The Discussion of Convertible Bond Valuation

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ABSTRACT

With the development of financial analysis techniques, computer technology, and artificial intelligence, the valuation methods for convertible bonds in the financial markets are constantly evolving and developing. This article discusses various approaches to valuing convertible bonds, including the Black-Scholes model, the Binomial model, the Monte Carlo model, and artificial intelligence techniques like machine learning. The article provides a detailed overview of each valuation method and analyzes the strengths and weaknesses of each approach.

KEYWORDS

Convertible bond; Valuation

1. INTRODUCTION

In corporate finance, occupy a unique space, by offering companies and investors a hybrid financial instrument, convertible bonds combines elements of both debt and equity. As corporations seek innovative ways to raise capital and investors pursue investment diversification, the valuation of convertible bonds is a critical area of study. Traditional valuation methods provide a solid foundation, however the multifaceted and dynamic nature of convertible bonds demands innovative approaches to accurately assess their worth in today's financial landscape. This essay delves into the valuation of convertible bonds, discusses both conventional methods and the cutting-edge techniques - machine learning. By examining the types of convertible bonds, the valuation models, and the potential of machine learning in enhancing valuation accuracy, this essay aims to provide a comprehensive understanding of convertible bond valuation in the context of modern finance.

2. LITERATURE REVIEW

2.1. Convertible Bond

Convertible bond is a corporate bond with a call option that gives bondholder the right to convert the bond into common stock of the issuer (Fabozzi, 2006). Because of the conversion provision, convertible bond is subordinate to company’s regular bonds and has a lower coupon rate, and companies can issue lower-quality debt with a lower interest which help them enter the credit market easily and reduce the cost of funds (Mayo, 2003, Faerber, 1999, Banks and Siegel, 2006). For investors, the reason of accepting the low quality and interest income bond is that the bond will appreciate if issuer’s stock price rises, investors sacrifice current income for possible future capital gains (Mayo, 2003).
2.2. Types of Convertible Bonds

There are many types of convertible bonds with various features. Firstly, convertible bond has two types of call options, one is European options which can be exercised only at maturity, and another is American options that can be exercised at any time up to the expiration date (Hull, 2009). Therefore, from the view of whether call options within convertible bond can be exercised earlier or only at maturity, convertible bonds can be divided into European-style and American-style convertibles (Goldman Sachs, 1994).

In addition, the callable convertible bond is one type of convertible bonds. This convertible bond is callable at issuer’s option which means call provision allows issuer to buy back the bond at a certain time with a predetermined prices; such call option is called an unprotected call (Fabozzi, 2006, Hull, 2009). Another kind of call option in convertible bond is the protected call meaning the convertible bond may only be called if the underlying stock price is above a certain price (Fabozzi, 2006).

Furthermore, some convertible bonds are putable (Fabozzi, 2006). These put features allow the bondholders to put back bonds to issuer for specific cash amounts on certain days (Goldman Sachs, 1994). This put option can be classified as hard put and soft put, the former means convertible bond must be redeemed for cash, while the latter means besides cash, convertible bond also can be redeemed for common stock or subordinated notes (Fabozzi, 2006).

Contingent convertible bond is a type of convertible bond that allows bondholder to convert into stock only if the underlying stock price exceeds a specified price for a certain number of trading days (Fabozzi, 2006). Exchangeable bonds give bondholders the right to convert bond into stocks of a company other than the issuers (Ibid.). Resettable convertible bond is a convertible that its conversion ratio can be reset according to the price of the underlying stock (Davis and Lischka, 1999).

2.3. The Importance of Convertible Bonds

Convertible bonds play an important role in corporate finance due to their unique blend of debt and equity characteristics, offering issuers and investors a flexible financing instrument. Compared with traditional debt instruments, convertible bonds provide a way to raise capital at lower interest rates for companies. Meanwhile, investors are willing to accept lower yields in exchange for the option to convert their bonds into equity in the future. This is attractive to the companies which seeking to finance growth initiatives or fund strategic investments without diluting existing shareholders’ ownership stakes or taking on excessive debt burdens. From an investor perspective, convertible bonds offer the opportunity for enhanced returns by participating in both the fixed-income and equity markets, thereby diversifying investment portfolios and potentially increasing overall risk-adjusted returns. Therefore, the unique features of convertible bonds make them an essential tool in corporate finance, offering benefits to both issuers and investors in the financial market.

3. VALUATION OF CONVERTIBLE BONDS

3.1. Black-Scholes Model

The Black-Scholes model can be used in pricing European-style convertible bond. Since a convertible bond can be seen as a straight bond plus a call option, the value of convertible bond is the total value of a straight bond and a call option, this method is called the component model or synthetic model (Martellini et al, 2003).

The valuation of the straight bond component is the same with a normal bond valuation that discounts the expected cash flows by appropriate discount rate (Bodie et al, 2008). The straight bond component’s value is the investment value of convertible bond (Mayo, 2003). The expected cash
flows contain periodic coupon interest payments to the maturity date and the par value at maturity (Fabozzi, 2006). Therefore, the bond value is:

\[
\text{Bond value} = \text{Present value of coupons} + \text{Present value of par value}
\]

\[
\text{Bond Value} = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}, \quad \text{where } T = \text{number of periods to maturity, } r = \text{interest rate}
\]

Then, the call option component is valued by Black-Scholes model. The Black-Scholes formulas are used to price European options on a non-dividend-paying stock at time 0 (Hull, 2009).

\[
c = S_0 N(d_1) - K e^{rT} N(d_2),
\]

\[
p = K e^{rT} N(-d_2) - S_0 N(-d_1)
\]

Where, function \(N(x)\) is the cumulative probability distribution for a standardised normal distribution, \(c\) = European call price, \(p\) = European put price, \(S_0\) = stock price at time zero, \(K\) = strike price, \(r\) = continuously compounded risk-free rate, \(\sigma\) = stock price volatility, \(T\) = time to maturity of the option.

Different from standard call options that the strike price is certain, the strike price of convertible bond’s call option will vary with the straight bond component’s value (Martellini et al, 2003). The stated conversion price (face value of the convertible bond/conversion ratio) can not be used as the strike price in Black-Scholes formulas, instead, the adjusted exercise price (investment value of convertible bond/conversion ratio) should be adopted (Martellini et al, 2003, Woodson, 2002).

\[
\text{Adjusted exercise price} = \frac{\text{Investment value}}{\text{Conversion ratio}}
\]

Assume a company issues a five year convertible bond with a par value of ¥1000 and pays 6% coupon annually. It is convertible at maturity into 40 shares. The risk-free rate is 7%. Nonconvertible debt of the same risk class yield 9%. The current stock price is ¥20. The volatility of the company’s stock is 20% per year.

With the bond pricing method, the value of straight bond component is ¥883.32 (Table 1).

\[
\text{Table 1. Bond Value Calculation}
\]

<table>
<thead>
<tr>
<th>Period (t)</th>
<th>Cash Flow</th>
<th>Present Value of Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>¥60</td>
<td>¥55.05</td>
</tr>
<tr>
<td>2</td>
<td>¥60</td>
<td>¥50.50</td>
</tr>
<tr>
<td>3</td>
<td>¥60</td>
<td>¥46.33</td>
</tr>
<tr>
<td>4</td>
<td>¥60</td>
<td>¥42.51</td>
</tr>
<tr>
<td>5</td>
<td>¥1,060</td>
<td>¥688.93</td>
</tr>
<tr>
<td>Bond value (Investment value)</td>
<td>¥883.32</td>
<td></td>
</tr>
</tbody>
</table>
Then, Adjusted exercise price = Convertible bond’s investment value / Conversion ratio
= Value of the straight bond component / Conversion ratio
= ¥883.32 / 40
= ¥22.08

The value of a call option is:
\[
c = S_0 N(d_1) - K e^{-rT} N(d_2),
\]

\[
S_0 = ¥20, K = ¥22.08, r = 7\%, \sigma=20\% \text{ and } T = 5.
\]

\[
c = 20 N(d_1) - 22.08 e^{-0.07*5} N(d_2)
\]

\[
d_1 = \frac{\ln( S_0 / K ) + ( r + \sigma^2 / 2 )T}{\sigma\sqrt{T}}
\]

\[
d_2 = \frac{\ln( S_0 / K ) + ( r - \sigma^2 / 2 )T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}
\]

\[
d_1 = \frac{\ln(20 / 22.08) + (0.07 + 0.2^2 / 2) \cdot 5}{0.2 \cdot \sqrt{5}} = 0.78
\]

\[
d_2 = 0.78 - 0.2 \cdot \sqrt{5} = 0.33
\]

\[
N(d_1) = N(0.78) = 0.7823, \quad N(d_2) = N(0.33) = 0.6293,
\]

\[
c = 20 \cdot 0.7823 - 22.08 \cdot e^{-0.07*5} \cdot 0.6293 = ¥5.85
\]

The total value of call option is multiplying call option price by the conversion ratio, ¥5.85*40=¥234. Therefore, the value of convertible bond is the sum of the value of the straight bond and the call option component, ¥883.32+¥234=¥1117.32.

However, the model that pricing convertible bond through Black-Scholes formulas has disadvantages. Firstly, Black-Scholes formulas are to price European option (Hull, 2009). Therefore, only the European-style convertibles can be valued by Black-Scholes formulas. Secondly, simply dividing convertible bond into a straight bond and an option ignores other embedded options in convertible bonds such as call and put provisions (Martellini et al, 2003). Moreover, the strike price of convertible bond’s option will change because straight bond component’s value varies with interest rate and credit spread in the future (Ibid.). But Black-Scholes formulas assume a constant interest rate and no default risk (Hull, 2009).

### 3.2. The Binomial Model

Binomial tree is introduced by Cox, Ross and Rubinstein in 1979, it is a diagram that shows different possible path that the stock price follows over the whole period of an option (Hull, 2009). In binomial tree, it assumes that the movement of stock price is a random work, and stock price has certain increase and decrease probabilities (Ibid.). Since an option on a stock is embedded in the convertible bond, binomial tree can be used to price the convertible bond (Martellini et al, 2003). There are some assumptions under binomial model of convertibles tree. It assumes that the future underlying stock price is the only factor that influence the convertible bond value, the stock price’s volatility, risk-free interest and issuer’s credit risk are constant (Goldman Sachs, 1994, Hung and Wang, 2002). The whole logic of using binomial tree to price convertible bond is to construct a binomial stock price tree, and then roll back through the tree from the terminal nodes to get convertible bond value (Ibid.).
The first step is to construct a binomial tree of the underlying stock prices with the assumption of a risk-neutral world (Goldman Sachs, 1994). The risk-neutral world means all investors require no extra return for bearing risks, the expected return on all assets is the risk-free rate (Hull, 2009). As Figure 1 shows, the price \( S \) in each node of the tree is the possible stock price at a certain time. With stock price volatility \( \sigma \), stock price \( S \) may move to a higher level \( S \cdot u \) with a probability of \( p \), or move to a lower level \( S \cdot d \) with a probability of \( (1-p) \) (Ibid.). Figure 2 represents a general binomial tree with more time steps.

![Figure 1. One-period binomial tree of stock price](image1)

After completing stock price tree, the next step is building binomial tree of convertible bond price (Figure 3). The holding value \( (H) \) equal to the expected present value of convertible bond value \( V_u \) and \( V_d \) in the following up- and down-nodes plus the present value of coupons paid in the period (Goldman Sachs, 1994). In order to know convertible bond value \( (V) \) at each node, it should compare the value given by the rollback \( (Q_1) \) (holding value) which assumes convertible bond is neither converted nor called, the call price \( (Q_2) \) and the value if conversion takes place \( (Q_3) \) at each of node, and get convertible bond value equal to \( \max \left[ \min (Q_1, Q_2), Q_3 \right] \) (Hull, 2009, Martellini et al, 2003). If put provision exist with put price \( (Q_4) \), the value equal to \( \max \left[ \min (Q_1, Q_2), Q_3, Q_4 \right] \) (Goldman Sachs, 1994).

![Figure 2. A general four-step binomial tree](image2)

![Figure 3. One-period binomial tree of convertible bond value](image3)
Figure 4. Binomial Tree of Convertible Bond Value, Discount rate and Conversion Probability

![Diagram]

V: convertible value, r: discount rate, P_{conv}: conversion probability

In convertible bond value tree, the appropriate discount rates that calculating the expected present values V_u and V_d should be determined. Different from stock price tree which is a risk-neutral valuation (Hull, 2009), the coupon payments and final principle return from convertible bond is risky because of issuer’s credit risk, the payoffs of convertible bond can not be just discounted simply by the risk-free rate (Goldman Sachs, 1994). In convertible bond value tree, the credit-adjusted discount rate is used as the discount rate which is a conversion probability weighted average of the risk-free rate and risky rate (Ibid.). When convertible bond is certain to convert, the appropriate discount rate is risk-free rate because bondholder get stock without default risk; if the convertible bond is certain to not convert, the risky rate (risk-free rate + issuer’s credit spread) is the discount rate (Ibid.). Therefore, to get the credit-adjusted discount rate, the conversion probability (p_{conv}) should be known firstly which is the probability of convertible bond at the corresponding node will convert into stock in the future. p_{conv} at one node is related with its following up- and down-nodes’ conversion probabilities, for instance p_{conv B}=p* p_{conv D}+ (1-p)*p_{conv E} (Figure 4). Then, the credit-adjusted discount rate (r) is p_{conv}*risk-free rate + (1-p_{conv})*risky rate.

The first valuation example is a convertible bond with no call and put provision. Assume one company issues a 5-year zero-coupon convertible bond with face value of ¥100. The conversion ratio is 1 and no calls and puts in this convertible bond. Detailed information and market variables are in Table 2.
Table 2. A Convertible Bond

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Face value:</td>
<td>¥100</td>
</tr>
<tr>
<td>Coupon:</td>
<td>Zero-coupon</td>
</tr>
<tr>
<td>Maturity:</td>
<td>5 years</td>
</tr>
<tr>
<td>Conversion ratio:</td>
<td>1</td>
</tr>
<tr>
<td>Current stock price:</td>
<td>¥100</td>
</tr>
<tr>
<td>Stock dividend price:</td>
<td>0%</td>
</tr>
<tr>
<td>Volatility:</td>
<td>25% per year</td>
</tr>
<tr>
<td>Risk-free rate:</td>
<td>6%</td>
</tr>
<tr>
<td>Credit spread:</td>
<td>400 basis point</td>
</tr>
</tbody>
</table>

The first step is to build a stock price tree with one-year time step, total step number is 5 steps. The started stock price is ¥100. To get future stock price, the parameters u and d should be calculated from stock price volatility (Hull, 2009):

\[
u = e^{\sigma \sqrt{\Delta t}}
\]

\[
d = 1/u = e^{-\sigma \sqrt{\Delta t}}
\]

Since \( \sigma = 25\% \), \( \Delta t =1 \), then \( u = 1.2840 \) and \( d = 0.7788 \).

The probability of an up movement (p):

\[
p = \frac{a-d}{u-d} = a = e^{\sigma \sqrt{\Delta t}}
\]

So, \( a = e^{0.0618} = 1.0618 \), and \( p = \frac{1.0618 - 0.7788}{1.2840 - 0.7788} = 0.5602 \), \( 1 - p = 1 - 0.5602 = 0.4398 \)

With u and d, the future stock prices in binomial tree are obtained (Figure 5). For instance, from current year to year 1, the stock price \( S_0 \) (¥100) may increase to ¥128.40 (¥100*1.2840) or move down to ¥77.88 (¥100*0.7788).

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P 349.034</td>
<td>K 271.828</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C 77.8801</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F 60.6531</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>J 47.2367</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>O 36.7879</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U 28.6505</td>
<td></td>
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</tr>
</tbody>
</table>

Figure 5. Binomial Stock Price Tree
The second step is to build the convertible bond tree (Figure 6) by constructing from terminal nodes back to the started node. The convertible value at each node equals to maximum of ¥100 and the value of stock price (stock price*conversion ratio). At maturity, convertible bond can be redeemed for ¥100 (face value ¥100 + coupon ¥0), or converted into stock. At node P, Q, R, the values of conversion into stock are ¥349.034(¥349.034*1), ¥211.7(¥211.7*1) and ¥128.40(¥128.40*1) which are larger than ¥100, therefore convertible bond value in P, Q, R are ¥349.034, $211.7 and $128.40 respectively. These nodes’ conversion probability (p_{conv}) (Figure 7) is 1 and the credit-adjusted discount rate (r) (Figure 8) is the risk-free rate (6%). For node S, T, U, their conversion values are less than ¥100, therefore convertible bonds are redeemed and the bondholders receive ¥100 and r is 10% (6% risk-free rate + 4% credit spread).

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>349.034</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>271.828</td>
<td>G</td>
<td>211.7</td>
<td>Q</td>
<td>211.7</td>
</tr>
<tr>
<td>D</td>
<td>164.872</td>
<td>L</td>
<td>164.872</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>129.643</td>
<td>H</td>
<td>130.748</td>
<td>R</td>
<td>128.40</td>
</tr>
<tr>
<td>A</td>
<td>102.823</td>
<td>I</td>
<td>91.753</td>
<td>S</td>
<td>100.00</td>
</tr>
<tr>
<td>C</td>
<td>86.6453</td>
<td>F</td>
<td>79.6776</td>
<td>N</td>
<td>90.48374</td>
</tr>
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<td></td>
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</tbody>
</table>

**Figure 6. Binomial Convertible Bond Price Tree**

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1</td>
<td></td>
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</tr>
<tr>
<td>K</td>
<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>D</td>
<td>0.914942</td>
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<td></td>
</tr>
<tr>
<td>B</td>
<td>0.77199</td>
<td>H</td>
<td>0.806591</td>
<td>R</td>
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</tr>
<tr>
<td>A</td>
<td>0.61182</td>
<td>E</td>
<td>0.58989</td>
<td>M</td>
<td>0.560218</td>
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<tr>
<td>C</td>
<td>0.40779</td>
<td>I</td>
<td>0.313844</td>
<td>S</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>0.175821</td>
<td></td>
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</tr>
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<td>J</td>
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</tr>
<tr>
<td>O</td>
<td>0</td>
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<tr>
<td>U</td>
<td>0</td>
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</tbody>
</table>

**Figure 7. Conversion Probability Tree**
Then, the value given by the rollback in year 4 is the sum of the present value of its up-node and down-node at year 5. For instance, ¥217.82(K) is the sum of present value ¥349.30(P) and ¥211.7(Q). Since in node P and Q, convertible bond is converted, \( p_{\text{conv}} \) are both 1 and \( r \) is \( 1\times 6\% + (1-1)\times 10\% = 6\% \). K’s holding value is 
\[
0.5602 \times (349.03 \times e^{-0.06\times1}) + 0.4398 \times (211.7 \times e^{-0.06\times1}) = ¥271.8.
\]
The ¥164.872(L) is calculated similarly. For node N and O, their following node S, T, U in year 5 are $100 with a conversion probability of 0. So, the $90.48(N and O) is from discounting $100 with 10% which is 
\[
0.5602 \times (100 \times e^{-0.1\times1}) + 0.4398 \times (100 \times e^{-0.1\times1}) = ¥90.48.
\]
Because ¥90.48 in N and O is larger than the corresponding stock price ¥60.65 and ¥36.79, convertible bonds value is ¥90.48 and it not be converted. The calculation for node M is little different. Convertible bond in M will evolve to into up-node R (¥128.4) or end up in down-node S(¥100) with no conversion. M’s holding value is 
\[
0.5602 \times (128.4 \times e^{-0.06\times1}) + 0.4398 \times (100 \times e^{-0.06\times1}) = ¥107.54
\]
that larger than stock pricing(¥100), the action of bondholder in M is to hold the convertible bond rather than converting it.

The valuation in year 3 is similar, but it should pay more attention on the credit-adjusted discount rate \( r \) that used to discount the present value of nodes in year 4. For example, holding value in I is the present value of ¥107.54(M) and ¥90.48(N). r in N is 10% because of \( p_{\text{conv}} = 0 \) (Figure 7). Before calculating \( r \) in M, it is necessary to M’s \( p_{\text{conv}} \) which is related with R and S’s \( p_{\text{conv}} \). So, M’s \( p_{\text{conv}} = 0.5602 \times 1 + 0.4398 \times 0 = 0.5602 \). Then according to \( p^\text{risk-free rate} + (1-p)^\text{risky rate} \), \( r_M \) is 
\[
0.5602 \times 6\% + (1-0.5602) \times 10\% = 7.759\%.
\]
Finally, I’s holding value is 
\[
0.5602 \times (107.54 \times e^{-0.07759\times1}) + 0.4398 \times (9048 \times e^{-0.1\times1}) = ¥91.75.
\]
Since ¥91.75 is larger than conversion value ¥77.88, convertible bond’s value in I is ¥91.75. Following by above valuation procedures, convertible bond value at start is ¥102.823.

Here is another example of convertible bond with call and put provisions. Assume a company issues a 3-year convertible bond with face value of ¥100 and a 6% annul coupon. The conversion ratio is 2. The current stock price is ¥50 with a 20% annual volatility. The risk-free is 5% and nonconvertible bond of the company yields 8%. The call price is ¥105. The convertible can be put at ¥120 in year 1 and year 2.
The procedures of building tree of stock price and convertible bond (Figure 9) are similar to the previous example, and the convertible bond value at maturity is ¥182.20(G), ¥112.14(H), ¥106(I and J). Then, the holding value of ¥149.18(D), ¥114.44(E) and ¥106.14(F) are obtained. The holding value(¥103.85) at F is the present values of I and J plus coupon ¥6 that received in year 3(0.5776*¥106e^{-0.08*1}+0.4224*¥106e^{-0.08*1}+¥6). Through same procedure, E’s holding value is ¥114.44. With the existing of put provisions, at E and F, bondholders can put bond back to issuer for a put price of ¥120 rather than holding it.

At node B, the holding value is ¥134.75(Q1), the call price is ¥105(Q2) and the conversion value is ¥122.14(Q3). According to max [min(Q1, Q2), Q3], convertible bond value at node B is max[min(¥134.75, ¥105), ¥122.14] = ¥122.14. At this node, issuer will call the bond for ¥105 and then bondholder is forced to convert it into stock with a value of ¥122.14 rather than being called by the issuer. Finally, the present value of the convertible bond is ¥110.66.

However, binomial model has limitations. The model is under the assumption that bondholder will take the optimal conversion and issuer is optimal to call back the bond, if they do not act optimally, the convertible bond value will be different from that given by the model (Martellini et al, 2003).
Additionally, binomial model assumes interest rate is constant in the future, this may influence convertible bonds valuation and this problem is more serious in long-term convertible bonds (Ibid.).

3.3. Monte Carlo Simulation

Monte Carlo simulation is a computational technique widely used in finance for valuing complex securities such as convertible bonds. Monte Carlo simulation can price convertible bonds whose embedded options are path-dependent and features are complex, such as floating and step-up coupons, soft-call provisions and reset clauses (Ammann et al, 2007, Lvov et al, 2004). Monte Carlo simulation is to simulate random paths of the underlying state variables, such as stock price, over relevant period in a risk-neutral world, and then calculate expected payoff from each sample path, finally average the discounted expected payoffs to get an estimate of the derivative’s value (Hull, 2009). Therefore, this simulation can obtain convertible bond’s expected payoff upon simulated paths of its underlying stock. The primary advantage of Monte Carlo simulation is its ability to model uncertainty by generating a large number of random samples from probability distributions for key input variables. This helps financial analysts to capture the full range of potential outcomes and assess the impact of various sources of risk in convertible bond valuations. Different from the traditional analytical methods, Monte Carlo simulation can handle nonlinear relationships and incorporate complex features such as path dependence. In addition, Monte Carlo simulation provides a framework for sensitivity analysis, allowing financial analysts to identify the key effective factors of value and evaluate the impact from changed input parameters on the output.

However, Monte Carlo simulation also has its limitations. First, generating a sufficient number of simulations, especially for complex securities with numerous input variables is a high computational intensity and time-consuming task. As a result, Monte Carlo simulation may not always be feasible for real-time decision-making. In addition, the accuracy of Monte Carlo simulation results is highly dependent on the quality of the input assumptions. Meanwhile, the appropriate chosen of the probability distributions to represent uncertainty also can influence the result accuracy of Monte Carlo simulation. To obtain the accurate results, financial analysts should avoid incorrect modeling assumptions and inadequate calibration of input parameters which may lead to biased or unreliable valuation estimates. Finally, the last challenging of using Monte Carlo simulation is the results explanation, because the outputs of Monte Carlo simulation typically consists of a huge number of simulated scenarios which requires careful analysis and interpretation. Although some limitations exist, Monte Carlo simulation is a useful financial evaluation method, which providing a flexible and comprehensive method to assessing risk and modeling uncertainty in complex financial products such as convertible bonds.

3.4. Machine Learning

With the development of computer technology and artificial intelligence, machine learning has been increasingly applied in the field of finance in recent years. It is feasible to use machine learning is convertible bond valuation since machine learning. Compared with traditional valuation methods, machine learning is able to handle large and complex datasets with high dimensionality. Machine learning can consider and analyze factors which conventional valuation models cannot handle, such as historical market data, company financial information. Nowadays, the financial market is dynamic and rapidly changing, by using machine learning models, analysts can find the nonlinearities and interactions between various variables, and provide more accurate value estimates of convertible bond. Furthermore, besides standard financial metrics and market data, machine learning models also can incorporate alternative data sources from financial market, economic environment and society, such as investor sentiment analysis, macroeconomic indicators, news sentiment analysis and social media trends. This broader data integration will result a more comprehensive assessment of convertible bond valuation.
However, in daily evaluation work, in order to achieve the effective and accurate analysis from machine learning models, financial analysts must obtain the large and high-quality datasets which maybe time- time-consuming and resource-intensive. Moreover, machine learning models may be complex and opaque, sometimes it is difficult for financial analysts to interpret the factors that impacting their predictions. Therefore, when machine learning holds is used for convertible bond valuation, financial analysts must consider carefully about the data quality, model interpretability to ensure the reliability of model results.

4. CONCLUSION

In conclusion, the common valuation models for convertible bonds include traditional methods such as the Black-Scholes model, the Binomial model, and the Monte Carlo model, as well as artificial intelligence techniques like machine learning. The Black-Scholes model is mainly used in pricing European-style convertible bond. The binomial tree is to price convertible bond by constructing a binomial stock price tree, and then roll back through the tree from the terminal nodes to get convertible bond value. Monte Carlo simulation is a computational technique which can price convertible bonds. It simulates the fluctuations in the price of convertible bonds by generating random paths, computing the convertible bond prices along each path, and finally obtaining the valuation of the convertible bonds by averaging these prices with weights. Machine learning techniques can enhance the accuracy and robustness of convertible bond valuation. Through leveraging vast datasets and complex algorithms, machine learning techniques capture the nonlinear relationships between influencing factors and the convertible bond prices. Machine learning provides a comprehensive approach to pricing convertible bonds.

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