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Bifurcation Characteristics Analysis of Gear System Considering Time-varying Gear Backlash

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ABSTRACT

Dynamical models for super gear systems have been established, considering both fixed gear backlash and time-varying gear backlash. The focus is on analyzing the modeling process of the time-varying gear backlash. Based on this, a six degrees of freedom bending-twisting dynamical model for super gears has been developed. Then, using rotational speed as the control variable, numerical simulations have been conducted to compare the bifurcation characteristics of super gear systems with fixed gear backlash and time-varying gear backlash. The results indicate that the overall trends of the bifurcation characteristics for both systems are consistent, with their vibration periods decreasing as the rotational speed increases, leading to a gradual stabilization of the system. However, the bifurcation characteristics of the super gear system considering the time-varying gear backlash exhibit a distinct leading trend, showing significant jump phenomena at relatively low rotational speeds.

KEYWORDS

Gear System; Time-varying Gear Backlash; Bifurcation Characteristics.

1. INTRODUCTION

The vibrations generated by gears are one of the key factors affecting the operational performance of the entire mechanical system. As the demand for lower vibrations increases, the mechanisms, prediction, and control of gear system vibrations have garnered widespread attention [1-5]. However, the mechanisms of gear vibrations are complex and closely related to nonlinear dynamic behavior. Therefore, establishing accurate dynamic models is of great significance. Liu et al. [6] adopted Newmark-β method to investigate the gear-shaft system with dynamic center distance and backlash. Zhu et al. [7] built a planetary gear dynamic model with multi backlash and mesh stiffness. Yi et al. [8] considered this backlash model and analyzed the nonlinear dynamics of the gear system with time-varying backlash.

From the above analysis, many studies have considered gear backlash in dynamic models, but they without analyze the bifurcation characteristic of time-varying gear backlash gear model. Bifurcation is a useful method for analyzing the vibrational response of a system, and it is defined as a fundamental change in the qualitative behavior of a dynamic system as its parameters vary. When the system is in a bifurcation state, it can severely affect the stability and reliability of the system, directly leading to damage in the gear system. Therefore, this study investigates the bifurcation characteristics of the system under both fixed and time-varying clearance conditions. The research findings provide a basis for the design and manufacturing of straight gears in the future.

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2. DYNAMIC MODEL OF A SPUR GEAR SYSTEM

2.1. Time-varying Gear Backlash

To meet the requirements of good lubrication and prevent tooth sticking due to tooth deformation and thermal expansion, a certain gear clearance must be preserved between non-working teeth and wings.

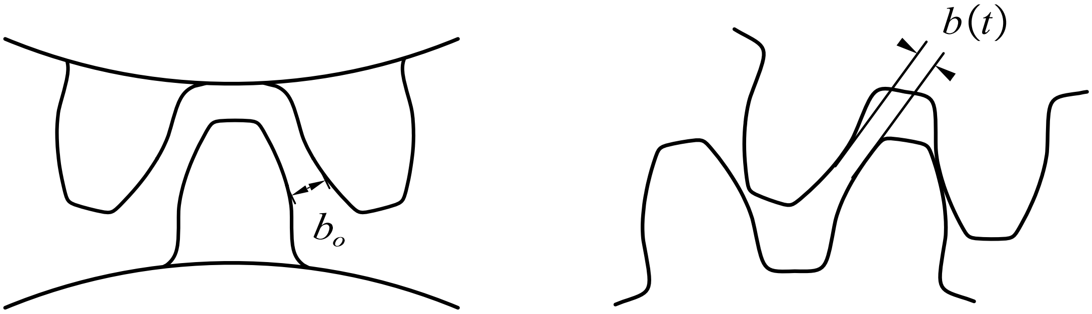
$$b_t = b_0 + \Delta b_t \quad (1)$$

The backlash in gear transmission system can be divided into two parts: constant backlash and time-varying backlash. Fix backlash b_0 is caused by tooth thickness deviation and installation center distance error, also known as initial backlash or design backlash as show in Fig 1(a). The time-varying clearance can be measured along the line of action and calculated from the circular section, where $b(t)$ represents the clearance along the direction of the line of action, and as a time-varying parameter, is mainly caused by various deformations at the point of mesh, as show in Fig 1(b). In the past nonlinear dynamic models, the time-varying characteristics of the backlash are often ignored. It can be calculated as:

$$\Delta b_t = L_0\left(\mathrm{inv}(\alpha) - \mathrm{inv}(\alpha_0)\right) \quad (2)$$

Here,

$$\mathrm{inv}(x) = \tan(x) - x \quad (3)$$



(a) Fix backlash (b) Time varying backlash

Fig 1. The different gear backlash model

2.2. Dynamic Gear Equation

A six-DOF lumped parameter model of a pair of involute spur gear transmission system arranged horizontally is illustrated in Fig. 1. In this model, the shafts, pinion and gear, except for the teeth, are assumed to be rigid, and the shafts are supported by a pair of flexible ball bearings that can be modeled as linear springs and viscous dampers. Dynamic analysis is made in the plane of gear pair, and any out-of-plane motion is neglected. Here, θ_i and T_i are the nominal angular velocity and the torsional torque applied of gear i; $k_{x,yi}$ is the bearing stiffnesses in the x and y directions, respectively; $c_{x,yi}$ is the corresponding bearing damping.

Based on the theory of viscoelasticity, the dynamic mesh force consists of elastic force and damping force along the LOA and can be written as:

$$F_{\mathrm{m}} = k_{\mathrm{m}} f\left(\delta, b_t\right) + c_m f_1\left(\delta, b_t\right) \quad (4)$$

where, $k_m$and c_m are time-varying mesh stiffness and damping, $f(\delta,\ b_t)$ and $f_1(\delta,\ b_t)$ are the nonlinear functions for backlash and relative speed respectively and can be expressed as:

$$f\left(\delta,b_t\right)=\begin{cases}\delta-\operatorname{sign}\left(\delta\right)b_t & \left|\delta\right|>b_t\\ 0 & \text{else}\end{cases},\quad f_1\left(\delta,\ b_t\right)\begin{cases}\dot{\delta}-\operatorname{sign}\left(\delta\right)\dot{b}_t & \left|\delta\right|>b_t\\ 0 & \text{else}\end{cases} \tag{5}$$

here, superscript $(\cdot)$ denotes derivative with respect to time.

The dynamic differential equation of the system is established as follows.

$$m_p\ddot{x}_p+k_p^x x_p+c_p^x\dot{x}_p+F_m\sin\alpha=0 \tag{6}$$

$$m_p\ddot{y}_p+k_p^y y_p+c_p^y\dot{y}_p+F_m\cos\alpha=0 \tag{7}$$

$$I_p\ddot{\theta}_p\left(t\right)+F_mR_p=T_p \tag{8}$$

$$m_g\ddot{x}_g+k_g^x x_g+c_g^x\dot{x}_g-F_m\sin\alpha=0 \tag{9}$$

$$m_g\ddot{y}_g+k_g^y y_g+c_g^y\dot{y}_g-F_m\cos\alpha=0 \tag{10}$$

$$I_g\ddot{\theta}_g\left(t\right)-F_mR_g=-T_g \tag{11}$$

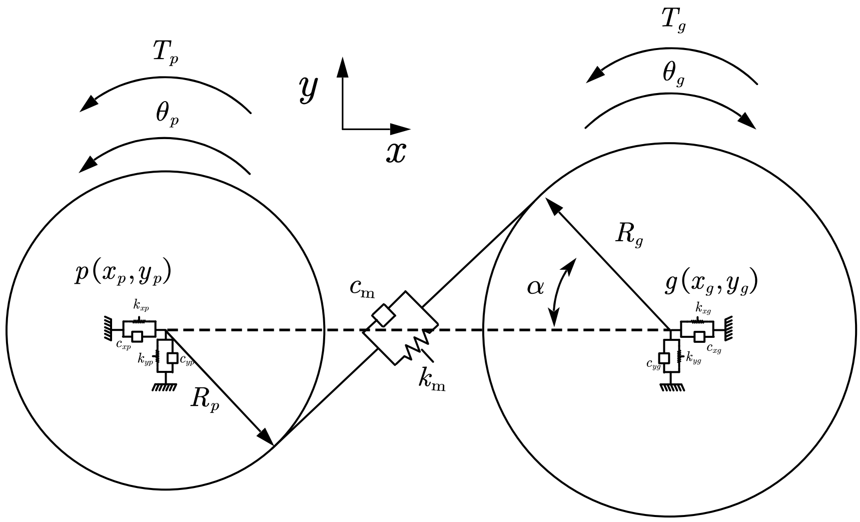


Fig 2. Gear dynamical model

3. NUMERICAL SIMULATION AND ANALYSIS

Table 1. The main parameters

Parameter	Pinion/Gear
Number of teeth	27/41
Tooth width (mm)	10
Module (mm)	2.5
Modulus of elasticity (GPa)	207
Initial pressure angle α_0(°)	20
Mesh damping ratio ξ_m	0.25
Backlash(m)	1E-4

The gear pair system is a non-linearity system. To fully research the systematic bifurcation and chaos, rotational speed of pinion is selected as variable parameters. The main parameters are listed in Table 1.

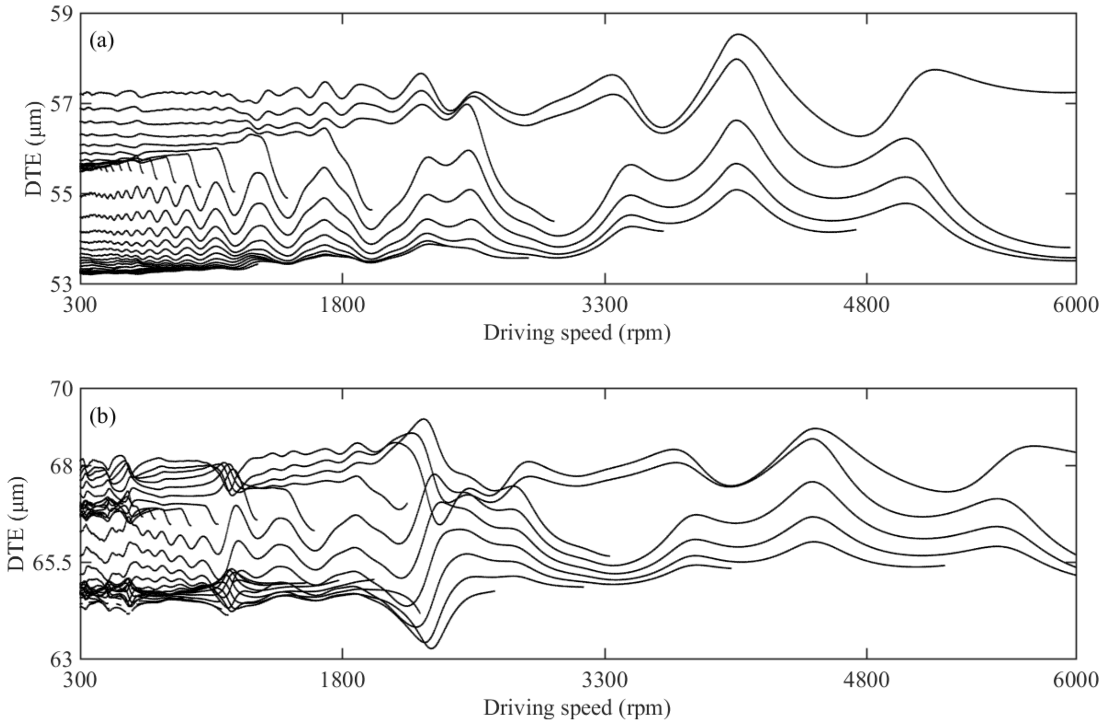


Fig 3. The bifurcation of different gear backlash model (a) fixed gear backlash (b) time varying gear backlash

As show in Fig 3, the bifurcation results of the gear system with fixed gear backlash and time varying gear backlash are shown respectively. First, in the low-speed range, the bifurcation lines of the gear system that considers time-varying gear backlash are densely clustered, and compared to the bifurcation results of the fixed gear backlash system, its curves fluctuate more frequently and with greater amplitude, especially in the rotational speed range of 300 to 400. When plotting the bifurcation diagram, local maxima during periodic motion at that rotational speed are selected, meaning that for each vibration cycle within a mesh cycle, data will be recorded once in the bifurcation diagram. Therefore, in the low-speed range, the gear system with time-varying gear backlash exhibits multiple vibration cycles at each speed, and the vibration impact characteristics are significantly stronger than those of the gear system that only considers fixed gear backlash. As the rotational speed increases, the bifurcation lines on the bifurcation diagrams of both gear systems become markedly fewer, with many lines appearing to abruptly break at certain speeds, and these broken lines do not reappear. The reason behind this phenomenon is that the rotational speed provides enough kinetic energy for the gears to complete the meshing process with fewer collision occurrences during engagement, resulting in fewer bifurcation lines represented on the bifurcation diagram.

Furthermore, at higher speeds, the bifurcation lines of both gear systems become increasingly smooth, indicating an improvement in the stability of the gear meshing process. Although multiple vibration cycles still exist within a meshing process, the variation in vibration periods of the gear systems between successive rotational speeds becomes more gradual, representing a stable meshing state for the gear system. However, it is notable that the gear system considering time-varying gear backlash reaches a sudden change earlier than the gear system with fixed gear backlash. That is, a noticeable increase in bifurcation results occurs at a speed of 2000 for the time-varying clearance gear system, while the fixed gear backlash gear system only shows a significant sudden change at a speed of 4000.

4. CONCLUSION

This work builds a six degrees of freedom dynamic model for a spur gear pair system which consider time-varying gear backlash. Then, with the help of various data method, bifurcation and chaos characteristics of the ystem are researched. Some beneficial conclusions are obtained as follows:

The bifurcation results of the gear system considering time-varying gear backlash and the gear system with fixed gear backlash show a consistent overall trend. Both bifurcation curves gradually decrease with increasing rotational speed, indicating that the vibrational impact characteristics of the system are gradually weakening, and higher speeds are more conducive to maintaining the stability of the gear system's transmission.

Compared to the fixed gear backlash gear system, the gear system with time-varying gear backlash exhibits a clear leading trend, meaning that the significant jump phenomena that occur at higher speeds in the fixed gear backlash gear system happen at lower speeds in the time-varying clearance gear system. At lower rotational speeds, the time-varying gear backlash gear system demonstrates more frequent vibrational impact behavior.

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