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Calculation and Programming of Motion Coordinates for 5-axis Machining of Cubic B-splineinterpolating Surface

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ABSTRACT

The control point of cubic B-spline interpolation surface is calculated inversely and the back cutter face is fitted according to the given profile value points. The movement coordinates of each axis of the five-axis grinding machine after the grinding wheel grinding are calculated. Using MATLAB software to draw the given original data, then calculate the μ-direction and ω-direction control points through the B-spline difference, and then generate the standard B-spline surface. According to the grinding principle of the five-axis grinding machine, coordinate changes are made, and finally the grinding track of the grinding wheel is calculated and visualized.

KEYWORDS

Matlab; B-spline Interpolation Calculation; Grinding Motion.

1. INTRODUCTION

With the gradual increase in the demand for gear, the precision requirements for gear are also getting higher and higher [1][2]. CAM system of 5-axis CNC machine tool is widely used in gear machining because of its high machining precision and complex surface machining [3]. As one of the important links of NC machining CAM system, post-processing technology is responsible for converting the output file data of the system into NC code that can be recognized by NC machine tool, and plays a very important role in improving the precision of NC machining. Taking the actual machining data as an example, this paper carries out simulation research in MATLAB environment, and realizes the calculation and visualization of grinding wheel grinding track through the given profile value points of the back of the cutter with different teeth.

2. REVERSE CALCULATION OF CONTROL POINTS

Let the grid of known type value points be Pij (i=1,2,... ,n; J = 1, 2,... n) where m and n are the type value points of μ direction and ω direction respectively, then the obtained polygon vertex mesh should be VIJ (I=1,2,... m+2; J = 1, 2,... ,n+2)[4], its algorithm is as follows:

Calculate n polygons in the direction of μ:

$$
\begin{bmatrix}
9 & -3 & & & & & \\
1/4 & 7/12 & 1/6 & & & & \\
 & 1/6 & 2/3 & 1/6 & & & \\
 & & \ddots & \ddots & \ddots & & \\
 & & & \ddots & \ddots & \ddots & \\
 & & & & 1/6 & 2/3 & 1/6 \\
 & & & & & 1/6 & 7/12 & 1/4 \\
 & & & & & & -3 & 9
\end{bmatrix}
\begin{bmatrix} V_{2J} \\ V_{3J} \\ V_{4J} \\ \vdots \\ \vdots \\ V_{m-1,J} \\ V_{m,J} \\ V_{m+1,J} \end{bmatrix}
=
\begin{bmatrix} 6P_{1J} \\ P_{2J} \\ P_{3J} \\ \vdots \\ \vdots \\ P_{m-2,J} \\ P_{m-1,J} \\ 6P_{m,J} \end{bmatrix}
\tag{1}
$$

$$
\begin{cases} V_{1J} = P_{1J} \\ V_{m+2,J} = P_{mJ} \end{cases} J = 1,2,\cdots,\text{n} \tag{2}
$$

From equations (1) and (2), n polygons in the direction of μ can be obtained, whose fixed point grid is VIJ (I=1,2,... m+2; J = 1, 2,... ,n+2), QIJ=VIJ (I=1,2,... m+2; J = 1, 2,... n+2).

Calculate m+2 polygons in the direction of ω:

$$
\begin{bmatrix}
9 & -3 & & & & & \\
1/4 & 7/12 & 1/6 & & & & \\
 & 1/6 & 2/3 & 1/6 & & & \\
 & & \ddots & \ddots & \ddots & & \\
 & & & \ddots & \ddots & \ddots & \\
 & & & & 1/6 & 2/3 & 1/6 \\
 & & & & & 1/6 & 7/12 & 1/4 \\
 & & & & & & -3 & 9
\end{bmatrix}
\begin{bmatrix} V_{I2} \\ V_{I3} \\ V_{I4} \\ \vdots \\ \vdots \\ V_{I,n-1} \\ V_{I,n} \\ V_{I,n+1} \end{bmatrix}
=
\begin{bmatrix} 6Q_{I1} \\ Q_{I2} \\ Q_{I3} \\ \vdots \\ \vdots \\ Q_{I,n-2} \\ Q_{I,n-1} \\ 6Q_{I,n} \end{bmatrix}
\tag{3}
$$

$$
\begin{cases} V_{I1} = P_{I1} \\ V_{I,n+2} = P_{I,n} \end{cases} I = 1,2,\cdots,\text{m}+2 \tag{4}
$$

From equations (3) and (4), m+2 ω-oriented polygons with n+2 vertices can be obtained, which form a polygon network VIJ (I=1, 2, ... m+2; J = 1, 2, ... n+2). The B-spline surface defined by the mesh can be interpolated to define m×n type value points.

3. CUBIC B-SPLINE INTERPOLATION SURFACE FITTING IS DONE BY CONTROL POINTS

If there are n control points and N-1 curve in the μ direction, and m control points and M-1 curve in the ω direction, then the coefficient matrices of each section are:

μ-direction segment 1:

$$
B_{qn1} = \begin{bmatrix} -1 & 7/4 & -11/12 & 1/6 \\ 3 & -9/2 & 3/2 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{5}
$$

μ-direction segment 2:

$$B_{qn2} = \begin{bmatrix} -1/4 & 7/12 & -1/2 & 1/6 \\ 3/4 & -5/4 & 1/2 & 0 \\ -3/4 & 1/4 & 1/2 & 0 \\ 1/4 & 7/12 & 1/6 & 0 \end{bmatrix} \quad (6)$$

μ-direction segment 3 to n-3:

$$B_{qni} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} i = (3,4,\cdots n-3) \quad (7)$$

μ-direction segment n-2:

$$B_{qnn-2} = \begin{bmatrix} -1/6 & 1/2 & -7/12 & 1/4 \\ 1/2 & -1 & 1/2 & 0 \\ -1/2 & 0 & 1/2 & 0 \\ 1/6 & 2/3 & 1/6 & 0 \end{bmatrix} \quad (8)$$

μ-direction segment n-1:

$$\mathrm{B}_{qnn-1} \begin{bmatrix} -1/6 & 11/12 & -7/4 & 1/4 \\ 1/2 & -5/4 & 3/4 & 0 \\ -1/2 & -1/4 & 3/4 & 0 \\ 1/6 & 7/12 & 1/4 & 0 \end{bmatrix} \quad (9)$$

ω-direction segment 1:

$$B_{qm1} = \begin{bmatrix} -1 & 7/4 & -11/12 & 1/6 \\ 3 & -9/2 & 3/2 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

ω-direction segment 2:

$$B_{qm2} = \begin{bmatrix} -1/4 & 7/12 & -1/2 & 1/6 \\ 3/4 & -5/4 & 1/2 & 0 \\ -3/4 & 1/4 & 1/2 & 0 \\ 1/4 & 7/12 & 1/6 & 0 \end{bmatrix} \quad (11)$$

ω-direction segment 3 to m-3:

$$B_{qmj} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} j = (3,4,\cdots m-3) \quad (12)$$

ω-direction segment m-2:

$$B_{qmm-2} = \begin{bmatrix} -1/6 & 1/2 & -7/12 & 1/4 \\ 1/2 & -1 & 1/2 & 0 \\ -1/2 & 0 & 1/2 & 0 \\ 1/6 & 2/3 & 1/6 & 0 \end{bmatrix} \tag{13}$$

ω-direction segment m-1:

$$\mathrm{B}_{qmm-1} \begin{bmatrix} -1/6 & 11/12 & -7/4 & 1/4 \\ 1/2 & -5/4 & 3/4 & 0 \\ -1/2 & -1/4 & 3/4 & 0 \\ 1/6 & 7/12 & 1/4 & 0 \end{bmatrix} \tag{14}$$

Cubic B-spline interpolation surface fitting formula is as follows:

$$P(\mu,\omega) = [U][B_{qni}][P][B_{qmj}]^T [W]^T \tag{15}$$

Where i= 1,2,... , n-1; j= 1,2,... , m-1;

4. SURFACE FEATURE ANALYSIS

Gaussian curvature of cubic B-spline interpolation surface is calculated.

Gaussian curvature formula:

$$K = k_1 k_2 = \frac{LN - M^2}{EG - F^2} \tag{16}$$

Where E=ru·ru; F=ru·rv; G=rv·rv; L=ruu·n; M=ruv·n; N=rvv·n; n=ru x ru.

5. MOVEMENT CALCULATION OF EACH AXIS OF FIVE-AXIS GRINDING MACHINE

The five-axis CNC grinding machine model and coordinate conversion diagram are shown in Figure 1 and Figure 2.

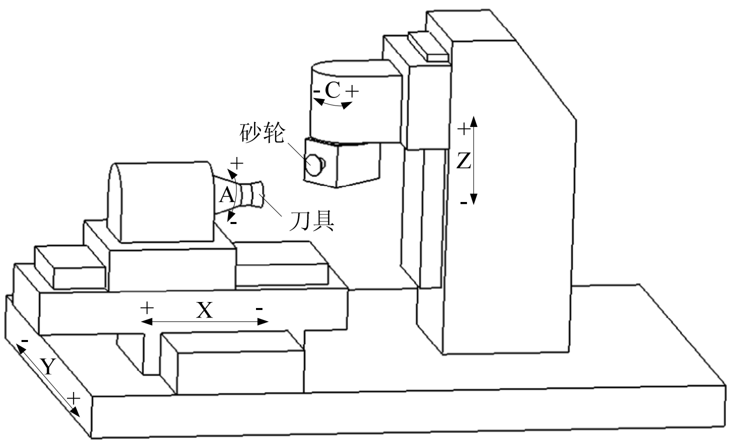

Figure 1. Five-axis CNC grinding machine model

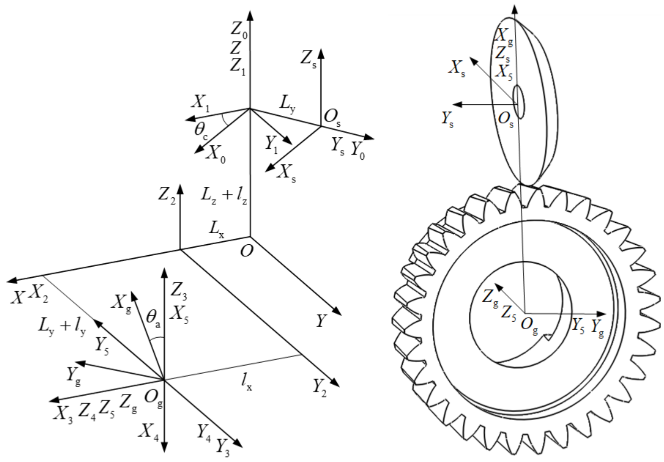

Figure 2. Schematic diagram of coordinate transformation

Among them,

Ss={Os; xs, ys, zs}; The coordinate system is fixed with the grinding wheel;

S={O; x, y, z}; The coordinate system is firmly connected with the machine tool;

Sg={Og; xg, yg, zg}; The coordinate system is fixed with the grinding wheel;

S0, S1, S2, S3, S4, S5 are the transitional coordinate systems in motion, respectively.

Lx, Ly, Lz are the starting installation positions of each shaft of the workpiece and grinding wheel machine, respectively.

lx, ly, lz are the distance that each axis of the machine tool moves when the machine tool moves;

θc and θa are the rotational momentum of the C axis and the A axis, respectively.

Sg moves Ly negatively along the Y-axis to the transformation matrix of S0[5]:

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -L_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

Transformation matrix of S0 rotated θc to S1 about the z axis:

$$T_2 = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 & 0 \\ -\sin\theta_c & \cos\theta_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

The transformation matrix of S1 moving negatively along the z axis - (Lz+lz) to S:

$$T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -(L_z + l_z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The transformation matrix for S to move Lx to S2 along the X-axis:

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & L_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

S2 moves lx along the X-axis and Ly+ly along the Y-axis to the transformation matrix of S3:

$$T_5 = \begin{bmatrix} 1 & 0 & 0 & l_x \\ 0 & 1 & 0 & L_y + l_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

Transformation matrix of S3 rotated 90° about the Y-axis to S4:

$$T_6 = \begin{bmatrix} \cos\frac{\pi}{2} & o & \sin\frac{\pi}{2} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

Transformation matrix of S4 rotated 180° about the z axis to S5:

$$T_7 = \begin{bmatrix} \cos\pi & \sin\pi & 0 & 0 \\ -\sin\pi & \cos\pi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

Transformation matrix of S5 rotated θa to Sg about the z axis:

$$T_8 = \begin{bmatrix} \cos\theta_a & -\sin\theta_a & 0 & 0 \\ \sin\theta_a & \cos\theta_a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Therefore, the workpiece coordinates are expressed in the grinding wheel coordinate system as follows:

$$\begin{bmatrix} x_{sg} \\ y_{sg} \\ z_{sg} \\ 1 \end{bmatrix} = T_1 T_2 T_3 T_4 T_5 T_6 T_7 T_8 \begin{bmatrix} x_g \\ y_g \\ z_g \\ 1 \end{bmatrix} \tag{25}$$

Since the transformation of the vector is independent of the movement of coordinates, the parametric equation of the normal vector ns of the main back tool face in the grinding wheel coordinate system is as follows:

$$\begin{bmatrix} n_{xs} \\ n_{ys} \\ n_{zs} \\ 1 \end{bmatrix} = T_2 T_6 T_7 T_8 \begin{bmatrix} n_{xg} \\ n_{yg} \\ n_{zg} \\ 1 \end{bmatrix} \tag{26}$$

The grinding surface parameter equation of the grinding wheel is:

$$\begin{cases} x_s = r_{s1} \sin\varphi \\ y_s = 0 \\ z_s = -r_{s1} \cos\varphi \end{cases} \tag{27}$$

Where, rs1 is the distance between N point and Ys axis; Is the Angle between the rs1 and Zs axes.

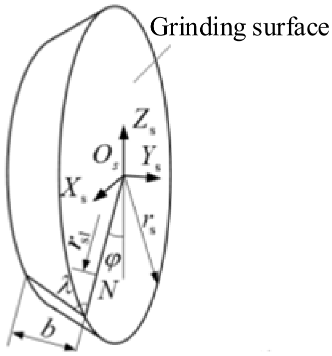

Figure 3. Grinding wheel

As can be seen from the figure, the unit normal vector nd of the grinding surface of the grinding wheel is:

$$\begin{cases} n_{xd} = 0 \\ n_{yd} = -1 \\ n_{zd} = 0 \end{cases} \tag{28}$$

According to the principle of normal vector coincidence between the grinding surface of the grinding wheel and the workpiece tangent plane at the grinding point, the following relationship can be obtained:

$$\mathrm{n_s} = C_0 n_\mathrm{d} \tag{29}$$

Where, C0 is a constant.

Solution (19) can be obtained:

$$\begin{cases} \theta_a = \arctan(\dfrac{n_{xg}}{n_{yg}}) \\ -n_{xg} \sin\theta_a \sin\theta_c - n_{yg} \cos\theta_a \sin\theta_c + n_{zg} \cos\theta_c = 0 \end{cases} \tag{30}$$

From this, θc, θa can be solved, and lx, ly, lz can be solved by bringing back formula (26).

6. APPLIED RESEARCH

Table 1, 2, 3, 4 and 5 are the type value points of five blades on the back surface of the cutting knife. According to the research in the above section, Matlab program is written to inversely calculate the control point of cubic B-spline interpolation surface and fit the back cutter surface.

The resulting diagram of five blades is shown in Figure 4.

After fitting, the fitting surface of the tooth scraper is shown in Figure 5.

Table 1. First blade

X	Y	Z
68.378897326818972	-1.259326212581802	-9.959889496035888
68.793200229499050	-1.321722586819270	-9.904913683919174
69.291004771621985	-1.413181198159756	-9.832796483571261
69.870254511978445	-1.539018673222349	-9.741742274444780
70.528592116800212	-1.704408142693446	-9.630031458237443
71.263370092056192	-1.914361729964907	-9.496026066993228
72.071662805165857	-2.173714007137551	-9.338174915851351

Table 2. Second blade

X	Y	Z
68.304124106560863	-1.793060167904543	-11.371810460181882
68.703433194638180	-1.853758869393619	-11.318618189778991
69.186533190861709	-1.943217321496496	-11.248373371264321
69.751381383589944	-2.066778410287497	-11.159269668271653
70.395632676677081	-2.229642694362555	-11.049576452467527
71.116650142237816	-2.436850796709126	-10.917644461111161
71.911516865704243	-2.693266739801668	-10.761911007914730

Table 3. Third blade

X	Y	Z
68.227519506461093	-2.326633130503716	-12.783932224409652
68.611719999979044	-2.385564985731745	-12.732557802451041
69.079997964242736	-2.472946824116331	-12.664222558567104
69.630326042743846	-2.594147392920103	-12.577109461296766
70.260371782922661	-2.754394501227568	-12.469476886112252
70.967509267623896	-2.958757126298695	-12.339664294537874
71.748830785287808	-3.212128632764635	-12.186097501141893

Table 4. Fourth blade

X	Y	Z
68.149130967041089	-2.860045428361674	-14.196250999790688
68.518107728507090	-2.917142943378773	-14.146728143169220
68.971449386352703	-3.002372983499624	-14.080338951489450
69.507141750050650	-3.121130324189327	-13.995255806070009
70.122865370751654	-3.278669828209534	-13.889726130699009
70.816005742977225	-3.480088658103831	-13.762078142290616
71.583665419715743	-3.730309380093169	-13.610726125864232

Table 5. Fifth blade

X	Y	Z
68.069004112601021	-3.393297616639314	-15.608763054314878
68.422647010308694	-3.448494239991295	-15.561124746345502
68.860939506140085	-3.531498850191813	-15.496717403816559
69.381882150430997	-3.647731846528681	-15.413702847716831
69.983168839083817	-3.802474957520171	-15.310317592867424
70.662196841439084	-4.000853369315599	-15.184878642932512
71.416078128364546	-4.247818727284488	-15.035788863866456

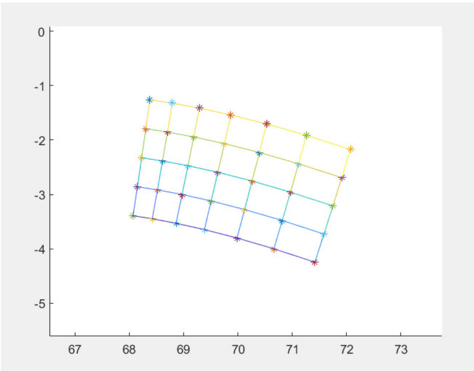

Figure 4. Five blades

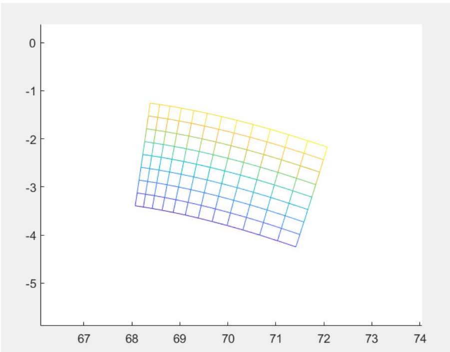

Figure 5. Fitting surface of the scraping tool

The waveform diagram composed of Gaussian curvature of each point is shown in Figure 6.

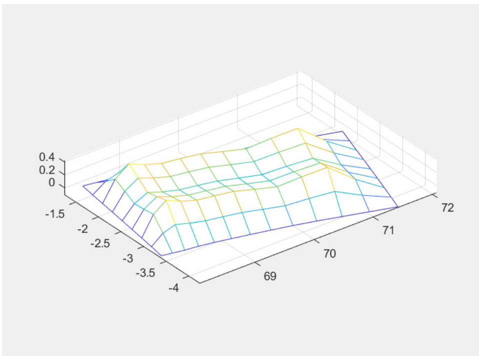

Figure 6. Gaussian curvature at each point

After coordinate conversion, the position relationship between the grinding wheel and the scraping tool face is shown in Figure 7.

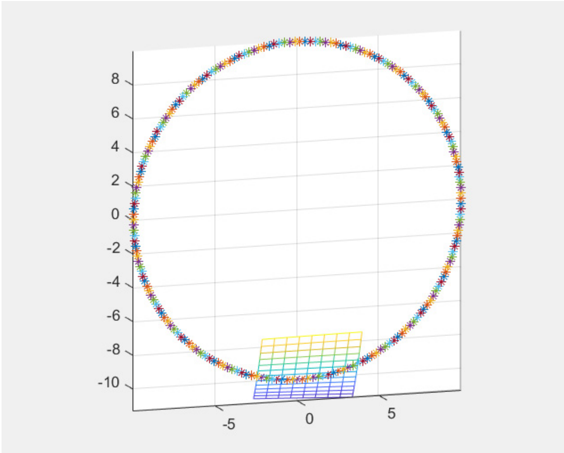

Figure 7. Position of grinding surface of grinding wheel and scraper surface

7. SUMMARY

The control point of cubic B-spline interpolation surface is calculated inversely and the back cutter face is fitted according to the given profile value points. The movement coordinates of each axis of the five-axis grinding machine after the grinding wheel grinding are calculated. Using MATLAB software to draw the given original data, then calculate the μ-direction and ω-direction control points through the B-spline difference, and then generate the standard B-spline surface. According to the grinding principle of the five-axis grinding machine, coordinate changes are made, and finally the grinding track of the grinding wheel is calculated and visualized.

CONFLICTS OF INTEREST

The authors declare that they have no conflict of interest.

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