results. According to equation (3), the values of relevant parameters in the model are set in advance as $\mu_0 = 0.0012$, $\mu_a = 0.02$, $\sigma = 0.03$, $\sigma_a = 0.03$, $\sigma_0 = 0.03$, $\sigma_\varepsilon = 0.003$ and $b = -0.52$, the number of samples is $N = 9$, the step stresses are 3.25kg, 4.25kg, 5.25kg, and the measurements are taken 5 times (except the initial value) under each stress level, and the intervals between the measurements are 30h respectively, and the measured values of the simulated 9 industrial robots in terms of the distance repeatability accuracy degradation are shown in Fig. 1, and the data are shown in Table 1, which are shown in Fig. 1. The data are shown in Table 1.

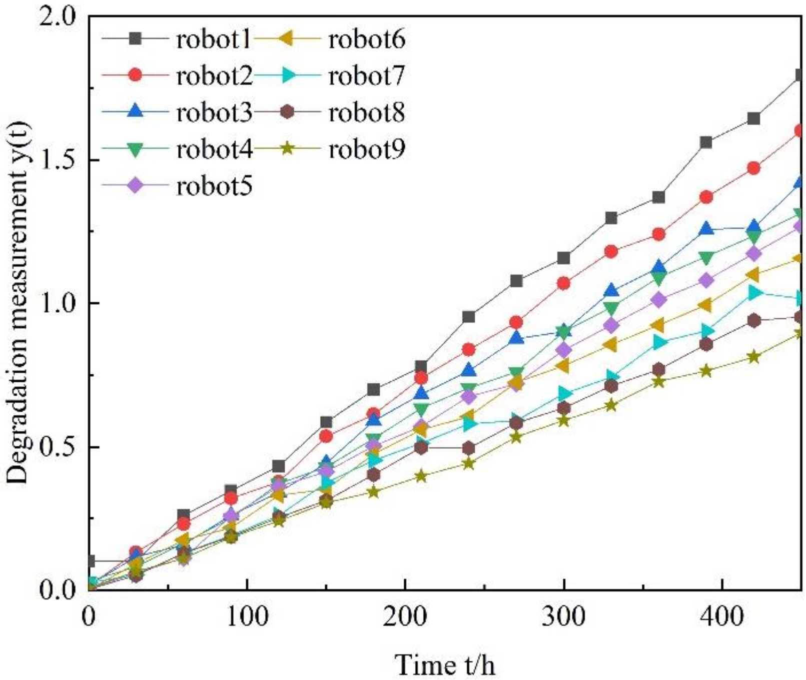

Figure 1. Distance Repeatability Accuracy Degradation Volume Measurements Simulation Trend.

The first step is to define the prior distribution of the unknown parameter $\boldsymbol{\theta}$. The defined values are $\mu_0 \sim N(0, 10^6)$, $\mu_a \sim N(0, 10^6)$, $\sigma_0^{-2} \sim Ga(0.001, 0.001)$, $\sigma_a^{-2} \sim Ga(0.001, 0.001)$, $\sigma^{-2} \sim Ga(0.001, 0.001)$, $\sigma_\varepsilon^{-2} \sim Ga(0.001, 0.001)$. MCMC sampling is performed in OpenBUGS using Gibbs sampling algorithm. The parameter estimation results are shown in Table 2.

Table 1. Distance Repeatability Accuracy Degradation Measured Value Simulation Data.

Time/h	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7	Sample 8	Sample 9
$t=0$	0.1011	0.0142	0.0176	0.0322	0.0063	0.0028	0.0264	0.0035	0.0077
$t=30$	0.1025	0.1322	0.1169	0.0812	0.0599	0.0930	0.0534	0.0523	0.0663
$t=60$	0.2602	0.2308	0.1592	0.1647	0.1122	0.1749	0.1294	0.1292	0.1110
$t=90$	0.3466	0.3201	0.2622	0.2512	0.2580	0.2180	0.1913	0.1872	0.1842
$t=120$	0.4325	0.3775	0.3415	0.3712	0.3613	0.3307	0.2608	0.2540	0.2418
$t=150$	0.5851	0.5356	0.4431	0.4287	0.413	0.3526	0.3747	0.3124	0.3050
$t=180$	0.6985	0.6129	0.5904	0.5291	0.5023	0.4743	0.4528	0.4027	0.3427
$t=210$	0.7781	0.7390	0.6824	0.6340	0.5707	0.5592	0.5114	0.4978	0.3978
$t=240$	0.9522	0.8381	0.7645	0.7039	0.6752	0.6043	0.5793	0.4954	0.4427
$t=270$	1.0778	0.933	0.8766	0.7613	0.7192	0.7252	0.5912	0.5831	0.5344
$t=300$	1.1575	1.0691	0.9016	0.9011	0.8369	0.7824	0.6847	0.6351	0.5924
$t=330$	1.2959	1.1807	1.0410	0.9856	0.9233	0.8562	0.7423	0.7123	0.6449
$t=360$	1.3700	1.2401	1.1241	1.0908	1.0124	0.9237	0.8639	0.7688	0.7278
$t=390$	1.5609	1.3694	1.2565	1.1627	1.0798	0.9935	0.9031	0.8572	0.7644
$t=420$	1.644	1.4709	1.2648	1.2349	1.1735	1.0997	1.0371	0.9401	0.8132
$t=450$	1.7947	1.6009	1.4205	1.3145	1.2684	1.1565	1.0169	0.9520	0.8976

After obtaining $\hat{\boldsymbol{x}}_0$ and $\hat{\boldsymbol{a}}$ in accordance with Bayesian theory and the MCMC-Gibbs sampling method, the four parameters μ_0 , μ_a , σ_0 and σ_a were estimated again using the Bootstrap method, where $M=1000$. Eventually, the parameter estimates were obtained as shown in Table 3.

The real and simulated values in Table 3 are sequentially substituted into Eqs. (16), (17) and (11), and the probability density and reliability function images of the industrial robot are shown in Fig. 2 after setting the failure threshold $D=10$, under normal operating conditions.

By comparing the simulated and real values, it is found that the difference between the two is very small, and furthermore, the probability density function intuitively shows that the simulation accuracy is high, which proves that the parameter estimation method of the joint Bayesian theory, the Markov chain Monte Carlo method, and the Bootstrap method is able to implement an accurate estimation of the model parameters.

From the reliability function image, it can be seen that the two curves of real and simulated values fit well, indicating that the established accelerated degradation reliability model based on the multi-uncertainty Wiener process can well interpret the degradation process of industrial robots.

Table 2. Parameter estimation results

Parameters	Averages	Standard Deviation	MC Errors	2.5%	Midpoint	97.5%
μ_0	0.0011	0.004612	5.38×10^{-4}	0.0009	0.0012	0.0015
μ_a	0.0253	0.1081	4.133×10^{-5}	0.0136	0.0252	0.0368
σ_0	0.0345	0.003707	2.287×10^{-5}	0.0241	0.0344	0.0447
σ_a	0.0326	0.01817	2.702×10^{-5}	0.0235	0.0325	0.0415
σ	0.0325	0.1293	2.325×10^{-4}	0.0264	0.0324	0.0384
σ_ε	0.0036	0.005634	6.041×10^{-4}	0.0015	0.0035	0.0055
b	-0.5136	0.9163	0.01631	-0.8811	-0.5023	-0.1235
a_1	0.0212	0.002872	1.495×10^{-5}	0.0149	0.0202	0.0255
a_2	0.0191	0.002826	1.272×10^{-5}	0.0143	0.0196	0.0249
a_3	0.0216	0.002903	1.373×10^{-5}	0.016	0.0213	0.0266
a_4	0.0197	0.002882	1.48×10^{-5}	0.0147	0.0200	0.0253
a_5	0.0187	0.002849	1.442×10^{-5}	0.0132	0.0185	0.0238
a_6	0.0194	0.002847	1.359×10^{-5}	0.0143	0.0196	0.0249
a_7	0.0208	0.002868	1.318×10^{-5}	0.0157	0.0210	0.0263
a_8	0.0210	0.002857	1.363×10^{-5}	0.0153	0.0206	0.0259
a_9	0.0201	0.002853	1.324×10^{-5}	0.0167	0.0220	0.0273
$x_{1,0}$	0.0024	0.05821	3.42×10^{-4}	-0.003	0.0023	0.0076
$x_{2,0}$	0.0025	0.05581	2.591×10^{-4}	-0.0029	0.0024	0.0077
$x_{3,0}$	0.0003	0.05632	2.537×10^{-4}	-0.0055	-0.0002	0.0051
$x_{4,0}$	0.0029	0.05587	2.487×10^{-4}	-0.0025	0.0028	0.0081
$x_{5,0}$	0.0002	0.05607	2.933×10^{-4}	-0.0051	0.0002	0.0055
$x_{6,0}$	0.0014	0.05730	3.115×10^{-4}	-0.004	0.0013	0.0066
$x_{7,0}$	0.0022	0.05610	2.602×10^{-4}	-0.0032	0.0021	0.0074
$x_{8,0}$	0.0013	0.05661	2.806×10^{-4}	-0.0041	0.0012	0.0065
$x_{9,0}$	0.0033	0.05666	2.634×10^{-4}	-0.0019	0.0034	0.0087

Table 3. Accelerated degradation model parameter estimates

Parameter	μ_0	μ_a	σ_0	σ_a	σ	σ_ε	b
Real Value	0.0012	0.02	0.03	0.03	0.03	0.003	-0.52
Simulation value	0.0011	0.02156	0.0292	0.0373	0.0362	0.0034	-0.526

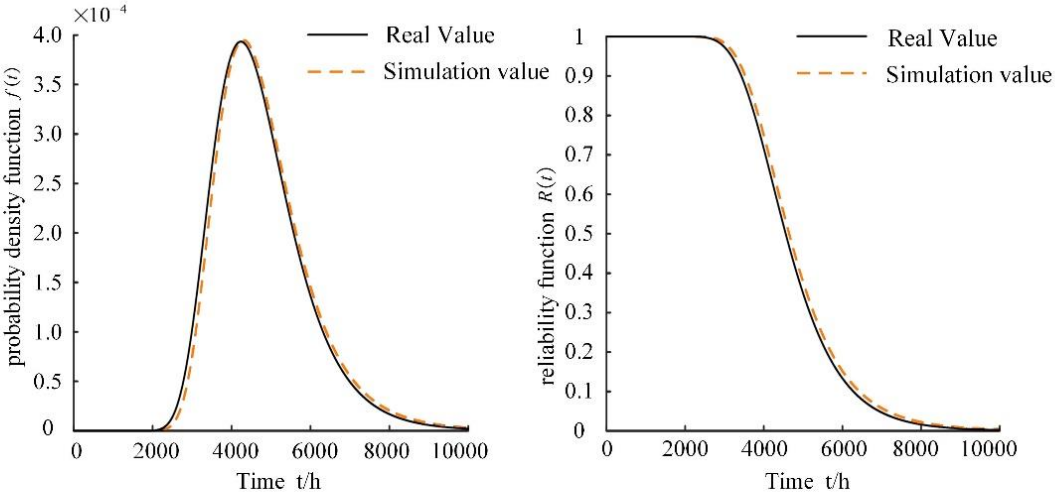

Figure 2. Plot of probability density and reliability function.

7. CONCLUSIONS

Firstly, the reliability index and acceleration model are introduced, and the polynomial acceleration model is analyzed and selected as the acceleration model for performance degradation modeling; secondly, a degradation model based on the Wiener process with multiple uncertainties is established, and the model simultaneously takes into account that the initial value of the performance degradation and degradation rate of different industrial robots are different for different individuals with different measurement errors, and the time to failure is defined based on the first attainment time; and again, the relationship between the degradation model and the acceleration model based on the Wiener process is established. The relationship between the degradation model of Wiener process and the acceleration model is established, and the accelerated degradation reliability model based on the Wiener process with multiple uncertainties is constructed. Finally, to address the problem of insufficient data available for the accelerated degradation test of industrial robots, Bayesian theory, Markov chain Monte Carlo method and Bootstrap method are applied to estimate the unknown parameters, and the validity of the parameter estimation method and the constructed performance degradation model are verified by Monte Carlo simulation data.

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