WEP Warwick Evans Publishing

International Journal of Global Economics and Management

ISSN: 3005-9690 (Print), ISSN: 3005-8090 (Online) | Volume 5, Number 1, Year 2024
DOI: https://doi.org/10.62051/ijgem.v5n1.08
Journal homepage: https://wepub.org/index.php/IJGEM/index

A Study on the Pricing of Convertible Bonds in the Chinese Market Based on the LSMC Model

Jiajie Qiu *, Xiangxin Tao, Yanting Ji

College of Zhejiang University of Science and Technology, Zhejiang, China

*Corresponding Author: Jiajie Qiu

ABSTRACT

Convertible bonds, which possess both stock and option characteristics, currently lack a sufficiently effective pricing method for the complex and specific clauses found in Chinese convertible bonds. This study employs structured model pricing and numerical experiment methods for the pricing of Chinese convertible bonds. The structured model selected is the Black-Scholes model, while the numerical experiment methods include the binomial tree pricing model and the Least Squares Monte Carlo (LSMC) model. A proposal is made to introduce an implied volatility model in the LSMC model pricing method, incorporating the Heston model to consider the impact of the implied volatility of the underlying stock in the real market, and further adjusting the LSMC model by estimating the volatility of the underlying stock using the implied volatility model. Empirical results indicate that numerical methods have higher pricing accuracy than structured models, with the LSMC model demonstrating the highest accuracy. It is also found that there is a certain premium phenomenon in the Chinese market for convertible bonds. Furthermore, the accuracy of the model improves after the volatility adjustment, indicating that implied volatility has explanatory power for the specific clauses contained in convertible bonds, and this influence is reflected in pricing.

KEYWORDS

Pricing of Convertible Bonds; Black-Scholes Model; LSMC Model; Binomial Tree Model; Heston Model

1. INTRODUCTION

Convertible bonds are a hybrid type of bond that combines the characteristics of conventional bonds and equity, and embeds multiple options. Therefore, accurately pricing them is an extremely challenging task. The various clauses contained in convertible bonds classify them as American derivative securities. Further analysis of the exercise conditions of these American options reveals their interdependence, such that if one option is exercised, the entire convertible bond will be terminated. In the Chinese market, issuers of convertible bonds may redeem the bonds or adjust the conversion price under certain conditions as stipulated in the terms. Therefore, these nested options implied by the clauses cannot be separately handled. Hence, convertible bonds are complex American derivative instruments that depend on multiple underlying assets such as stock prices and volatility. They possess exceptionally intricate value forms and strong path-dependent characteristics, thus endowing the accurate pricing of convertible bonds with significant academic value and practical significance.

The first convertible bond was issued by the Erie Railway Company in the United States in 1843, and since then, the convertible bond market has rapidly developed, making convertible bonds an important financial instrument in the capital market. The development of the convertible bond market in China started relatively late compared to foreign markets, with the issuance of the first convertible bond "Baoan Convertible Bond" in 1992 when the Chinese securities market was launched. With the continuous improvement of relevant laws and regulations related to convertible bonds in line with the development of the Chinese securities market, enterprises have begun to explore the use of convertible bonds for financing. After the trough of the primary market following the equity division reform in 2005, and the wave of bond redemptions during the bull market in 2015, the refinancing regulations and credit subscription system implemented in 2017 re-stimulated the demand for convertible bond issuance. This ushered in a golden period for the convertible bond market, with a steady increase in the issuance volume and continuous expansion of the market size. Accurately estimating the value of convertible bonds is conducive to the healthy development of the Chinese market economy.

2. CORRELATION THEORY

2.1. Literature on Structured Models for Convertible Bond Pricing

After the establishment of the Black-Scholes option pricing theory by Merton (1974), Black, and Scholes (1973) [1], Ingersoll (1977) and Brennan & Schwartz (1977) [2] applied the Black-Scholes model to the pricing of convertible bonds. They regarded convertible bonds as a combination of ordinary bonds and call options, and the value of convertible bonds was seen as a function of the company's value and time. Assuming that the company's value follows a geometric Brownian motion, they derived the partial differential equation that convertible bonds satisfy using the Black-Scholes model. Taking into account the embedded option characteristics of convertible bonds, they utilized the principle of no arbitrage to determine the optimal conversion and redemption strategies for convertible bonds, thereby establishing the boundary conditions and terminal value conditions for the partial differential equation. Finally, they computed the value of convertible bonds using numerical methods. However, they did not consider the impact of the implied volatility of convertible bonds as an option. Heston (1993) [3] attempted to better describe market phenomena by introducing the concept of stochastic volatility. The Heston model assumes that asset prices follow a geometric Brownian motion, while volatility follows a mean-reverting stochastic process. This setting allows the volatility to fluctuate over time and also demonstrates a tendency to regress towards its long-term mean, depicting the dynamic changes in volatility. Since the maturity of foreign convertible bonds mostly exceeds 10 years, it is unreasonable to assume that the long-term interest rate is constant, and convertible bonds carry a certain default risk. Therefore, Tsiveriotis and Fernandes (1998) [3] decomposed the value of convertible bonds into two parts: cash flows for equity and fixed-income parts. Considering the risk perspective, the fixed-income cash flow is affected by default risk, thus the risk-free rate adjusted for the risk premium is used as the discount rate for model calculation. The cash flow for the equity part is directly discounted using the risk-free rate, as the study assumes that the cash flow for the equity part is risk-free without default risk.

2.2. Experimental Literature on the Pricing of Convertible Bonds

In addition to the aforementioned analytical approaches, numerical model pricing methods have also been applied in the study of convertible bond pricing. Compared to the pricing method of numerical partial differential equations, Monte Carlo simulation can more effectively determine the value of convertible bonds under boundary conditions. The flexible dynamic simulation characteristics of Monte Carlo simulation also enable it to better handle special terms that conventional models cannot address, thus achieving a more precise replication of the real environment. Buchan (1997) [4] was the first to apply the Monte Carlo simulation method to convertible bond pricing, combining the conversion holding value under different stock prices at different simulation nodes to price convertible bonds. However, Monte Carlo requires simulation of all possible early exercise times, resulting in low computational efficiency. Therefore, Longstaff and Schwartz (2001) [5] first proposed the use of the Least Squares Monte Carlo (LSMC) model pricing method, using the least squares regression on each simulation path to estimate the value of cash flows, thereby reducing the number of simulation paths. This not only improves computational efficiency but also enables path dependency handling. Subsequently, the LSMC model has been widely used in pricing American options. C. Wilde and A.H. Kind (2005) [6] proposed and empirically studied a Monte Carlo simulation-based convertible bond pricing model. The theoretical research on convertible bonds in China started relatively late, mainly built on the basis of foreign convertible bond pricing theory, combined with specific conditions of the convertible bond market in China for model modification and empirical analysis. Therefore, this paper focuses on summarizing the numerical methods for pricing domestic convertible bonds. Lai Qinan et al. (2005) [7] believe that the Chinese convertible bond market has certain particularities. They used a binomial tree to handle the impact of redemption clauses by adding an upper limit to the value of convertible bonds, and adding a lower limit to handle the impact of put options and special downward adjustment clauses. However, in the binomial tree model, as the number of time steps increases, the number of nodes in the binomial tree grows exponentially, posing significant computational challenges, and the model is highly sensitive to parameters such as volatility and interest rates, with slight parameter changes leading to significantly different pricing results. Tang Wenbin (2008) [8] and Zhao Yang (2009) [9] were the first to use the LSMC method to study the pricing of Chinese convertible bonds, but did not consider the potential impact of parameters such as volatility and default probabilities. Subsequently, scholars enriched the LSMC convertible bond pricing model based on the terms and market conditions of convertible bonds. For instance, Luo Xin and Zhang Jinlin (2020) [10] studied the default probability of convertible bonds and incorporated downward adjustment clauses into the model. Chang Hui (2019) [11] improved the accuracy of volatility estimation for the Black-Scholes model by using the GARCH model, and then applied it to convertible bond pricing. Ma Changfu et al. (2019) [12] argued that trigger conditions such as soft redemption and soft put options should not be simply overlooked, thus they used a two-factor lattice model for stock prices and interest rates, considering default risk. The results showed that when using the implied volatility of the model, the pricing of convertible bonds could be more accurate. Ni Wufan and Li Mingsheng (2023) [13] derived the implied volatility of convertible bonds by constructing a binomial tree model and empirically found that the implied volatility of convertible bonds enhances the uncertainty of convertible bond market prices. Combining with the national conditions, it is necessary to consider the impact of implied volatility on convertible bond prices when studying the domestic convertible bond market.

3. RESEARCH METHOD

3.1. Structured Models Based on the Black-Scholes Framework

The first pricing approach involves using the Black-Scholes pricing method to divide the value of convertible bonds into two parts: one part for the bond value and the other for the conversion option value. When convertible bonds are issued and enter the conversion period, and before the end of the term, if investors exercise the conversion option early, it is considered as a European call option, indicating the belief that the future increase in the company's stock price will bring higher returns and convert the bond into the company's stock. Another possibility is that investors continue to hold the convertible bond until it is redeemed by the company at maturity, at which point the convertible bond is considered a regular corporate bond. The Black-Scholes model is more suitable for pricing European call options, but it does not account for the impact of the special terms of convertible bonds on their prices. Although the direct use of the Black-Scholes model is not precise, for the Chinese convertible bond market, the triggering of redemption, put options, and downward adjustment clauses is relatively rare and does not significantly impact the value of convertible bonds. Therefore, using the Black-Scholes pricing method for domestic convertible bond pricing has a certain reference value, and the formula is as follows:

For the stock price, it is described using a geometric Brownian motion that satisfies the following stochastic differential equation (SDE):

$$dS_t = rS_t dt + \sigma S_t dW_t^Q \quad (1)$$

r is the risk-free interest rate, S_t is the risk-free interest rate, σ is the volatility of stock price returns, W_t is the standard Wiener process, Q is a risk-neutral measure.

The value of the convertible bond's conversion option C can be calculated using the Black-Scholes model with the following formula:

$$C = S_0 N(d_1) - Ke^{-r(T-t)} N(d_2) \quad (2)$$

Where K is the conversion price, T is the remaining period of the convertible bond, S_0 is the stock price of the convertible bond, and N represents the cumulative distribution function of the standard normal distribution:

$$d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \quad (4)$$

The value B of the convertible bond portion is obtained by discounting cash flows:

$$B = \sum_{t=1}^{T} \frac{C_t}{(1+r_i)^t} + \frac{F}{(1+r_i)^T} \quad (5)$$

C_t represents the annual coupon interest, r_i represents the market interest rate for corporate bonds, F represents the face value of the convertible bond, and T represents the remaining time until the maturity of the convertible bond.

According to the par value specified in the terms of the convertible bond at the time of issuance M and Conversion price K, The ratio of convertible bonds to equity m can be compute:

$$m = \frac{M}{K} \quad (6)$$

Therefore, the value of the convertible bond can be expressed as:

$$V = mC + B \quad (7)$$

3.2. Numerical Experiment Methods Based on LSMC Model

In the second pricing approach, the LSMC pricing method is employed. It processes each asset price path separately, and handles paths that comply with the corresponding terms accordingly. The method involves simulating regression of option values at each time point along each path using ordinary least squares from the end to the beginning, to compute the expected present value of future holding value, i.e., the anticipated holding value. This enables a comprehensive consideration of the impact of the rights contained in the special terms on the price when comparing the anticipated holding value with the immediate exercise value, and thus determines the optimal strategy of exercising or continuing to hold. Through this approach, cash flows at each time point along each path are obtained, and the present value of the convertible bond at specific times is computed as required. The specific steps are as follows:

First, assume that in a risk-neutral market, the underlying stock price movement of a convertible bond obeys a geometric Brownian motion:

$$dS_t = rS_t dt + \sigma S_t dW_t^Q \tag{8}$$

According to the stochastic process formula, the stock price at expiration can be expressed as:

$$S_T = S_t \exp[(r - \frac{\sigma^2}{2})(T - t) + \sigma(W_T^Q - W_t^Q)] \tag{9}$$

By utilizing the stochastic differential equation for stock price, N paths can be generated using Monte Carlo simulation. Each path is set with a step size of one trading day, encompassing a total of T time nodes.

Subsequently, for the generated N paths, they are to be analyzed separately according to the following provisions, and the paths that meet the specific conditions outlined are to be treated accordingly.

Put option clause: During a certain period, when the stock price falls below a certain proportion of the conversion price, investors are unable to gain higher returns by holding the convertible bonds. At this point, the right of exercise belongs to the investor, who is entitled to sell the convertible bonds back to the issuer at a predetermined price, known as "timely stop-loss". When this clause is triggered, the conversion value and expected holding value of the convertible bond are often lower than the sum of the face value of the convertible bond and the current interest. Assuming that the number of paths in N paths triggering the put option clause is *k*, the value of this portion is calculated using the sum of the face value of the convertible bond and the accrued interest, and the put option value is discounted. Finally, the average value of these *k* paths at the initial moment is calculated and denoted as *p*.

Call option clause: During a certain period, when the stock price exceeds a certain proportion of the conversion price, the issuer's own interests may be threatened. At this point, the right of exercise belongs to the issuer, who has the right to redeem the convertible bond from the investor at a predetermined price, i.e., the investor is forced to convert the bonds. When this clause is triggered, investors often take the maximum value between the conversion value of the convertible bond and the sum of the face value of the convertible bond and the current accrued interest as the final return on investment. Assuming that the number of paths in *N* paths triggering the call, option clause is *w*, the redemption value is discounted, and the average value of these w paths at the initial moment is calculated, denoted as *q*.

Downward revision clause: This clause only exists in China and a few other Asian countries. Similar to the put option clause, the conversion price downward revision clause is triggered when the stock price falls below a certain proportion of the conversion price, but its triggering condition is weaker than that of the put option clause. When the put option clause is triggered and the expected holding value is less than the put option value, the issuer faces a significant cash flow pressure from investors selling the bonds back to the company, in order to alleviate the financial burden, the issuer will downwardly revise the conversion price to increase the expected holding value to encourage investors to continue holding, while protecting the interests of both parties. When this clause is triggered, investors need to compare the expected holding value after the downward revision of the conversion price with the put option value, and the greater of the two is taken as the final return on investment. Assuming that the number of paths in *N* paths triggering the downward revision, clause is *x*, different thresholds are set to distinguish put option paths that may overlap with downward revision paths, and the value of the convertible bond after the conversion price revision is discounted. Finally, the average value of these *x* paths at the initial moment is calculated and denoted as *v*.

Subsequently, for paths not triggering any special clauses, a comparison is made at each time node from the end to determine whether to exercise the conversion right. For price paths that have not undergone any processing, the redemption value at the maturity date of the convertible bond is used for calculation. For ease of calculation, the LSMC model is adopted to estimate the expected continuation value through cross-sectional least squares polynomial regression.

$$Y_t = a + bX_t + cX_t^2 + \varepsilon \tag{10}$$

X_t is the convertible value at time t, Y_t is the current value of the convertible bond at time t. The dataset comprises the present value of future earnings along each path and X_t.Then, Minimize ε by fitting the equation and determine the model parameters a, b, and c. We compare Y_t with the current convertible value ECV_t: if the Y_t is greater, we convert the bond at that time, otherwise, we continue to hold the convertible bond. This process is repeated for all paths to finalize the selection.

Finally, discount the value of these alternative paths back to the initial time and take the average, denoted as *u*. After obtaining the optimal execution strategy for each path, discount the profits of all paths and their corresponding interest cash flows to the initial time, and take the average. This is the theoretical price of the convertible bond based on LSMC theory.

$$C = \frac{pk + qw + xv + (N - k - w - x)u}{N} \tag{11}$$

3.3. Numerical Experiment Methods Based on Binary Tree Model

3.3.1. Binary tree single-period model

The binary tree model is a commonly used model for simulating stock price movements. It assumes that the movement of stock prices is composed of a series of small binary movements, thus simulating the changes in the stock over a specific time period. The steps of the binary tree model involve dividing the time period from the option purchase date to the expiration date into several equal intervals. It is specified that within each time interval, the stock price can only have two possible changes: either up or down. Under this setting, the binary tree model can illustrate the changing paths of the underlying asset price during the option's lifespan. On the option's expiration date, since the asset price is known, the value of the option is also determined. Through backward calculation, the present value of the option can be derived based on the option value at each node.

Suppose the stock price is S_0 at time t_0. At time t_1, according to the assumptions, the price of the stock may rise to S_u or may fall to S_d.Thus the price of the European-style option C_0 at the moment may rise to C_u or may fall to C_d:

$$Cu = \max(0, S_u - X) \tag{12}$$

$$Cd = \max(0, S_d - X) \tag{13}$$

At time t_1, when the stock price rises, the portfolio value is expressed as:

$$Vu = mS_u - C_u \tag{14}$$

At time t_1, when the stock price rises, the portfolio value is expressed as:

$$Vd = mS_d - C_d \tag{15}$$

According to the no-arbitrage principle, if the initial value of two portfolios is equal, their ending value is equal $V_u = V_d$, thus:

$$m = \frac{C_u - C_d}{S_u - S_d} \tag{16}$$

m corresponds to the number of shares that need to be bought to sell an option. According to the principle of risk-neutral pricing, under the setting of no arbitrage in the market, the portfolio rate of return should be equal to the risk-free interest rate *r* in the market, so the equation is derived:

$$(mS - C)(1 + r) = mS_u - C_u \tag{17}$$

According to the above equation, the formula of binary tree option pricing model is finally derived:

$$C = p\left(\frac{C_u}{1+r}\right) + (1-p)\left(\frac{C_d}{1+r}\right) \tag{18}$$

3.3.2. Convertible bond pricing model based on multi-period binary tree

The binomial pricing model for convertible bonds primarily involves simulating the movement of the underlying stock price over the bond's tenure using a binomial tree model. After simulating the stock price at each time node, the convertible bond value at each node is calculated based on the bond's redemption, put, and conversion provisions. Finally, applying the principle of risk-neutral pricing, the convertible bond price at the initial time point is backwardly derived. An N-step binary tree model is established based on the underlying stock price on the date of issuance of the convertible bond:

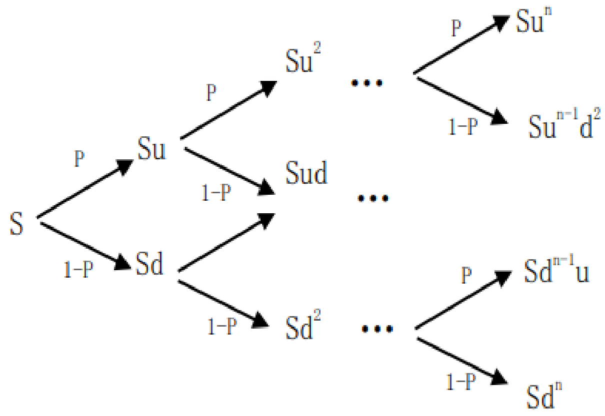

Figure 1. N-step binary tree stock price simulation

4. EMPIRICAL ANALYSIS

4.1. Data Collection and Processing

In order to validate the pricing model, the selection of convertible bond pricing samples should prioritize standardized issuance terms and a significant issuance volume to ensure representativeness and effectiveness. Although convertible bonds are subject to credit default risk, as of December 31, 2022, Chinese convertible bonds have maintained a flawless record since their inception in 1992, which is unparalleled worldwide. Therefore, this article does not consider the impact of default risk on convertible bond pricing.

Following the aforementioned sample selection criteria, this study conducts numerical calculations and research on convertible bond pricing using the sample of Hailan Convertible Bonds, which has an issuance size of 3 billion yuan and includes redemption provisions, put provisions, and downward revision provisions. Hailan Convertible Bonds were issued by Hailan Fashion Group Co., Ltd., with a credit rating of AA+ for both the issuer and the bond itself. The specific convertible bond terms for Hailan Convertible Bonds are presented in Table 4-1.

Table 1. Terms of Hailan Convertible Bonds Issuance Date

Heilan Convertible Debt Terms	
The name of the convertible bond	Hailan Convertible Bonds
Issuance size (in 100 million yuan)	30
Denominadion (in yuan)	100
date of listing	2018-7-31
duration of existence (in years)	6
conversion period	2019-01-21 to 2024-07-12
Initial Conversion Price (in Yuan)	6.10
Coupon Rate	0.3%, 0.5%, 0.8%, 1.0%, 1.3%, 1.8%

The specific special terms and conditions of Heilan Convertible Bonds are as follows:

(1) Redemption Terms:

When either of the following circumstances occurs, the Board of Directors of the company shall have the right to decide to redeem all or part of the outstanding convertible bonds at a price equal to the face value of the bonds plus accrued interest up to the current period: a) During the conversion period, if the closing price of Company A's shares on at least 15 trading days out of a consecutive 30 trading days is not less than 130% (inclusive) of the current conversion price. b) The remaining balance of the convertible bonds issued this time is less than 30 million yuan.

(2) Put Option Clause

During the last two interest-bearing years of the convertible bonds issued this time, if the closing price of the company's stock falls below 70% of the current conversion price for any consecutive 30 trading days, the bondholders have the right to put their entire or partial holdings of convertible bonds back to the company at a price equal to the face value plus accrued interest up to the current period.

(3) Downward Revision Clause

During the tenure of the outstanding convertible bonds issued this time, if the closing price of the company's stock falls below 80% of the current conversion price for at least 15 trading days within any consecutive 30 trading days, the Board of Directors shall have the authority to propose a downward revision plan for the conversion price and submit it for a vote at the company's shareholder meeting.

In the empirical analysis, the estimated price of Hailan Convertible Bonds from the listing date to December 1, 2023, is calculated using the LSMC pricing model and compared with the Black-Scholes pricing model. Initially, this study compares the stock price data of Hailan Fashion from July 31, 2018, to December 1, 2023, consisting of a total of 1327 trading days, with the data of Hailan Convertible Bonds. There are 31 days of missing data, accounting for 2.3% of the total trading days. The study adopts a deletion approach to handle the missing values. The closing price trend of Hailan Convertible Bonds' underlying stock price on trading days is depicted in Figure 2, and the volatility of the logarithmic stock price returns is illustrated in Figure 3. From the figures, it can be observed that the underlying stock price of Hailan Convertible Bonds exhibits stable fluctuations, and the logarithmic stock returns display a certain level of volatility, with a relatively fixed fluctuation range, demonstrating a degree of stability. However, there are also instances where the volatility has continuously decreased or increased over a period of time.

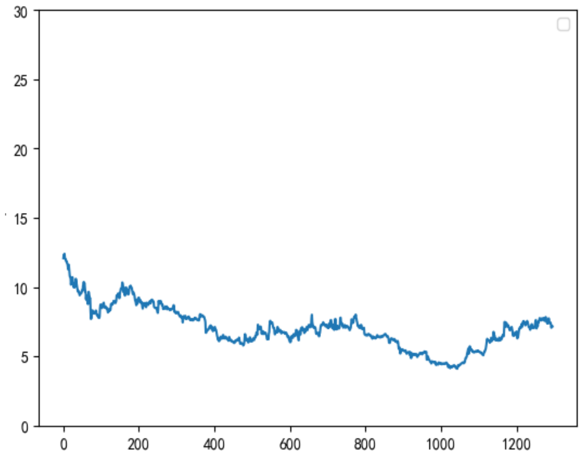

Figure 2. Stock price movements

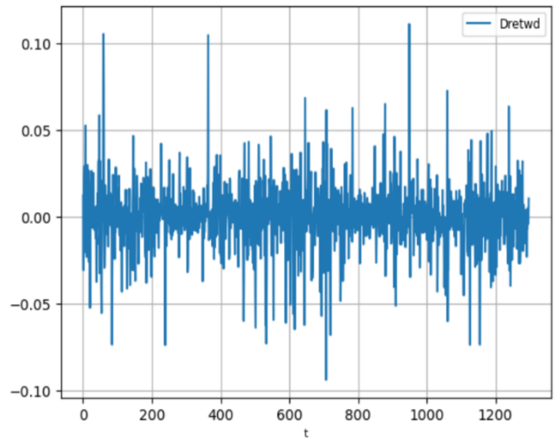

Figure 3. Logarithmic return time series

4.2. Model Parameter Estimation

Based on the previous analysis, it is evident that the parameters required for pricing convertible bonds include: the underlying stock price, stock volatility, risk-free interest rate, remaining tenure, conversion price, put price, and call price. Among these, the remaining tenure, conversion price, put price, and call price are all based on the data from the issuance prospectus. The risk-free interest rate is determined using the yield of the ten-year government bond as the benchmark. The estimation of volatility adopts the historical volatility method, with the specific steps as follows:

First, calculate the daily logarithmic returns of the stock price R_i:

$$R_i = \ln(\frac{S_i}{S_{i-1}}) \tag{19}$$

Then calculate the overall average logarithmic return:

$$\bar{R_i} = \frac{1}{n}\sum_{i=t-n}^{t} R_i \tag{20}$$

Finally, estimate the historical volatility:

$$\sigma_{t+1}^2 = \frac{1}{n-1}\sum_{i=t-n}^{t}\left(R_i - \bar{R}\right)^2 \tag{21}$$

Where n is the number of samples used to calculate historical volatility, this article selects 30 trading days, and then calculates the annualized volatility using the example of 252 trading days per year:

$$\hat{\sigma} = \sigma * \sqrt{252} \tag{22}$$

4.3. The Actual Optimization Strategy of the LSMC Pricing Method

According to the actual issuance terms of Heilan convertible bonds, the optimized execution strategy is formulated.

Table 2. Convertible bond clauses optimize execution strategies

Clause triggers	condition	Holder Strategy	earnings
Redemption at expiration date (T)	$mS_T \le C_T$	Redemption Price: Redemption	C_T
	$mS_T > C_T$	Execution of equity transfer	mS_T
Redemption is triggered before expiration date (t)	$mS_t \le C_t$	Forced to redeem early	C_t
	$mS_t > C_t$	Forced to transfer shares	mS_t
Trigger a buyback before expiration date(t)	$ECV_t \le mS_t \le C_t$	Proactive resale	C_t
	$mS_t \le C_t \le ECV_t$	Keep holding	/
A downward revision is triggered before expiration date(t)	$C_t \le \hat{ECV}_t \le \hat{m} S_t$	Execution of equity transfer	$\hat{m} S_t$
	$C_t \le \hat{m} S_t \le \hat{ECV}_t$	Keep holding	/

4.4. Pricing Performance

Based on the selection of parameters and optimization strategies, we will use the Black-Scholes pricing model, the binomial tree pricing model, and the Least Squares Monte Carlo (LSMC) pricing model to price the Hai Lan convertible bond. In the LSMC pricing method, the determination of the optimal timing for special clauses is heavily influenced by the comparison of current yield to the yield in the next period. Therefore, the number of steps should not be too large. We will choose to simulate 2,000 paths with 252 steps. The stock price simulation results are shown in Figure 4.

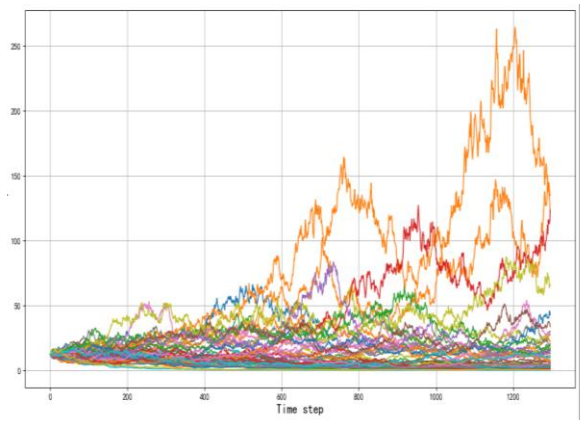

Figure 4. Stock price simulation

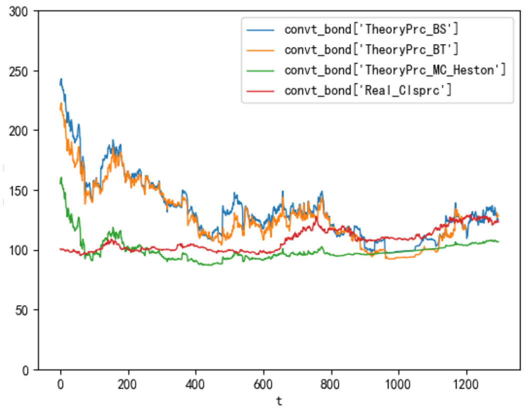

Figure 5. Price comparison

The estimated price of the convertible bond within the corresponding date of the LSMC model is obtained by simulation, and the estimated price is plotted with the actual price in Figure 5 by comparing it with the binary tree pricing method and the Black-Scholes pricing method.

4.5. Model Testing

We will conduct a series of tests to evaluate the accuracy of the Black-Scholes pricing model, the binomial tree pricing model, and the LSMC pricing model. These tests include calculating the mean squared error (MSE), heteroscedasticity-adjusted mean squared error (HMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). By controlling the estimation time window, we will compare the accuracy of the models based on the chosen loss function.

$$MSE = \frac{1}{n}\left(\sum\left(\tilde{p} - p\right)^2\right) \tag{23}$$

$$MAE = \frac{1}{n}\sum\left|\tilde{p} - p\right| \tag{24}$$

$$MAPE = \frac{1}{n}\sum\left(\frac{\tilde{p} - p}{p}\right)\times 100\% \tag{25}$$

$$HMSE = \frac{1}{n}\sum\left(1 - \frac{\tilde{p}}{p}\right)^2 \tag{26}$$

Where n denotes the number of days for the model to be evaluated, $\tilde{p}$ represents the theoretical price, P denotes the actual price. The four loss function values of the model, MSE, HMSE, MAE, and MAPE, are summarized in Table 3.

Table 3. Model accuracy comparison

	MSE	HMSE	MAE	MAPE
B-S	1551.8272	1.1185	28.1125	0.1868
LSMC	180.0445	0.7322	10.4423	0.0993
Binary tree	221.1556	0.9516	15.1646	0.1565

After the calculations, it has been observed that the values of the four loss functions for the experimental pricing methods are both lower than those of the Black-Scholes pricing model. This indicates that the numerical experimental pricing methods exhibit higher accuracy. This is attributed to the fact that the Black-Scholes pricing model only considers convertible bonds and cannot account for the influence of exotic options contained in the special terms of convertible bonds on model pricing.

Furthermore, in the numerical experiments, the accuracy of the LSMC pricing model is higher than that of the binomial tree pricing model. This is due to the significant impact of step size selection on the precision of the results in the binomial tree experiment. A smaller step size can improve accuracy but also increases the computational workload. In the experiment, a step size exceeding 1000 steps was used, which has a certain impact on the pricing accuracy of the binomial tree model. However, the least squares Monte Carlo method can mitigate this problem to some extent, resulting in the highest pricing accuracy.

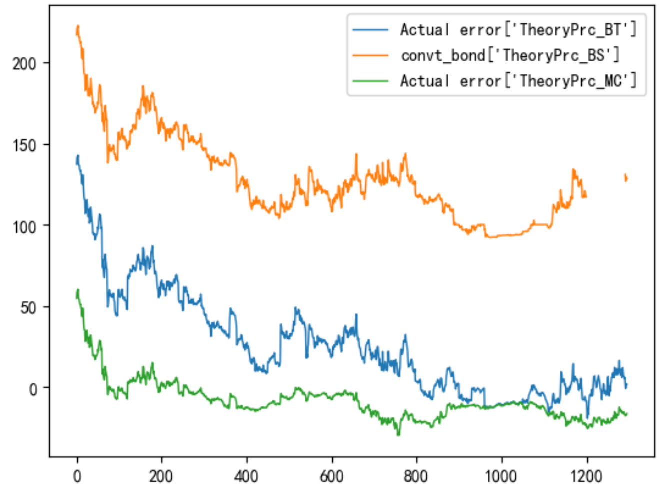

Figure 6. Comparison of the errors of the three models

Consequently. In comparing the pricing results of the three models, it is evident from the graph that the Black-Scholes model significantly overestimates the pricing of convertible bonds in the pre-issuance period. This is attributed to the fact that prior to the conversion period of the convertible bond, the market price of the underlying stock of Hailan Convertible Bonds is much higher than the conversion price. The Black-Scholes model assumes a constant conversion price as the exercise price, resulting in a severe overestimation of the value of the call option. Conversely, while the numerical experiments exhibit a lower overestimation compared to Black-Scholes during the pre-issuance period, the theoretical prices still significantly exceed the actual prices. This is due to the higher market price of the underlying stock at the time of the bond's listing, which enhances the value of conversion. However, as the bonds have not entered the conversion period, investors do not have the right to exercise the conversion privilege, and the redemption clause has not been triggered, hence investors cannot convert the bonds into ultimate returns, resulting in an "inflated" value.

Subsequently, as the conversion period commences, the theoretical prices under the numerical experimental pricing method approach the actual prices, slightly below the actual price of the convertible bond, with an average error of 3.34%. This to some extent confirms the existence of a "high premium" in the domestic convertible bond market. The higher the premium, the greater the disparity between the market price of convertible bonds and their conversion value, weakening the equity characteristics of the convertible bonds.

5. CONCLUSION

As a compound bond with embedded multiple options, the pricing of convertible bonds relies more on numerical simulations. In this paper, the numerical experimental pricing method and structural model pricing method were adjusted for the Chinese market, specifically for pricing convertible bonds in the Chinese market containing downward adjustment clauses. Empirical analysis was conducted using the convertible bonds issued by Hailan Home, comparing the binomial tree model and LSMC model with the traditional structural model Black-Scholes pricing method that incompletely considers the special terms. The Mean Squared Error (MSE), Mean Absolute Error (MAE), HMSE, and HMAE of the estimated results of different models were compared, validating the necessity of numerical experiments in handling the special terms in the pricing of convertible bonds and the effectiveness of the LSMC model pricing method.

REFERENCES

[1] Black F, Scholes M. The pricing of options and corporate liabilities [J]. Journal of political economy, 1973, 81(3): 637-654.

[2] Brennan M J, Schwartz E S. Convertible bonds: Valuation and optimal strategies for call and conversion [J]. The Journal of Finance, 1977, 32(5): 1699-1715.

[3] Heston SL. A closed-form solution for options with stochastic volatility with applications to bond and currency options. The review of financial studies. 1993 Apr 1; 6(2):327-43.

[4] Tsiveriotis K, Fernandes C. Valuing convertible bonds with credit risk [J]. The Journal of Fixed Income, 1998, 8(2): 95.

[5] Buchan, Melissa Jane. Convertible bond pricing: Theory and evidence. Harvard University, 1997.

[6] Longstaff, Francis A., and Eduardo S. Schwartz. "Valuing American options by simulation: a simple least-squares approach." The review of financial studies14.1 (2001): 113-147.

[7] Wilde C, Kind A H. Pricing convertible bonds with Monte Carlo simulation [J]. Available at SSRN 676507, 2005.

[8] Lai Qinan, Yao Changhui, Wang Zhicheng. An empirical research on the pricing of convertible bonds in China [J]. Journal of Financial Research, 2005(09): 105- 121.

[9] Tang Wenbin, Zhang Xiaoyong. Research on the LSM pricing model of convertible bonds and its application [J]. The Theory and Practice of Finance and Economics, 2008, 29(154):50-53.

[10] Zhao Yang, Zhao Lichen. Research on the pricing of convertible bonds based on Monte Carlo simulations [J]. Journal of Systems Engineering, 2009, 24(5):621-625.

[11] Luo Xin, Zhang Jinlin. Research on the pricing of Convertible bonds with downward revision of conversion price based on Monte Carlo method [J]. Financial Theory ＆ Practice, 2020(10):45-53.

[12] Chang Hui. Research on the pricing of convertible bonds in China based on the B-S option pricing model [D]. Shandong University, 2019.

[13] Ma Changfu, Xu Wei, Yuan Xianzhi. Research on the pricing of Chinese convertible bonds based on two-factor willow [J]. Systems Engineering-Theory & Practice, 2019, 39(12): 3011-3023

[14] Ni Wufan, Li Mingsheng. Measurement of implied volatility of convertible bonds and its intrinsic value assessment: A case study of convertible bonds of listed commercial banks [J]. Price:Theory & Practice, 2023(08): 154-157+210.