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Modeling Method of Meshing Stiffness of High Speed Spur Gears

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ABSTRACT

As one of the most important dynamic excitation sources, the gear time-varying mesh stiffness is the key parameter of gear system dynamic model. So how to calculate the gear mesh stiffness accurately is of great importance. Based on Euler beam theory, this paper proposes a primitive calculation algorithm (PCA) to calculate the time-varying mesh stiffness of spur gears. A simplified gear model is developed based on nonlinear strain-displacement relationships, and a finite element program is used to simulate the gear's meshing stiffness. The rationality of this method was verified through finite element analysis. The results indicate that mesh stiffness plays a crucial role in controlling mesh deformation. In view of this, this study lays a theoretical foundation for further analysis of the vibration and noise of gears.

KEYWORDS

Time-varying Mesh Stiffness; Gear Modeling.

1. INTRODUCTION

Spur gears are typical transmission elements in industrial machinery, automotive applications, and aerial engineering.Mesh stiffness waveforms are main sources of internal excitations [1-3], generally used as key inputs to dynamic models for gear vibrations and loads [4-7].Therefore, appropriate mesh stiffness representations lay foundation for dynamic analysis and optimal designs of high-speed spur gears.

Various methods have been developed for mesh stiffness calculation, such as experimental method; FEM and potential energy method.Yang et al. [8] proposed a potential energy method to calculate the tooth deformation, which embodied the tooth deformation subjected to Hertzian contact, bending, and axial compression.The experiment is simple and reliable in grid stiffness evaluation, but the measurement equipment is expensive and difficult to operate [9, 10]. FEM can also accurately predict mesh stiffness, but finite element method is usually time consuming due to repetitive work in pre-processing.An in-depth study and critical review of FE-based methods for mesh stiffness estimation in spur gears has recently been carried out by Zheng et al [11].Compared with experiments and FEM, analytical methods are more flexible to reveal the correlation between gear parameters and mesh stiffness.

In this paper, a simplified gear element model is derived by using the expression of nonlinear strain displacement relationship and variational formula. The finite element program is compiled and the meshing stiffness of the gear affected by mesh force is analyzed. Quasi-static and dynamic solutions are compared to existing models, FEM.Results demonstrated the reasonable accuracy of this model.

2. ORGANIZATION OF THE TEXT

2.1. Gear Strain Energy and Kinetic Energy

The motion process of the gear is divided into two stages, the first stage is its own rotational motion, the second stage is due to centrifugal force and caused by deformation displacement. The gear is assumed to be a variable section beam with large plane rotation and small deformation, as shown in Fig 3, one of the elements can be approximated as equal section. The gear rotation is controlled by the ψ rotation Angle and is represented by two coordinate systems, each denoted as $e_i^{'} \otimes e_j^{'}$ and $E_i^{'} \otimes E_j^{'}$, respectively.

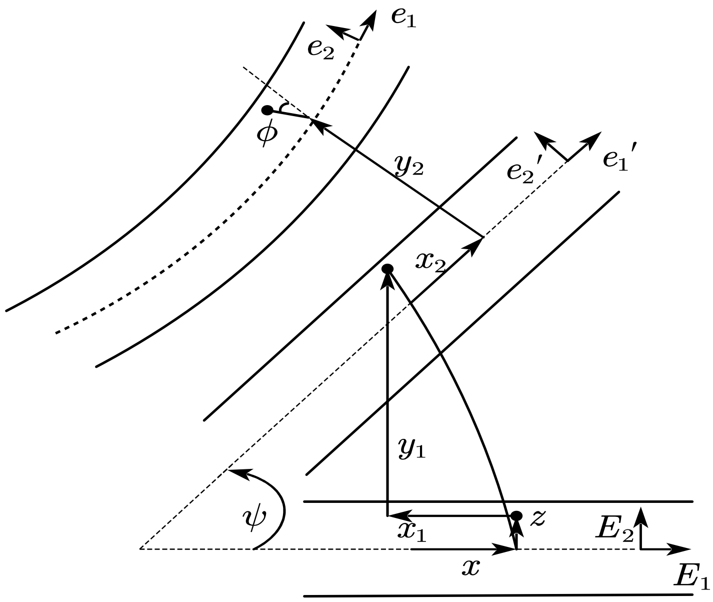

Fig 1. Schematic diagram of the equivalent beam of the tooth

According to the above assumptions, the rotation angle of the equivalent beam gear section ϕ small. Therefore, the position vector X for a given point in the coordinate system is

$$X = \begin{bmatrix} x - x_1 + (x_2 + z\phi)\cos\psi - y_2 \sin\psi \\ z + y_1 + (x_2 + z\phi)\sin\psi - y_2 \cos\psi \end{bmatrix} \tag{1}$$

Based on the assumption that the shear strain is negligible and the transverse normal stress contribution is negligible, only the axial component of the strain tensor $\varepsilon_{xx} \equiv E_{11}$ is considered in this paper. The potential energy of a gear can be expressed as:

$$U_i = \frac{1}{2}\int E\varepsilon_{xx}^2 dV \tag{2}$$

According to the longitudinal displacement u and transverse displacement S of the rotating gear in the illustrated plane, the kinetic energy T is expressed as:

$$T = \frac{1}{2}\rho\int \left(\dot{S}^T\right)^T \dot{S}^T dV \tag{3}$$

2.2. Gear Equations of Motion

Using the expressions for strain and kinetic energies presented above, a variational formulation is used to write the equations of motion. Hence, from Hamilton's principle

$$\delta\int_{t_1}^{t_2}(U-T)dt=0 \tag{4}$$

The discretization uses Lagrange linear shape function to discretize the axial displacement u and Hermitian cubic function to discretize the lateral deflection v. This leads to six degrees of freedom $S^T = [u_1, v_1, \beta_1, u_2, v_2, \beta_2]$, where (β_1, β_2)=$(\acute{v}_1, \acute{v}_2)$. Using Hamilton's principle (13), combined with the discretized strain and kinetic energies, leads to the following equations of motion

$$M\ddot{X}+C\dot{X}+K_eX=F \tag{5}$$

The symmetric translational mass matrix M corresponding to the $\dot{s}$ velocity vector can be expressed as:

$$M=\frac{\rho LA}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22L & 0 & 54 & -13L \\ & & 22L & 0 & 13L & -3L^2 \\ & & & 140 & 0 & 0 \\ & & SYM & & 156 & -22L \\ & & & & & 4L^2 \end{bmatrix} \tag{6}$$

The symmetric elastic stiffness matrix of gears can be expressed as:

$$K_e=\frac{E}{\Delta L}\begin{bmatrix} A & 0 & 0 & -A & 0 & 0 \\ & \frac{12I}{\Delta L^2} & \frac{6I}{\Delta L} & 0 & -\frac{12I}{\Delta L^2} & \frac{6I}{\Delta L} \\ & & 4I & 0 & -\frac{6I}{\Delta L} & 2I \\ & & & A & 0 & 0 \\ & sym & & & \frac{12I}{\Delta L^2} & -\frac{6I}{\Delta L} \\ & & & & & 4I \end{bmatrix} \tag{7}$$

The damping matrix of the gear calculated by assuming that it is a combination of the K and M for the Riley damping as follows:

$$C=\alpha_M M+\beta_K\left(K_v+K_v\right) \tag{8}$$

here, α_M is mass matrix coefficient β_K is stiffness matrix coefficient.

2.3. Calculation of Meshing Stiffness

The meshing stiffness is calculated by evaluating the displacement and forces at each meshing point of the gear teeth. The initial conditions assume zero velocity and acceleration, and the displacement matrix is iteratively calculated using an average acceleration method. This process yields the dynamic displacement matrix, from which the single-tooth dynamic stiffness for both the driving and driven gears is derived. The total dynamic meshing stiffness of the gear pair is then determined based on these individual stiffness values.

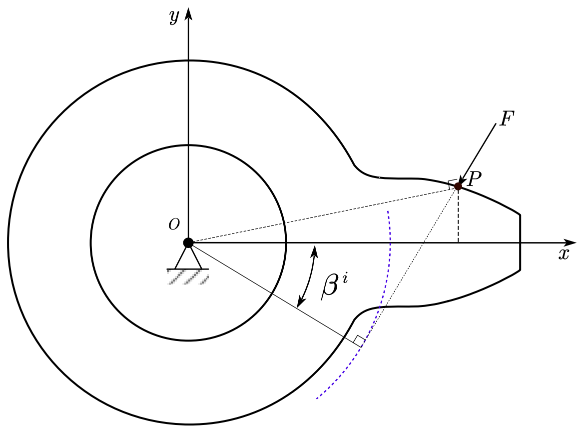

Fig 2. Single tooth model

Solve it using the average acceleration method. For the definition of initial conditions, the initial velocity matrix $\dot{X}_1$ and initial acceleration matrix $\ddot{X}_1$ corresponding to the first meshing point of the gear teeth set in this algorithm are 0, and the displacement matrix is calculated using the traditional statics method, which can be expressed by the following formula:

$$X_1 = \frac{F_1}{K} \tag{9}$$

In the formula, $F$is the external load matrix when the meshing force acts on the meshing point.

After setting the above parameters, use the average acceleration method to iteratively calculate $\{X_i\}, \{\dot{X}_i\}, \{\ddot{X}_i\}$ in equation (1) until the meshing force stops at the last meshing point. At this point, the$\{X_i\}$ obtained through iterative calculation is the dynamic displacement matrix related to the rotational speed. Extract the deformation values Δx_i and Δy_i of the i-th meshing point in the x and y directions from the constantly iterating $\{X_i\}$. Then, the single tooth dynamic stiffness value k_{pi} of the driving gear at this point can be represented by Δx_i and Δy_i as follows:

$$k_{pi} = \frac{F}{\sqrt{\Delta x_i^{\,2} + \Delta y_i^{\,2}}} \tag{10}$$

Similarly, the single tooth dynamic stiffness value k_{gi} of the driven gear at the i^{th} meshing point can be obtained. Finally, the comprehensive dynamic meshing stiffness of the gear pair related to the rotational speed during single tooth meshing can be represented by k_{pi} and k_{gi} as follows:

$$k_{ms} = \frac{1}{\dfrac{1}{k_p} + \dfrac{1}{k_g}} \tag{11}$$

3. RESULT VERIFICATION

In this section, the accuracy of stiffness calculation was verified. The parameters of the gear pair are detailed in

Table 1. The static stiffness of a one-stage spur geared system is compared with previous results by using the FEM [10, 12] and the PEM [36]. Note that for the comparison of the test results, both the PCA and fem calculated results use Euler beam elements in order to make a comparison of test results.

Table 1. Parameters of spur gear system

Parameter	Pinion/Gear
Number of teeth	29/39
Tooth width (mm)	10
Module (mm)	2.5
Modulus of elasticity (GPa)	207
Initial pressure angle $\alpha_0(°)$	20
Initial position angle $\beta_0(°)$	20
Mesh damping ratio ξ_m	0.25
Backlash(m)	1E-4

It can be observed in Figure 6 that there is a significant difference between EM and FEM, PM. The results obtained from the PCA calculations are in good agreement with those obtained by the ANSYS software. There are only minor differences between the PCA and ANSYS calculations, which is due to the fact that ANSYS utilizes numerical approximations in order to obtain approximate values. Although this method improves the computational speed, it sacrifices a certain amount of accuracy in the results. In order to improve the computational accuracy, in this study, the integral expression of each element is integrated exactly to obtain a more accurate overall stiffness matrix of the gear. Meanwhile, comparing the results with FEM [5], it can be seen that PCA performs better than EM, which further proves the correctness of PCA.

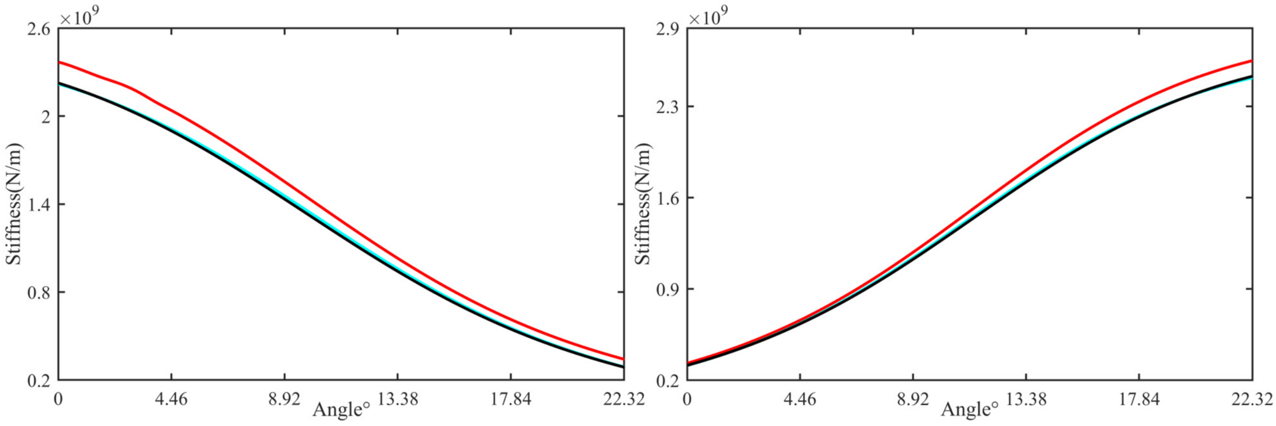

Fig 3. Static stiffness comparison in PM(—), FEM(—) and EM(—)

FEM is a precise numerical analysis technique that breaks down the gear into smaller elements, calculating the stresses, strains, and displacements of each one. In contrast, the blue curve in the figure represents the results from the PCA, which closely follows the FEM results. Particularly in terms of the overall trend and peak stiffness variations, PCA aligns well with FEM, providing nearly identical insights into mesh behavior. The key advantage of PCA lies in its ability to achieve this high level of accuracy while being significantly more computationally efficient.Meanwhile, the red curve in the figure illustrates the outcomes of the EM, which show a clear deviation from both FEM and PCA. This difference arises because EM tends to oversimplify the complex mechanical interactions involved in gear meshing. By neglecting nonlinear effects and dynamic changes, EM fails to capture the true variations in mesh stiffness, leading to less reliable predictions.

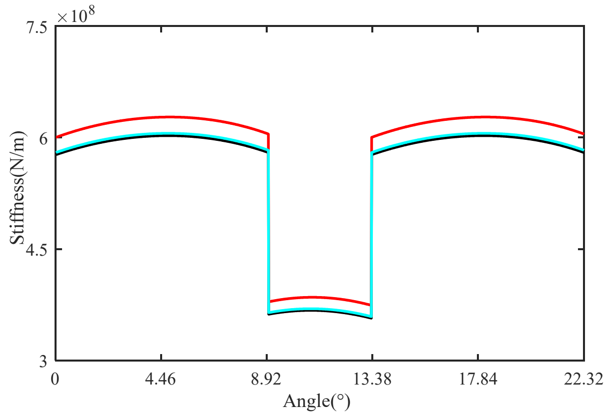

Fig 4. Mesh stiffness comparison in PM(—), FEM(—) and EM(—)

4. SUMMARY

This paper presents a reliable and computationally efficient approach to calculating the dynamic meshing stiffness of spur gears.The PCA method offers greater accuracy and efficiency compared to FEM and PEM, providing a robust tool for analyzing high-speed gear systems. The study's results show that the inclusion of centrifugal effects is critical for understanding gear behavior under high-speed conditions, and the PCA method offers a significant advancement in gear design and analysis.

CONFLICTS OF INTEREST

The authors declare that they have no conflict of interest.

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This is the place to fill in information about funds, sponsors, etc. that need to be thanked.

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