

# Multi-Objective Particle Swarm Optimization Algorithm based on Position Vector Offset

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## ABSTRACT

In response to the issue that particle swarm optimization algorithms tend to fall into local optima when dealing with multi-objective optimization tasks, a multi-objective optimization algorithm based on particle swarm is proposed. This algorithm is based on the relationship between the position vectors of particles, changing the selection and movement strategies of particles to find the true Pareto front. Firstly, two additional position vectors are added around the iterating particles to enhance their search capability; then, a non-dominated vector archive is established to record the non-dominated solutions of the iterating particles and the additional position vectors, increasing particle diversity. Finally, additional position vectors with high fitness are selected to produce a shift in the iterating particle's position, accelerating particle convergence. Comparing this algorithm with dMOPSO, SMPSO, NMPSO, and MOPSOCD algorithms, simulation experiments show that the proposed PVPSO algorithm has stronger optimization ability.

## KEYWORDS

Multi-Objective Optimization; Particle Swarm Optimization; Vector Offset.

## 1. INTRODUCTION

Many real-world industrial problems can be modeled as multi-objective optimization problems (MOPs), such as data clustering [1], ship scheduling design [2], and multi-label classification [3]. Addressing these issues has always been a challenging task. Therefore, a large number of researchers in the field of multi-objective optimization have been developing and applying intelligent algorithms. Among them, the Particle Swarm Optimization (PSO) algorithm has superior global search capabilities, parallel processing, and high-dimensional problem-solving abilities. It is not restricted by constraint conditions and has strong robustness to noise, making it widely applicable. However, when dealing with multi-objective optimization problems, the PSO algorithm tends to fall into local optima. To address this, researchers have helped the algorithm escape from this dilemma through various means, such as improving mutation strategies and enhancing search strategies. To improve the quality of the optimal solution set in the decision space, Li Guoqing and others [4] located the Pareto front in the population and added a network search strategy, searching the particle swarm on the grid to enhance the algorithm's search capability; Zhang Yunfeng and others [5] adaptively divided the population into multiple sub-populations and updated them with different strategies to increase population diversity.

Based on the above theoretical research, to further enhance the search capability of the MOPSO algorithm and alleviate the issue of the algorithm easily falling into local convergence, this paper proposes a multi-objective particle swarm optimization algorithm based on position vector offset (PVPSO). The algorithm enhances particle search capability by adding position vectors, establishes a non-dominated vector archive to preserve particle diversity, and applies additional vectors with high fitness to influence iterating particles, accelerating particle convergence. Finally, the optimization performance of the PVPSO algorithm is evaluated by comparison with four existing classic PSO algorithms.

## 2. MULTIPLE OBJECTIVE PARTICLE SWARM OPTIMIZATION

Although the PSO algorithm has achieved success in various optimization tasks, it remains a heuristic method. Initially, there was not much research on its application to multi-objective optimization. In 2002, the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm was proposed by C.A. Coello Coello et al.[6]. It utilizes an external archive to maintain particle diversity, aiming to address the PSO algorithm's inability to handle multi-objective optimization problems.

MOPSO draws parallels with evolutionary algorithms, employing the Pareto sorting scheme and establishing an archive of discovered non-dominated vectors. It uses a global attraction mechanism to encourage particles to converge towards global non-dominated solutions. The specific approach is as follows: First, after initializing the population and the velocities of the particles, they are evaluated to find and store the positions of particles with non-dominated vectors. Subsequently, a hypercube of the search space is constructed, using these hypercubes to establish a coordinate system, with each particle's coordinates defined by the objective function values. Finally, a repository truncation operation is established to maintain the scale of relatively optimal solutions in the repository, stopping the run when the maximum number of cycles is reached. The method for updating particle positions is seen in equation:

$$POP[t+1]= POP[t]+ VEL[t] \quad (1)$$

when a particle's current position is better than the positions in the repository, the particle's position updating method is seen in equation:

$$P_{best}[t]= POP[t] \quad (2)$$

Where  $POP[t]$  and  $VEL[t]$  respectively represent the position and velocity of the particle at iteration  $t$ ; represents the non-dominated vector solutions saved in the repository.

A major drawback of the particle swarm algorithm is that the population may converge prematurely. The fundamental principle behind this problem is that the algorithm cannot guarantee that the optimal solution is located on the line between the global best position and the individual best position. Another reason for this problem is the too rapid flow of information between particles, leading to a loss of particle diversity and increasing the possibility of falling into local optima. Moreover, the PSO algorithm has weak local search capabilities, and many studies have shown [7] that the algorithm tends to fall into local optima during the later stages of convergence when dealing with optimization problems of different characteristics. Achieving a good balance between diversity and convergence is important for the particle swarm algorithm to overcome these shortcomings.

### 3. IMPROVED MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION ALGORITHM

In response to the shortcomings of the particle swarm algorithm in terms of diversity and convergence ability, the PVSPSO algorithm was proposed. The algorithm process is as follows:

- 1) Initialize the position vectors  $X$  and velocity vectors  $V$  of the particle swarm.
- 2) Initialize the non-dominated vector archive  $A$ .
- 3) While  $t \leq t_{\max}$  do
- 4) Calculate the fitness values of the particles, including  $x_{il}(t)$  and  $x_{ir}(t)$ .
- 5) Update the personal best  $p_{\text{best}}$  and the global best  $G_{\text{best}}$  according to fitness.
- 6) Update position and velocity based on  $X$  and  $V$ .
- 7) Update the non-dominated vector archive, retaining non-inferior solutions.
- 8)  $t=t+1$
- 9) end while
- 10) Output the archive.

PVSPSO enhances the particle diversity and convergence of the particle swarm optimization algorithm by incorporating an enhancement formula into the PSO algorithm. During the convergence process of the algorithm, the formula can generate new position vectors around the iterating particles. The formula is described as follows:

$$x_i(t+1) = x_i(t) + v_i(t) + \mu[\alpha x_{il}(t) + \beta x_{ir}(t)] \quad (3)$$

$$x_{il}(t) = x_i(t) \left\{ 1 - \frac{[G_b - x_i(t)] \|v_i(t)\|_2}{\|G_b - x_i(t)\|_2} \right\} \quad (4)$$

$$x_{ir}(t) = x_i(t) \left\{ 1 + \frac{[G_b - x_i(t)] \|v_i(t)\|_2}{\|G_b - x_i(t)\|_2} \right\} \quad (5)$$

Where:  $x_i(t)$  and  $v_i(t)$  represent the position and velocity of the particle at iteration  $t$ ,  $x_{il}(t)$  and  $x_{ir}(t)$  are two additional position vectors around the iterating particle;  $\alpha$  and  $\beta$  are steering factors;  $\mu$  is a random number with a value range of  $[0, 0.1]$ .

As shown in the formulas above, in PVSPSO, each particle has two additional position vectors. These vectors can be within the space between the current particle and the global best position or outside of it, ensuring the exploration and utilization of the space around the particles. Particles are defined by individuals moving in the search space, and the search process is based on the following three rules:

Each particle (including the two additional position vectors) has a memory, capable of remembering the better solutions they have encountered.

The current position of the particle needs to participate in the fitness evaluation with the two additional position vectors, retaining the relatively optimal steering factor that is not zero.

In each iteration, every particle will be influenced by the two additional position vectors, resulting in a slight position shift towards the global best position (or in the opposite direction).

Taking a two-dimensional space as an example, the new position of a particle influenced by additional position vectors with high fitness is shown in Figure 1. This figure demonstrates how the proposed

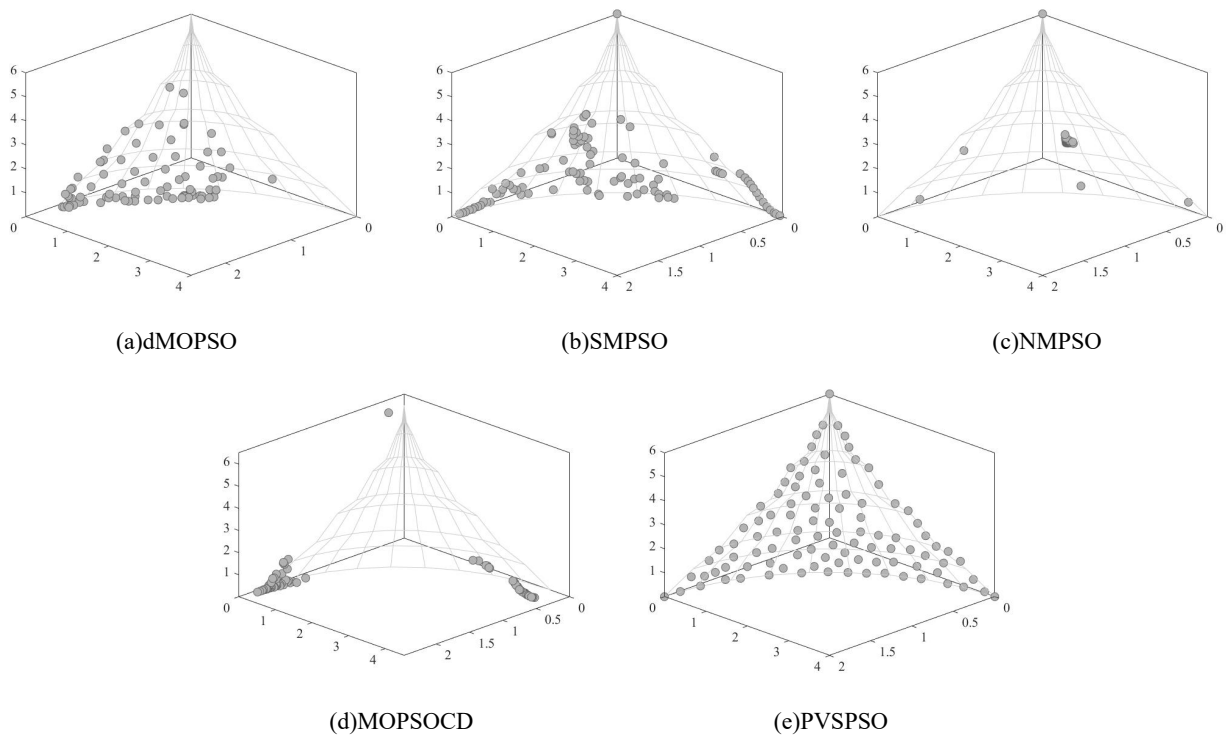
formula defines new potential solutions in the search space and affects the position of iterating particles.

At each iteration, new position vectors generated by PVSPSO are evaluated, and they can only influence the iterating particle if their fitness is superior to that of the iterating particle and another position vector. Figures 1b and 1c respectively show how  $x_{ir}(t)$  and  $x_{il}(t)$  cause a slight shift of the current particle position towards the global best position (or in the opposite direction). From this, it can be seen that the PVSPSO algorithm adds more search points around the particle, resulting in higher particle diversity, a broader search space, and better efficiency. Moreover, the PVSPSO algorithm accelerates the convergence speed to the global best position in the later stages of iteration by evaluating fitness and preserving the optimal solutions obtained from these position vectors, enhancing the algorithm's convergence. When the termination conditions are met, the PVSPSO algorithm stops the optimization process, at which point the number of iterations has reached the set maximum value.

#### 4. EXPERIMENTAL RESULTS AND ANALYSIS

To test the performance of the PVSPSO algorithm, it was simulated and tested with four PSO algorithms (dMOPSO [8], SMPSO [9], NMPSO [10], MOPSOCD [11]).

To more vividly demonstrate the performance of the PVSPSO algorithm, Figure 1 depicts the optimization results of the five PSO algorithms on the 3-objective WFG1 test function. From the figure, it can be observed that the NMPSO and MOPSOCD algorithms fall into local optima, with most particles concentrating in a small area of the true Pareto front; the dMOPSO and SMPSO algorithms only found parts of the true Pareto front, with still unexplored areas and uneven particle distribution. The PVSPSO algorithm found the vast majority of the true Pareto front and the particles are uniformly distributed, indicating that the PVSPSO algorithm has good diversity and convergence.



**Figure 1.** Optimization results of five PSO algorithms on the 3- objective WFG1 test function

## 5. CONCLUSION

A multi-objective optimization algorithm based on particle swarm optimization is proposed based on the relationship between iterative particles and global optimal particles, combined with additional position vectors. This algorithm enhances the diversity of particles by adding additional position vectors, and iterates particles to change their direction of travel based on high fitness position vectors, accelerating the convergence of the algorithm. The experimental results compared with four existing classic algorithms show that the PVPSO algorithm has good performance and the optimization results are evenly distributed.

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