

Analyzing Equitable Goods Distribution: Integrating Fair Division Principles and Game Theory

Lei Zhang^a, Chenmin Ni^b, Mengping Wang^c and Lisha Tang^d

School of International Business, Zhejiang Yuexiu University, Shaoxing, 312000, China

^a2641355574@qq.com, ^bcmni@163.com, ^c1932905171@qq.com, ^d2033177009@qq.com

ABSTRACT

This research focuses on the equitable allocation of indivisible goods among five agents, where each agent holds goods contributing to its profit. The dual objectives are to maximize all agents' collective profit and ensure the fairness of goods distribution. Fair distribution is identified as a significant social challenge. The study aims to explore and elucidate the fair relationships within goods distribution through the lens of game theory, a methodological framework that has seen rapid advancements in various fields. Utilizing game theory for this analysis is crucial for advancing social equity and improving public satisfaction with distribution outcomes.

KEYWORDS

Fair distribution, Equitable Allocation, Public Satisfaction, Game theory, Social Equity.

1. INTRODUCTION

1.1. Background of the study

The Earth is the cradle of mankind, but mankind cannot live in the cradle forever, and in the not-too-distant future, mankind migrates to Mars. Even if mankind is still no traces of civilization found on the surface of Mars, but imagination can always create the possibility of a new fantasy for mankind. Suppose that at the beginning of the 23rd century, humans have already immigrated and settled on Mars, the moon and other planets of the solar system. Numerous colonies and space stations carry out the transportation of goods to ensure the normal life of human beings on Mars. Under the guidance of the computer system HAL13, the Global Freight System (GFS) is transporting cargo between the colonies and the space station. Alice, Bob, Charlie, David, and Erin live on a remote research base on Mars. One day a transportation vessel arrives at the base unexpectedly, carrying a cargo that is apparently intended for some other recipient. But HAL13 denies the mistake, denies the arrival of the transport and the existence of the ship. Since it was not returnable, the goods had to be distributed to the researchers.

1.2. Issues to be studied

- (1) Problem 1 aims to analyze the relevant data in the list of goods and develop a mathematical model to develop a set of criteria for the fair distribution of goods; subsequently, through the interpretation of the results of the model, it is judged that assuming that all researchers involved in the distribution of the goods have an equal right to distribute the goods on the basis of giving the item to the person who likes it the most, and distributing all the goods to Alice and Bob, the allocation of goods of such a Fairness.

- (2) This question requires the addition of an influence factor to determine whether a change in the criteria for the fair distribution of goods would occur if social interactions were considered, i.e., if the five researchers could compete or cooperate with each other to achieve some collective goal, under such conditions.
- (3) Question 3 suggests that since the subjective value of goods may vary with other items that the person owns or does not own, modify the value in the list of goods based on the researcher's ownership of other goods.

2. MODEL ASSUMPTIONS AND NOTATION

2.1. Model assumptions

The study in this paper is based on the following basic assumptions:

- (1) It is assumed that all researchers involved in the distribution of goods have equal rights.
- (2) Assume that all five researchers are perfectly rational.
- (3) It is assumed that researcher satisfaction depends on the subjective value of each unit of goods to the researcher.
- (4) Assuming that the slight disparities arising from the allocation process are negligible.
- (5) It is assumed that the relevant data consulted are true and reliable.

2.2. Description of the symbol

Table 1 Notion

Notion	Clarification	Notion	Clarification
N	A finite set of players	R	Perceptual level
i	Player	a_i	Victim i
A_i	Denotes the set of strategies that a player can choose from	x_j	Total quantity of goods of type j
u_i	Yield function	t_{ij}	Subjective value of the jth good for the ith researcher
G	Strategic gaming	x_{ij}	The ith researcher is assigned to the jth good
L	Market leader	π_j	Weight of jth goods
F_i	Follower		

3. MODELING AND SOLVING OF THE FIRST PROBLEM

3.1. Equitable distribution-based modeling and solving[7]

3.1.1. Preparation of the model[12]

Based on the question and the given data, we know that 30 pieces of goods will be allocated to five people, Alice, Bob, Charlie, David, and Erin. Then we can initially build the resource fair allocation model to allocate the goods. The resource fair allocation model is used as our initial simple model. We analyze it below:

(1) Modeling of equitable distribution of resources

Let there be N parties participating in the resource allocation, and the number of people in party i is a_i ($i = 1, 2, \dots, N$), denoted $a = \sum_{i=1}^N a_i$. Let there be a total of p goods available for allocation, and the goods allocated to party n are x_j ($j = 1, 2, \dots, N$), and clearly $\sum_{j=1}^N x_j = p$ and x_j are integers. The number of representatives per unit of goods of party n is $\frac{a_i}{x_j}$, and the number of representatives per unit of goods in the whole allocation scheme is $\frac{a}{p}$, and there is a deviation between the two $\frac{a}{p} - \frac{a_i}{x_j}$, according to which an integer programming model of the goods allocation problem can be established.

$$\left\{ \begin{array}{l} \min Z = \sum_{i=1}^N \left(\frac{a}{p} - \frac{a_i}{x_j} \right) \\ \sum_{j=1}^N x_j = p \\ x_j \text{ is an integer } (j = 1, 2, \dots, p) \end{array} \right. \quad (3-1)$$

Z is the degree of inequity in distributing the goods, and the optimal solution to the model is to minimize Z to the goods distributed to each of the five individuals found at x_j , and then calculate the number of goods distributed to each of the disaster victims based on the ratio.

(2) Description of the weights of the value of goods[20]

Since different goods have different subjective values to each individual (i.e., the value of the goods to them personally). For this characteristic of the goods we divided them into three main categories to establish the weights. Based on the data, using excel, Table 2 (the high and low value of the goods study) was obtained.

Table 2 High and low value of research on goods

Value category	Freight
High research value	Pet dog-butterfly hybrid (in cryostasis), Length of silk cloth, Lightsaber (out of order), etc.
Medium research value	Table cutlery set, Space bow-tie, Tableware set, Albanian keyboard, etc.
Average research value	Box of survival food packs, Handyman toolset, Automatic chicken counter, etc.

Based on the broad category to which the goods x_j belong, the goods x_j are given a reasonable weight π_j on the basis of the principle of positive correlation between the level of value and weight, stipulating $1 > \pi_j > 0$.

Based on the broad category to which the goods x_j belong, the goods x_j are given a reasonable weight π_j on the basis of the principle of positive correlation between the level of value and weight, stipulating $1 > \pi_j > 0$.

(3) Assessment of value

Different people have different subjective values for goods, and we introduce the function I_n to show the value of the n th person for the j th goods. In order to describe the subjective value of each person for various goods, assume that robot n has been assigned to the goods as $X_{i1}, X_{i2}, \dots, X_{im}$, we define this moment I_n as:

$$J_n = \sum_{j=1}^X \lambda_j (t_{ij} - x_{ij}), (\lambda_j = \pi_j / S_j). \quad (3-2)$$

(4) Description of the distribution scheme

In order to account for the distribution of outgoing goods, we created a matrix A of type $N \times M$:

$$A = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1M} \\ X_{21} & \dots & \dots & \dots \\ \dots & \dots & X_{ij} & \dots \\ X_{N1} & \dots & \dots & X_{NM} \end{bmatrix}. \quad (3-3)$$

Here x_{ij} denotes the quantity assigned to the n th person shipment x_j .

where the row vector of $A \vec{c}_i$ represents the goods that have been assigned to researcher n and the column vector table of $A \vec{d}_j$ represents the goods that have been assigned out.

(5) Description of the value objective function

Let y denote the total value of the goods, then the objective function is **maxy**.

1) Let the value generated by the allocation of goods x_j to researchers a_n be $y_{j,n}$:

Then there are assumptions that can be made: $y_{j,n} = 1 \times \lambda_j \times J_n$.

After all the goods have been distributed, the final total value is obtained as:

$$y = \sum_{j=1}^X y_j. \quad (3-4)$$

Thereby.

$$y = \sum_{j=1}^X y_j 1 \times \lambda_j \times J_n. \quad (3-5)$$

The objective function is further expressed as:

$$\max \sum_{j=1}^X y_j 1 \times \lambda_j \times J_n. \quad (3-6)$$

(6) Constraints

The constraints of this question are that there are a finite number of various goods and that the goods are indivisible, i.e., each good can be given to only one researcher. That is, the allocation of the good X_j is equal to the supply S_j .

By means of the column vector \vec{d}_j of the matrix A of (3-3), the constraints can be formulated as follows:

$$\sum_{i=1}^N x_{ij} \leq S_j. \tag{3-7}$$

Also additional constraints: the researcher a_i gets the amount of goods X_j not greater than its total amount t_{ij} , i.e.:

$$x_{ij} \leq t_{ij}. \tag{3-8}$$

3.1.2. Solving the model 错误!未找到引用源。

- (1) Sort the goods according to the subjective value of the different goods. It may be useful to set the order after sorting to X_1, X_2, \dots, X_j .
- (2) Method of distribution of goods

The goods are allocated according to the order of the sorted goods X_1, X_2, \dots, X_j . Assuming that $j-1$ species have been allocated, the j th method of allocation of goods is given:

- 1) Let the first $j-1$ distribution of goods be matrix A':

$$A' = \begin{bmatrix} x'_{11} & \dots & x'_{1,j-1} & 0 & \dots & 0 \\ x'_{21} & \dots & x'_{2,j-1} & 0 & \dots & \dots \\ \dots & \dots & \dots & 0 & ? & \dots \\ \dots & \dots & \dots & 0 & ? & \dots \\ \dots & \dots & \dots & 0 & ? & \dots \\ x'_{N1} & \dots & x'_{N,j-1} & 0 & \dots & 0 \end{bmatrix}. \tag{3-9}$$

- 2) MATLAB 错误!未找到引用源。 The program solves directly:

In the case of split goods, the objective function is an N-element quadratic function about $x_{1,j}, x_{2,j}, \dots, x_{i-1,j}, x_{i,j}$ and the constraints are $\sum_{i=1}^N x_{ij} \leq S_j$. At this time, the MATLAB program can be used directly to find the optimal solution.

3.1.3. Based on utility functions[14] and satisfaction 错误!未找到引用源。 Making material allocations

The criterion for equitable distribution is that there should not be too great a disparity in the value of goods received by researchers. Such disparities must be reasonable and justified by the researchers' research.

- (1) The utility function describes the satisfaction of a researcher with the subjective value of a good obtained for a given unit of measurement. The utility function[13] is as follows:

$$U(x) = \begin{cases} w \log x, & a = 1 \\ w(1-a)^{-1} x^{1-a}, & a > 1 \end{cases} \quad (3-10)$$

Where x denotes the goods obtained by the researcher and w denotes the subjective value of the goods obtained by the researcher, it is proved through Matlab results that when $a=1$, the utility function $U(x) = w \log x$ also known as the logarithmic function, the logarithmic utility function provides proportional fairness among the researchers; and when $a > 1$, the utility function $U(x) = w(1-a)^{-1} x^{1-a}$ also known as the negative exponential function, provides a-fairness.

- (2) Assume that the share of goods is $x_j (j=1,2,\dots,n)$, X is the amount of goods allocated to the researcher, the subjective value of the researcher is $w_j (j=1,2,\dots,n)$ and the total value is W . The value of a unit amount of goods to the researcher can be obtained as $A_i = \frac{W}{X}$.

Assuming that all goods are allocated to a researcher, the value to the researcher of a unit amount of goods is $AZ_i = \frac{W_{iz}}{X_{iz}}$.

The distribution of goods is fair when $A_i = \frac{W}{X}$ and $AZ_i = \frac{W_{iz}}{X_{iz}}$ is similar, i.e., when there is a similar level of satisfaction among researchers.

3.1.4. Basis for distributive justice[11]

To summarize, the goods were divided into 3 categories as in (Appendix 1), the first group (low subjective value) distributed the goods to the highest subjective value among the researchers, the second group (medium subjective value) distributed the goods to the middle subjective value among the researchers, and the third group (high subjective value) distributed the goods to the one closest to the average of their subjective values.

3.2. If all goods are distributed to Alice and Bob, the distribution is based on giving the item to the person who likes it best. Is such an allocation of goods fair?

- (1) If all the goods are allocated to Alice and Bob, the allocation is based on giving the goods to the person who likes it the most. the goods allocated to Alice and Bob respectively are as in (Appendix 2). Based on the fair distribution model and satisfaction algorithm, the formula for

the value of a unit amount of goods to the researcher is applied: $A_i = \frac{W}{X}$ and

$$U(x) = \begin{cases} w \log x, & a = 1 \\ w(1-a)^{-1} x^{1-a}, & a > 1 \end{cases}$$

Using excel, the sum of the value of the allocated goods to them individually is obtained in Table 3 below.

Table 3 Total value of goods to them personally

Name and Surname	Alice.	Bob.
The total value of the goods to them personally	836	3015
Value of unit of goods to them	64.3	177.4

According to excel,

$$\text{Alice's } AZ_1 = \frac{W_{1z}}{X_{1z}} = 69.6 > 64.3, \text{ Bob's } AZ_2 = \frac{W_{2z}}{X_{2z}} = 114.1 < 177.4.$$

From this, it follows that Alice's unit of goods is worth much less to them than Alice's total average subjective value. Bob's unit of goods is worth much more to them than Bob's total average subjective value. So this distribution is unfair.

4. MODELING AND SOLVING OF THE SECOND PROBLEM

4.1. Restatement and assumptions of question two

If social interaction is factored in, the criteria for what we consider to be a fair distribution of goods will change. Social interaction is the process by which individuals take social action towards others and others take reactive social action - that is, we are constantly aware of the effects of our actions on others and, conversely, the expectations of others influence most of our own behavior. In its main form, symbolic interactionism, social interaction is generally characterized by the following three elements.

- (1) There must be two or more interacting subjects.
- (2) Some form of contact between interacting subjects must occur.
- (3) The parties involved in the interaction consciously take into account the "meaning" represented by the "symbols" of the action.

In this question, Alice, Bob, Charlie, David, and Erin are clearly influenced by the social interactions between them on the issue of fair distribution of goods, and the action of "symbols", i.e., fair distribution of goods, should be changed according to the change of "meaning" and the criteria of distribution. Then, the distribution criteria should also change with the change of "meaning". Let us first assume that:

- (1) All five researchers were perfectly rational.
- (2) Negligible micro-gaps arising from the distribution process.

It can be explored in terms of its being in a competitive and cooperative relationship, respectively. The specific arguments are as follows.

4.2. Distributional game models considering social interactions[10]

4.2.1. Application of non-cooperative game models 错误!未找到引用源。

In this question, assuming that the five researchers are in a competitive, i.e., non-cooperative relationship, the benefit of the goods received by any one of them in their distribution must imply a loss for one or even several of the others. Therefore, we can base our answer argument on these.

Formalization of the equitable distribution of commodities:

$$G = \{\{1,2,3,4,5\}, \{A_i\}_{i=1}^N, \{u_i\}_{i=1}^N\}. \quad (4-1)$$

Player Sets: $N = \{1,2,3,4,5\}$ denotes the five player strategy set of Alice, Bob, Charlie, David and Erin: $\{A_i\}_{i=1}^N$.

Thereby, we can obtain the value of the corresponding payoff function according to each person's satisfaction with different commodities $u_i(A_i, A_{i+1})$ and we can also find the game[9] result that satisfies the optimal responses of these five people at the same time, i.e., we can find the Nash equilibrium of the mixed-optimal strategy[15][16], i.e., for $\forall i$, the game result that satisfies $a_i^* \in B_i(a_{-i}^*)$. Take one of the commodities as an example, through excel and spss, the payoff matrix[19] is drawn as Table 4.

Table 4 Gain matrix on scanning quark microscopy

Scanning microscope	quark	Alice	Bob	Charlie	David	Erin
Alice		200, 200	200, -800	200, -600	200, 1100	200, -1000
Bob		800, -200	800, 800	800, -600	800, 1100	800, -1000
Charlie		600, -200	600, -800	600, 600	600, 1100	600, -1000
David		1100, -200	1100, -800	1100, -600	1100, 1100	1100, -1000
Erin		1000, -200	1000, -800	1000, -600	1000, -1100	1000, 1000

From this, we can find that when there is a competitive conflict among these five people, their fair distribution results will be affected by individual subjective factors, i.e., by social interaction factors, and therefore, the criteria for the fair distribution of their goods will be changed, and instead of distributing them in the same way as they were originally distributed, they will be distributed by looking for Nash equilibriums in the mixed-strategy game in order to seek optimal responses to these five researchers.

4.3. Distributional Game Model Considering Reciprocal Behavior - The Stackelberg Game Model[17]

4.3.1. Application of the Stackelberggame model错误!未找到引用源。

In the traditional Stackelberg game model[18], both the leader's and the follower's gains are affected by the leader's decisions. Therefore, this can also be seen as a distributional scheme between the leader and his followers, i.e., it can be viewed as a distributional game.

In the application of this question, we assume that all five people are perfectly rational, and consider the follower as the decision maker, i.e., he/she does not only care about the value of the goods allocated to him/her (temporarily measured according to the satisfaction level of the five people in the data given in the question), but also compares the value of the others, so as to promote the fairness of the goods allocated to him/her, and the leader as the perfectly rational decision maker. For the leader's behavior $l \in L$, the follower can predict the maximum value of the commodity that he or she will get under full rationality as: $\pi_F(l) = \max_{f \in F} \pi_F(l, f)$. And the maximum value of the leader's good

is $\tilde{\pi}_L(l) = \pi_L(l, \tilde{f}(l))$. Therefore, this allocation is the result of the leader proposing a corresponding scheme first, and then the followers deciding the orientation of the final outcome through rational judgment. This can be seen more as a kind of cooperation. At the same time, the leader's allocation scheme creates a perception of fairness to the followers, and we assume that the degree of this

perception is R , which is a function of $\tilde{\pi}_F(l)$ and $\tilde{\pi}_L(l)$. Thus, we can find $R = R\left(\tilde{\pi}_F(l), \tilde{\pi}_L(l)\right)$. That is, R is based on a comparison of the follower's optimal prediction of the value of the good obtained from the leader's allocation.

It is now assumed that the follower and the leader are in a cooperative relationship, and that the follower will adopt positive reciprocity (cooperation) decision-making behavior if he or she will appreciate the leader's goodwill. When one of them receives more than expected and adopts positive reciprocity (cooperation), it favors the leader, and vice versa. From this, to understand this reciprocal behavior more clearly and intuitively, we can construct the utility function:

$$Q_F(l, f) = \pi_F(l, f) + \mu\Delta\pi_F + \eta\Delta\pi_L. \quad (4-2)$$

(μ is positive and η is a function of the follower's perceived level of fairness (level of cooperative intent), i.e.: $\eta = \eta\left(\tilde{\pi}_F(l), \tilde{\pi}_L(l)\right)$).

Where the expression $\pi_F(l, f)$ is the value of the good that the follower ultimately allocates when he or she engages in reciprocal behavior, i.e., cooperation, the expression $\mu\Delta\pi_F$ is the cost to the follower of adopting the behavior (cooperation), and $\eta\Delta\pi_L$ denotes the increase in utility resulting from the follower's adoption of the reciprocal behavior.

And based on the previous equation (14), we can conclude that

$$\Delta\pi_F = \pi_F(l, f) - \tilde{\pi}_F(l), \quad (4-3)$$

$$\Delta\pi_L = \pi_L(l, f) - \tilde{\pi}_L(l). \quad (4-4)$$

Suppose η is on the interval $[-1, 1]$. $|\eta|$ The larger it is, the deeper the follower perceives fairness, i.e., the greater the intention to cooperate. When $\eta > 0$, the follower receives a greater value of goods than expected, it will be grateful to the leader, for positive reciprocity, and in this question, we regard it as positive cooperation; while when it is less than 0, it is negative reciprocity, i.e., negative cooperation. But when $\eta = 0$, then the result is neutral.

Therefore, we can get the utility function expressed by the objective function maximization formula for the follower to adopt the behavior as:

$$\max_{f \in L} Q_F(l, f) = \pi_F(l, f) + \mu\left(\pi_F(l, f) - \tilde{\pi}_F(l)\right) + \eta\left(\pi_L(l, f) - \tilde{\pi}_L(l)\right). \quad (4-5)$$

Based on this equation (4-6), the leader's objective function to maximize the value of the goods he receives can be expressed as: $\max_{l \in L} \pi_L(l, f)$, and the corresponding utility function equation is

$$\text{s.t. } \max_{f \in F} Q_F(l, f) = \pi_F(l, f) + \mu\left(\pi_F(l, f) - \tilde{\pi}_F(l)\right) + \eta\left(\pi_L(l, f) - \tilde{\pi}_L(l)\right). \quad (4-6)$$

If and only if $\eta = 0$, the follower equation

$$\max_{f \in L} Q_F(l, f) = \pi_F(l, f) + \mu \left(\pi_F(l, f) - \tilde{\pi}_F(l) \right) + \eta \left(\pi_L(l, f) - \tilde{\pi}_L(l) \right). \quad (4-7)$$

The optimal decision expressed is equivalent to Eq. $\tilde{f}^{(l)} = \arg \max_{f \in F} \pi_F(l, f), \forall l \in L$, i.e., this is a special case.

Therefore, in this model, we can find that according to the cooperative relationship, i.e., when the collective goal of fair distribution of goods is achieved through cooperation, it leads to a change in the standard of fair distribution of goods. Due to the different perspectives of the leader and the follower, the allocation standard gradually changes to the degree of the follower's fairness perception, i.e., the magnitude of the cooperation intention, which then indirectly affects the result of fair allocation. In conclusion, when these five people cooperate to realize this collective goal of fair distribution of goods, the criteria of distribution will change.

4.4. Summary of question two

Through the establishment of the non-cooperative game model and the Steinberger game model, we can conclude that, whether the five researchers are in a state of competition or cooperation to achieve the collective goal, due to the existence of social interaction factors, will eventually lead to a change in the standard of the fair distribution of commodities, and as a third party we judge the distribution standard has a certain gap. Therefore, when the researchers fairly distribute the relevant commodities, the possibility of reaching a state of complete rationality is extremely small, and it is difficult to keep the distribution standard constant at one, and it is impossible for fair distribution to be absolutely fair. Therefore, the five researchers can either compete with each other or cooperate to achieve certain collective goals, but no matter what relationship they are in, the goal of allocating their commodities will change to some extent.

5. MODELING AND SOLVING OF THE THIRD PROBLEM

5.1. Processing and improvement of data[6]

According to the original data, we can first set the value of goods as x tentatively as four categories, namely $0 < x \leq 10$, $10 < x \leq 50$, $50 < x \leq 100$, $x > 100$. Based on the different values of these thirty commodities to each person to filter out the value of different categories for different researchers, and then to modify the data for redistribution. We start from the largest value, through excel and spss to get, the following table 5, 6, 7, 8, 9 is the five researchers each think the largest value of goods.

Table 5 Table of commodities that Alice considers to be of greater value

No.	Freight	Alice	Bob	Charlie	David	Erin
3	Length of silk cloth	110	70	50	90	80
5	Electronic thermometer (lab grade)	200	310	200	320	300
6	Pet dog-butterfly hybrid (in cryostasis)	180	50	-50	0	200
8	Space suit	200	700	450	550	550
14	Box of rare paper books	120	80	90	150	170
16	Scanning quark microscope	200	800	600	1100	1000
17	Web videostreaming gear	150	50	300	100	100
24	Antique uPhone 17 (good condition)	200	300	340	125	150

26	High-school student correction tool	200	250	150	400	500
----	-------------------------------------	-----	-----	-----	-----	-----

Table 6 Table of Commodities Considered by Bob to be of Higher Value

No.	Freight	Alice	Bob	Charlie	David	Erin
5	Electronic thermometer (lab grade)	200	310	200	320	300
8	Space suit	200	700	450	550	550
13	Holodeck access key card	10	110	110	30	40
16	Scanning quark microscope	200	800	600	1100	1000
24	Antique uPhone 17 (good condition)	200	300	340	125	150
26	High-school student correction tool	200	250	150	400	500

Table 7 Table of commodities Charlie considers to be of greater value

No.	Freight	Alice	Bob	Charlie	David	Erin
5	Electronic thermometer(lab grade)	200	310	200	320	300
8	Space suit	200	700	450	550	550
13	Holodeck access key card	10	110	110	30	40
16	Scanning quark microscope	200	800	600	1100	1000
17	Web videostreaming gear	150	50	300	100	100
24	Antique uPhone 17(good condition)	200	300	340	125	150
25	Lightsaber (out of order)	50	100	220	110	70
26	High-school student correction tool	200	250	150	400	500

Table 8 Table of commodities that David considers to be of high value

No.	Freight	Alice	Bob	Charlie	David	Erin
5	Electronic thermometer(lab grade)	200	310	200	320	300
8	Space suit	200	700	450	550	550
14	Box of rare paper books	120	80	90	150	170
16	Scanning quark microscope	200	800	600	1100	1000
24	Antique uPhone 17(good condition)	200	300	340	125	150
25	Lightsaber (out of order)	50	100	220	110	70
26	High-school student correction tool	200	250	150	400	500

Table 9 Table of commodities considered by Erin to be of greater value

No.	Freight	Alice	Bob	Charlie	David	Erin
5	Electronic thermometer(lab grade)	200	310	200	320	300
6	Pet dog-butterfly hybrid(in cryostasis)	180	50	-50	0	200
8	Space suit	200	700	450	550	550
14	Box of rare paper books	120	80	90	150	170
16	Scanning quark microscope	200	800	600	1100	1000
24	Antique uPhone 17(good condition)	200	300	340	125	150
26	High-school student correction tool	200	250	150	400	500
29	"We were known as BTS" memoirs (books)	70	40	100	10	120

From this we can find that for the four items numbered 5, 8, 16, 24 and 26, the value of the goods is higher in the eyes of all five people, then we can assign them to the person who thinks that each item is worth the most, and thus we can make a pairing, 26-E, 24-D, 16-C and 8-B. However, we then find that this method is not entirely fair to Alice, and the subjective value of the goods will change

accordingly, so we need to change the value of the Electronic thermometer (lab grade) numbered 5, and the results are as follows, see Table 10. fair, and the subjective value of the goods will change, so we will need to change the value of the Electronic thermometer (lab grade) good numbered 5, and the results are as follows, see Table 10.

Table 10 Values of Electronic thermometer (lab grade) commodities for No. 5

No.	Freight	Alice	Bob	Charlie	David	Erin
5	Electronic thermometer(lab grade)	360	310	200	320	300

Thereby, the first allocation is completed. By analogy, the goods with larger subjective values are assigned to these five researchers for a reasonable and fair allocation by appropriately changing their values. As a result, we used excel to organize the commodities that all five researchers considered to be of similar value under different intervals of subjective value, as shown in Tables 11, 12, 13, and 14.

Table 11 Table of commodity numbers of different researchers for subjective values $x > 100$

No.	No.	No.	No.	No.
3	5	5	5	5
5	8	8	8	6
6	13	13	14	8
8	16	16	16	14
14	24	17	24	16
16	26	24	25	24
17		25	26	26
24		26		29
26				

Table 12 Table of commodity numbers of different researchers for subjective values $50 < x \leq 100$

No.	No.	No.	No.	No.
10	1	1	3	3
29	3	10	4	15
	4	14	17	17
	14	22	22	23
	21	29	23	25
	23		28	
	25			

Table 13 Table of commodity numbers of different researchers for subjective values $10 < x \leq 50$

No.	No.	No.	No.	No.
1	2	2	1	1
2	6	3	2	2
4	10	4	13	4
12	17	15	15	10
18	18	18	18	13
21	22	20	19	18
22	27	23		19
23	28	28		22
25	29	30		28
30				

Table 14 Table of commodity numbers of different researchers for subjective values $0 < x \leq 10$

No.	No.	No.	No.	No.
7	7	6	6	7
9	9	7	7	9
11	11	9	9	11
13	12	11	11	12
15	15	12	12	20
19	19	19	20	21
20	20	21	21	27
27	30	27	27	30
28			29	
			30	

The red markers in the table are the goods with similar subjective values in the eyes of the five people, and we can continue to allocate them according to the above method. The remaining goods can be modified according to the previous allocation, in order of subjective value from the largest to the smallest in order to achieve a better allocation effect. The new subjective value table is shown in Table 15 (the four colors of red, yellow, blue and green indicate the order of improvement, and the last row is the final allocation result).

Table 15 Improved record of subjective value magnitude of different commodities and distribution results

No.	Freight	Alice	Bob	Charlie	David	Erin	Assign objects
1	Handyman toolset	30	70	60	45	45	B
2	Box of survival food packs	20	22	25	23	15	C
3	Length of silk cloth	110	70	50	90	80	A
4	Computer memory banks	50	100	50	150	20	D
5	Electronic thermometer (lab grade)	360	310	200	320	300	A
6	Pet dog-butterfly hybrid (in cryostasis)	180	50	-50	0	200	E
7	Tableware set	7	6	5	5	6	A
8	Space suit	200	700	450	550	550	B
9	Space bow-tie	3	10	3	4	1	B
10	Rolled-up 300" flat TV	95	50	90	50	40	A
11	Table cutlery set	4	4	1	1	5	E
12	Summer shoes	15	20	7	5	10	B
13	Holodeck access key card	10	110	110	30	40	B
14	Box of rare paper books	120	80	90	150	170	E
15	Sundew-pumpkin seeds	5	3	15	30	100	E
16	Scanning quark microscope	200	800	600	1100	1000	C
17	Web videostreaming gear	150	50	300	100	100	C
18	Knit sweater	20	20	20	25	20	D
19	Adjustable-wavelength projector	5	8	7	45	35	D
20	Albanian keyboard	9	10	15	2	5	C
21	Foldable real estate (high tax)	50	75	-30	-50	-40	B
22	Bottle of spice melange	50	25	95	100	50	D
23	Automatic chicken counter	20	75	20	70	90	E

24	Antique uPhone 17 (good condition)	200	300	340	125	150	D
25	Lightsaber (out of order)	50	100	220	310	70	D
26	High-school student correction tool	200	250	150	400	500	E
27	Unsuspecting mechanical parts	50	30	5	7	5	A
28	Suspicious mechanical parts	3	45	50	70	45	C
29	"We were known as BTS" memoirs (books)	70	40	150	10	120	C
30	Luke's birth certificate	30	5	25	10	5	A

6. EVALUATION AND GENERALIZATION OF THE MODEL

6.1. Model evaluation

6.1.1. Advantages

- (1) quantitative analysis of the many influences involved in the model makes the paper convincing;
- (2) The established planning model can be closely connected with the actual situation, and the problem can be solved in combination with the actual situation, which makes the model have good generality and popularization.

6.1.2. Disadvantages

- (1) The processing of data is highly influenced by human factors;
- (2) Fewer influencing factors are considered and there may be some errors in dealing with the problem;
- (3) In giving criteria for fair distribution, only the most important aspects were analyzed, ignoring the influence of other objective factors and being overly idealistic;
- (4) Deficiencies such as low precision in calculating the satisfaction of researchers need to be further optimized.

6.2. Extension of the model

- (1) Consideration could be given to increasing the generalizability of the model, in terms of its assumptions, by using more sensitive data indicators of the assumptions' abatement;
- (2) Some of the methods of analyzing and solving problems in the model can still be used for other mathematical problems and general models;
- (3) It is also used more in other areas of real life, such as the fair distribution of seats, the fair distribution of cloud computing resources, and the distribution of substances in disaster areas.

REFERENCES

- [1] Shi Xiaohui, Using mathematical modeling to achieve optimal allocation of scarce resources--The LP problem as an example[J]. Encyclopedic Knowledge.2020.
- [2] Liu Shui Lin, Reflections on Law and Economic Ethics of Fair Distribution Issues[J]. Political Science.2020.
- [3] TANG Yun. Analysis of optimal price strategy of tourism supply chain based on Steinberg game model[J]. Business Times,2014.
- [4] Zou J,He W. Fuzzy construction and application of non-cooperative game model[J]. Journal of Chongqing Institute of Education,2008,21(06):8-9+15.Chang Wei,Xie Guangjun,Huang Chaofeng.
- [5] MATLAB 2007 Fundamentals and Improvement Electronic Industry Publishing House.

- [6] Niu Zhuoran, Research on Flood Disaster Trend Analysis and Material Allocation Based on Gray System Theory (Master's Thesis, Capital University of Economics and Business).2020.
- [7] Ma Tianhang. Journal of Gansu Institute of Administration,2018(04):46-56+66+127.
- [8] Mao Yushi. How are donated items distributed in rural areas? Rural Finance Research,2007(08):39-40.
- [9] Li Zhe. Problem research and distributed algorithm design based on non-cooperative game[D].Southwest University,2023.
- [10] Peng Chaojing. Research on benefit distribution of mobile payment cooperative alliance based on cooperative game[D].Shanghai University,2021.
- [11] Lv Zhufang. On the application of fair share distribution model of scarce inventory[J].Logistics Engineering and Management,2011,33(09):35-36.
- [12] Wei Bipeng, Shi Wenlei, Mo Jinglan. Journal of Hainan University of Tropical Oceanography,2022,29(02):64-73.
- [13] Tong Sirong, Sun Bingzhen, Zhao Meng, et al. Multi-attribute group decision model based on BWM and vision utility function[J].Journal of Systems Science and Mathematical Sciences,2023,43(10):2573-2597.
- [14] Zou Mucheng. M&A pricing analysis based on expected utility theory[D].Central South University,2009.
- [15] Weng Jie, Lin Bing, Chen Xing. Load balancing strategy for multi-edge server based on game theory[J].Computer Science,2023,50(S2):754-761.
- [16] Chen Hongwei, Zhang Qingjie, Ge Yuanzheng, et al. Nash Equilibrium Optimal Control Method for Formation Problem of Non-cooperative Group System[J/OL].Computer Engineering and Application:1-10[2024-03-01].
- [17] Information on https://blog.csdn.net/qq_43403025/article/details/133321576.
- [18] Information on <https://zhuanlan.zhihu.com/p/662019174>.
- [19] Jiang Bichuan. Research on strategy evolution dynamics with fuzzy game returns[D].University of Electronic Science and Technology of China,2023.
- [20] Jiang Bichuan. Research on strategy evolution dynamics with fuzzy game returns[D].University of Electronic Science and Technology of China,2023.