

Comparative Analysis of Volatility Forecasting Models for Carbon Emission Market

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ABSTRACT

It's an interesting question to consider whether time series models based on historical data or implied volatilities obtained directly from option prices are more efficient in forecasting future volatilities. According to a study on EUA options, when the forecast horizon is a week, implied volatilities are more efficient in predicting future volatilities. Additionally, the study suggests that the larger the options trading volume, the more information is contained in implied volatilities.

KEYWORDS

EUA; Realized Volatility; Implied Volatility; GARCH Model

1. INTRODUCTION

Predicting volatility becomes increasingly important as the European carbon emissions market continues to develop and diversify. The carbon emissions market is a primary tool to combat climate change by setting emission quotas that drive emission reductions. The market's complexity highlights the necessity of accurate volatility prediction for participants' decision-making.

Firstly, the expansion of commodities in the carbon emissions market increases market complexity. In addition to traditional carbon emission quotas, there are derivatives and trading instruments such as carbon futures and options. These various types of commodities are influenced by various factors, making volatility prediction more challenging.

Moreover, volatility prediction is vital for derivative pricing and hedging strategies. Investors engage in hedging and risk mitigation in the derivative market. Accurate volatility prediction can aid investors in developing better hedging strategies and pricing derivative contracts reasonably.

Therefore, as the European carbon emissions market evolves and the variety of commodities expands, the significance of volatility prediction research lies in providing investors and market participants with more precise and comprehensive market information. This helps them understand market dynamics better and formulate more effective investment and risk management strategies.

Our research will utilize three different types of volatility research to better understand the European carbon emissions market. Accurately predicting volatility is crucial for investors who want to make informed decisions. One popular method for predicting volatility is the GARCH model, which considers past volatility and other market data to forecast future volatility levels. Another method is to analyze realized volatility, which is a measure of actual volatility over a specific period. Implied volatility is the third method, and it is derived from options prices. This method reflects the market's expectations for future volatility. By understanding these different measures of volatility, investors can make more informed decisions and better navigate the European carbon emissions market.

These seem complex models are mainly divided into three categories: one is the use of historical information to predict future volatility, referred to as the historical information method, such as the ARCH family of models most commonly used in finance, and in recent years began to popular random volatility model (SV model); The other is based on the option price to launch the market's expectations of future volatility, that is, the implied volatility method. The historical information method tries to find the change law of volatility from the past sample period and then predict future volatility. This method has the following disadvantages: First, the rules summarized from the sample may be false rules, or there may be overfitting problems; Second, this method requires that history must repeat itself, that is, the law found from the sample must apply to the future; Finally, this approach does not take into account information other than historical information such as the latest information and changes in market conditions. As the information collection and distribution center of all participants, the price formed daily in the financial market reflects the expectations formed by the supply and demand parties after obtaining various information from historical data and the latest information. It contains the largest and most forward-looking pre-forecast information and is constantly updated and adjusted, so the implied volatility method has its unique advantages. Of course, the premise of the use of this method is that market participants are more rational, and market prices can objectively reflect investors' rational expectations for the future. Otherwise, options will contain all kinds of noise, and it is difficult to accurately predict future volatility by extracting the information of implied volatility from option prices.

Realized volatility refers to the actual degree of fluctuation in the price of an asset over a specific period. It reflects the extent of price variations and is typically calculated using historical data.

In the financial domain, realized volatility serves as a measure of asset price volatility and holds significant importance for portfolio management, risk assessment, and derivative pricing. Calculating realized volatility involves various methods, with the most common being the statistical analysis of historical returns on the asset price.

Therefore, which of these three forecasting methods is better is the subject of this paper.

It is interesting to note that early research on implied volatility mainly focused on individual stock markets. However, the information contained in the implied volatility based on individual stocks is not stable because individual stock options are traded less, and their prices contain a lot of noise. As a result, starting in the 1990s, research on implied volatility began to focus on index options instead, which have been shown to contain useful information for future volatility.

Despite this, different opinions exist on whether implied volatility contains all the historical information and whether it contains the information of other time series volatility. Some studies, such as those by Edrington and Guan (2002, 2005) and Martens and Zein (2004), have found that implied volatility does not fully include the volatility of ARCH family models and the volatility predicted based on historical information. On the other hand, Blair et al. (2001) and Szakmary et al. (2003) found that the implied volatility of options performed better than the forecast of time series and fully included the information of all-time series volatility.

It is also worth noting that the performance of implied volatility varies in different historical stages and that the length of the forecast period can significantly affect its performance.

This article compares the performance of different time series models using data from the December options market of the EU ETS. The models analyzed are represented by the GARCH model, realized volatility, and implied volatility models on various maturities. The article aims to explore which model is applicable in different situations. The structure of the article consists of four parts: the first part introduces the implied volatility model, the second part selects data and calculates different volatility, the third part presents empirical results, and the fourth part is the conclusion of this article.

2. IMPLIED VOLATILITY THEORY

Implied volatility is the volatility derived from the market price of options based on the option pricing formula (BS formula) proposed by Black and Scholes (1973), reflecting people's expectations for the future volatility of the underlying asset. According to risk-free arbitrage, Black and Scholes (1973) proved that option prices can be priced using the risk-neutral pricing method, thus obtaining an analytical solution to the option pricing formula:

$$C = SN(d_1) - Ke^{-r(T-t)}N(d_2) \quad (1-a)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \quad d_2 = d_1 - \sigma\sqrt{T-t} \quad (1-b)$$

Among them, C represents the option price at time t, S represents the stock price at time t, K is the exercise price, T is the maturity date, and r is the risk-free interest rate from time t to time T, σ the volatility of returns, where N represents a normal distribution. Equation (1) is the Black Scholes European option pricing formula, abbreviated as the BS formula.

The BS formula indicates that the option price at time t is related to S, K, r, T, and σ . The function of σ is implied volatility. Among them, S, K, r, and T are known. Once the market gives the price of the option, the market can infer the estimated volatility of the underlying asset from time t to time T based on the option price. This estimated value is the implied volatility.

The volatility of a base asset σ is unique. However, options with the same maturity date but different strike prices have different implied volatility, which creates a non-linear relationship between the strike price and volatility known as the volatility smile. There are different explanations for this phenomenon. Some attribute it to the non-normal distribution of returns and the randomness of volatility, while others point to market microstructure, issues with measurement error caused by liquidity, bid-ask spread, and minimum trading units, and investor risk preferences such as model risk, lottery premiums, and portfolio insurance.

Early research on BS implied volatility was enthusiastic about finding an optimal weighting mechanism to aggregate the implied volatility corresponding to different exercise prices (Bates, 1991). Since BS-implied volatility exhibits different shapes regarding exercise prices, it is difficult for a weighting mechanism to eliminate all pricing errors. The BS formula assumes that volatility is constant, but volatility is time-varying. According to the stochastic volatility model proposed by Hull and White (1987), if the underlying asset follows a price process:

$$dS = \phi S dt + \sqrt{V} S d\omega \quad (2)$$

$$dV = \mu V dt + \xi V dz \quad (3)$$

Among them, $d\omega$ is the instantaneous correlation coefficient with dz is ρ . Assuming that volatility risk is not priced and $\rho=0$, then the call option price at time t is:

$$p_t = \int BS(\bar{V}_t) h(\bar{V}_t | I_t) dV_t = E[BS(\bar{V}_t) | I_t] \quad (4)$$

$$\bar{V}_t = \frac{1}{T-t} \int_t^T V_t dt \quad (5)$$

$h(\bar{V}_t | I_t)$ is the density function of \bar{V}_t (i.e. average volatility) at time t, where I_t is the information set at time t. BS () is the option pricing formula of Black Scholes, where HW price is calculated based on the conditional distribution of the average volatility of BS price.

Cox and Rubinstein (1985) proved that for parity options, the BS formula is about volatility σ . The linear function of $E[BS(\sigma) | I_t] = BS[E(\sigma) | I_t]$ from which it can be concluded that:

$$p_t = E[BS(\bar{\sigma}_t) | I_t] = BS[E(\bar{\sigma}_t | I_t)] \quad (6)$$

Therefore, there are:

$$E[(\bar{\sigma}_t) | I_t] = BS^{-1}(p_t) \quad (7)$$

The implied volatility derived from parity options reflects the market's subjective view of volatility. As long as there is no pricing of volatility risk, the implied volatility of parity options is an impartial estimation of the future average volatility. Due to high trading volume and liquidity concerns, the implied volatility of parity options is frequently employed to anticipate future volatility.

3. SELECTION OF DATA AND CALCULATION OF VOLATILITY

3.1. Data Selection and Calculation of Implied Volatility

The early trading of EU ETS options was not very active, but trading volume increased during phase 3. Since higher trading volume provides more information and a larger sample size for empirical research, this article has chosen a sample period from May 2021 to March 2024.

Literature typically focuses on studying the volatility of a month. However, it is important to note that the longer the prediction period of the time series model, the greater the error rate will be. To overcome this issue, this article compares one week of data instead. It should be noted that shorter-term volatility predictions are rarely used in practice and have limited significance, therefore they will not be compared to shorter terms.

3.2. Selection and Prediction of GARCH Volatility Data

The GARCH model is the most popular and widely used among the many time series models used in finance. In this article, we compare the volatility of a simple GARCH, a commonly used time series model, with the implied volatility.

This article uses within-sample data to establish a model for predicting values outside of the sample, where the out-of-sample interval corresponds to the predicted interval of implied volatility. To obtain a better predictive ability of the GARCH model, a rolling estimation method is used to estimate the equation, which is to predict the weekly volatility for each week.

Select the return data 120 days before the current week and establish a GARCH model to predict each one, instead of building a single model to predict the volatility of all weeks. Due to the different models estimated during different periods, if only one model is used to predict weekly volatility, it will greatly reduce GARCH's predictive ability. The use of the rolling estimation method can better improve the predictive ability of the model.

Established GARCH (1, 1) model using daily data:

$$\begin{cases} R_t = \mu R_{t-1} + \varepsilon_t, & \varepsilon_t = \sigma_t z_t, \quad z_t \sim i. i. d. N(0,1) \\ \sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 + \beta \varepsilon_{t-1}^2(1,1), & \omega > 0, \alpha, \beta \geq 0 \end{cases} \quad (8)$$

Assuming that the first day of the week to be predicted starts from day t+1, the predicted residual α and the predicted variance σ_t^2 obtained from day t can predict the conditional variance σ_{t+1}^2 for day t+1. However, when predicting the conditional variance for the second day of the week (i.e. day t+2), there is a problem as the prediction starts from day t and the residual term at+1 for day t+1 is unknown. At this point, the predicted return on day t+1 is used instead of the true return on day t+1, resulting in the residual term at+1 on day t+1. By substituting it into the conditional variance model, the predicted variance ht+1 on day t+2 is obtained. By analogy, the variance of the return rate for each day of the

week can be predicted (sum up to obtain the variance of the return rate for that week), and this prediction method is called dynamic prediction. It can be imagined that replacing the true value with the predicted value of the mean equation yields results in each prediction error being incorporated into the subsequent predictions. The longer the prediction period, the worse the prediction results will be.

3.3. Calculation of Realized Volatility

To accurately represent the market's microstructure and reflect its volatility information, we need to calculate the realized volatility. This can be done by using daily data and considering all the information about the daily yield changes. Doing so can make a more precise estimation of the volatility.

Realized volatility σ^{RE} is used to measure the predictive ability and information contained in implied volatility, calculated based on the standard deviation of the daily return corresponding to the remaining term of the option:

$$r_t = \log P_t - \log P_{t-1} \quad (9)$$

$$\sigma^{RE} = \sqrt{\sum_1^n r_t^2} \quad (10)$$

Among them, n is the actual trading days, and r_t is the daily logarithmic rate of return.

4. THE STATISTICAL CHARACTERISTICS OF VOLATILITY

Table 1 presents the statistical characteristics of weekly implied volatility, GARCH volatility, and corresponding realized volatility in Table 1. In the weekly volatility, both GARCH and implied volatility deviate significantly from the mean of realized volatility.

Table 1. Descriptive statistics.

	n	mean	median	sd	min	max
σ^{RE}	150	0.003	0.002	0.005	0	0.5
σ^{GAR}	150	0.002	0.002	0.0008	0.0004	0.007
σ^{BS}	150	0.008	0.008	0.001	0.006	0.01

Examining its correlation with realized volatility, the results are shown in Table 2.

Table 2. Empirical results

	σ^{RE}	σ^{GAR}	σ^{BS}
σ^{RE}	1	0.04	0.1
σ^{GAR}	0.04	1	0.02
σ^{BS}	0.1	0.02	1

4.1. Empirical Results

Figure 1 displays a comparison of the volatility among the three models that we tested. We can observe that the volatility of GARCH is quite like the realized volatility. However, it's not sufficient to rely solely on visual observations. Moving forward, we will employ the inclusion regression method to assess the information present in GARCH volatility and implied volatility.

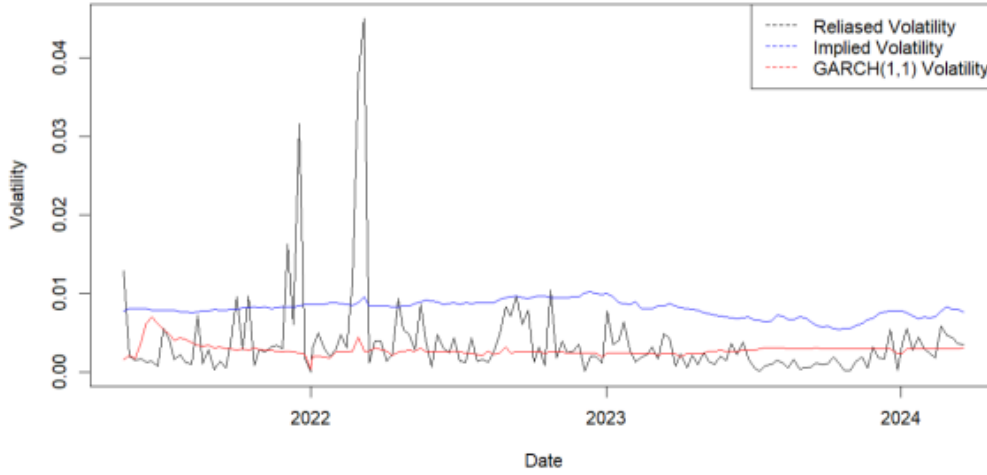


Figure 1. Comparison of GARCH volatility, implied volatility, and realized volatility

This article uses the inclusion regression method to test the information contained in GARCH volatility and implied volatility. Fair and Shiller (1990) provided a clear explanation for the study of comparing the information contained in different models using this method. Use X_{1t} and X_{2t} to represent the predictions of Model 1 and Model 2 on Y_t at t-1 time, respectively. Consider regression:

$$Y_t = \alpha + \beta X_{1t} + \gamma X_{2t} + u_t \quad (11)$$

If models 1 and 2 predict Y_t without any information at time t-1, then the estimated β, γ All should be 0; If two models contain independent information about Y_t at time t-1, then β And γ Neither should be 0; If both models contain information about Y_t prediction, but the information of Model 1 is completely included in Model 2, then β Should be 0, and γ Not 0; If two models contain the same information, i.e. X_{1t} and X_{2t} is completely correlated, then β and γ They cannot be recognized. This article considers regression models.

$$\sigma_t^{RE} = \alpha + \beta^{LRE} \sigma_t^{LRE} + \beta^{GAR} \sigma_t^{GAR} + \beta^{BS} \sigma_t^{BS} + \varepsilon_t \quad (12)$$

Among them, σ It is the volatility of asset returns, with subscript t indicating the observation date and lagged σ_t^{RE} means σ_t^{LRE} represents historical volatility.

4.2. Comparison of Information Contained in One-Week GARCH Volatility and Implied Volatility

In univariate regression, if a volatility does not include information about future volatility, then the slope coefficient β It should be 0. As shown in the table above, in univariate regression, the coefficients of realized volatility lagged by one period and implied volatility are significantly positive, indicating that both GARCH volatility and implied volatility contain predictive information for future volatility, while GARCH volatility does not contain predictive information for future volatility of carbon emissions. If volatility is an unbiased estimate of future realized volatility, then the slope coefficient β Should be 1, while the intercept term α is 0. To test the second hypothesis $H_0: \alpha = 0 \beta = 1$. Wald coefficient test, provide the p-value corresponding to the test statistic in the last column of the table. In the regression of historical volatility, GARCH volatility, and implied volatility, this assumption has been rejected at any traditional level of significance.

In univariate regression, the adjusted R2 corresponding to the regression of historical volatility is the highest (0.1464), followed by the adjusted R2 corresponding to implied volatility (0.0596), while the adjusted R2 corresponding to GARCH volatility is the lowest. This indicates that among these three types of volatility, implied volatility has the most explanation for future realized volatility and contains the most information, while GARCH volatility contains the least information.

Table 3. Univariate regression results of weekly fluctuation rate

Univariate regression	α	β^{rv}	β^{GAR}	β^{BS}	Adjusted R-squared	$P(\chi^2)$
Estimating	0.002 (4.5)	0.386 (5.1)				
Standard error	0.0005	0.0753			0.1464	0.0000
P value	0.0000	0.0000				
Estimating	0.0034 (1.9)		0.1657 (0.27)			
Standard error	0.0018		0.6053		-0.0062	0.0388
P value	0.0609		0.7847			
Estimating	-0.007 (2.1)			1.3877 (3.2)		
Standard error	0.0035			0.4293	0.0596	0.0000
P value	0.0361			0.0015		

In bivariate regression, the adjusted R2 (0.1697) highest series corresponding to the regression of historical volatility after implicit volatility, and the coefficients of both variables are significantly positive, indicating that GARCH volatility information is redundant, and its information is already included in both historical volatility and implicit volatility. In addition, in the two regressions including GARCH volatility, the coefficients of GARCH volatility are not significant, which further indicates that GARCH volatility information is ineffective for predicting future volatility(in Table 4).

Table 4. The bivariate regression results of the weekly fluctuation rate

Bivariate regression	α	β^{rv}	β^{GAR}	β^{BS}	Adjusted R-squared	$P(\chi^2)$
Estimating	0.0008 (0.4)	0.3917 (5.2)	0.5097 (0.9)			
Standard error	0.0017	0.0755	0.5612		0.1454	0.0000
Pvalue	0.6308	0.0000	0.3653			
Estimating	-0.0052 (1.5)	0.3429 (4.4)		0.9378 (2.2)		
Standard error	0.0033	0.0767		0.4149	0.1697	0.0000
Pvalue	0.1240	0.0000		0.0253		
Estimating	-0.0105 (2.4)		0.7044 (1.12)	1.5220 (3.4)		
Standard error	0.0044		0.6052	0.4441	0.0619	0.0000
Pvalue	0.0181		0.2463	0.0008		

In the three-variable regression, the coefficients of historical volatility and implied volatility are significant, while the coefficients of GARCH volatility are not significant, indicating that both implied volatility and historical volatility contain predictive information for future realized volatility, while GARCH volatility does not help predict future realized volatility.

Table 5. Trivariate regression results of weekly fluctuation rate

Trivariate regression	α	β^{rv}	β^{GAR}	β^{BS}	Adjusted R-squared	$P(\chi^2)$
Estimating	-0.0091 (2.1)	0.3435 (4.5)	0.8884 (1.6)	1.1086 (2.6)		
Standard error	0.0042	0.0764	0.5695	0.4271	0.1777	0.0000
Pvalue	0.0308	0.0000	0.1209	0.0104		

5. CONCLUSION

Compared with implied volatility, the GARCH model, which is the most commonly used in financial time series, has shown varying performance in predicting volatility. The empirical analysis in this article found that when predicting the volatility of the next week, the performance of the implied volatility is better than that of the GARCH model. The GARCH model is based on historical information to determine the future. Whether and when history will repeat itself is an uncertain problem.

When economic emergencies occur, predictions based on historical information become even more unreliable. Implied volatility is different from GARCH volatility in that it incorporates people's judgments about the future based on the current economic and financial situation, without if the future will inevitably repeat history. Many traders have given the trading price of options based on their judgment of the future, and based on this price, we have translated future volatility using the BS formula.

Therefore, when there are many participants in the options market and they are wise and mature, the option prices formed by trading will be more reasonable, and the implied volatility obtained from such option prices will be better than time series models in predicting future volatility. It can be imagined that the more participants, the greater the trading volume, and the more accurate people's judgments about the future.

The empirical results also support this point, as the implied volatility is better than the GARCH model in predicting volatility.

REFERENCES

- [1] Andersen, T.G., Bollerslev, T., & Lange, S. (1999). Forecasting financial market volatility: Sample frequency vis-à-vis forecast horizon. *Journal of Empirical Finance*, 6(5), 457-477. [https://doi.org/10.1016/S0927-5398\(99\)00013-4](https://doi.org/10.1016/S0927-5398(99)00013-4)
- [2] Bates, D.S. (1991). The crash of '87: Was it expected? The evidence from options markets. *The Journal of Finance*, 46(3), 1009-1044. <https://doi.org/10.1111/j.1540-6261.1991.tb03775.x>
- [3] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637-654. <https://doi.org/10.1086/260062>
- [4] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327. [https://doi.org/10.1016/0304-4076\(86\)90063-1](https://doi.org/10.1016/0304-4076(86)90063-1)
- [5] Canina, L., & Figlewski, S. (1993). The informational content of implied volatility. *The Review of Financial Studies*, 6(3), 659-681. <https://doi.org/10.1093/rfs/5.3.659>
- [6] Christensen, B.J., & Prabhala, N.R. (1998). The relation between implied and realized volatility. *Journal of Financial Economics*, 50(2), 125-150. [https://doi.org/10.1016/S0304-405X\(98\)00034-8](https://doi.org/10.1016/S0304-405X(98)00034-8)
- [7] Cox, J.C., & Rubinstein, M. (1985). *Options markets*. Englewood Cliffs: Prentice-Hall.
- [8] Ederington, L.H., & Guan, W. (2005). The information frown in option prices. *Journal of Banking & Finance*, 29(6), 1429-1457. <https://doi.org/10.1016/j.jbankfin.2004.05.037>
- [9] Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4), 987-1007. <https://doi.org/10.2307/1912773>

- [10] Fair, R.C., & Shiller, R.J. (1990). Comparing information in forecasts from econometric models. *American Economic Review*, 80(3), 375-389. <https://www.jstor.org/stable/2006672>
- [11] Gemmill, G. (1986). The forecasting performance of stock options on the London traded options market. *Journal of Business Finance & Accounting*, 13(4), 535-546. <https://doi.org/10.1111/j.1468-5957.1986.tb00516.x>
- [12] Hull, J., & White, A. (1987). The pricing of options on assets with stochastic volatilities. *The Journal of Finance*, 42(2), 281-300. <https://doi.org/10.1111/j.1540-6261.1987.tb02568.x>
- [13] Lamoureux, C.G., & Lastrapes, W.D. (1993). Forecasting stock-return variance: Toward an understanding of stochastic implied volatilities. *The Review of Financial Studies*, 6(2), 293-326. <https://doi.org/10.1093/rfs/6.2.293>
- [14] Martens, M., & Zein, J. (2004). Predicting financial volatility: High-frequency time-series forecasts vis-à-vis implied volatility. *Journal of Futures Markets*, 24(11), 1005-1028. <https://doi.org/10.1002/fut.20126>
- [15] Szakmary, A., Ors, E., Kim, J.K., & Davidson, W.N. (2003). The predictive power of implied volatility: Evidence from 35 futures markets. *Journal of Banking & Finance*, 27(11), 2151-2175. [https://doi.org/10.1016/S0378-4266\(02\)00323-0](https://doi.org/10.1016/S0378-4266(02)00323-0)