

Models of the Stackelberg Game for Progressive Pricing of Urban On-Street Parking Lots

Zixin He*

School of Traffic and Transportation, Chongqing Jiaotong University, Chongqing 400074, China

*Corresponding Author

ABSTRACT

This paper establishes the Stackelberg three-party game model of on-street parking price to solve the specific pricing strategy problem of on-street parking lots in a single city. In the model, the government acts as the leader to decide the parking price strategy, while the long-time parkers and short-time parkers act as the followers, deciding whether to park or not according to the price and their own utility, thus forming a tripartite game system. The model, through in-depth analysis of the utility function and response strategy of each participant, designs the functional expression of the progressive billing strategy, aiming to find the equilibrium point between the government's pricing strategy and the parkers' behavior, in order to further solve the parameters in the government's pricing strategy, and arrive at the optimal price of the parking lot balancing the interests of all parties, which provides a specific pricing basis for the system to formulate the pricing scheme of the on-street parking in a piece of the urban area.

KEYWORDS

Traffic engineering; On-street parking pricing; Stackelberg game modeling

1. INTRODUCTION

On-street parking is an important part of the urban transportation system, referring to the act of parking within the red line of urban roads, in a broader sense, on-street parking includes not only parking within the scope of the delineated parking line, but also parking outside the parking line. On-street parking facilitates parkers to get to their destinations faster, saves walking time as well as time for entering and exiting off-street parking lots, and is particularly important for meeting short-term parking demand. Parking pricing has become one of the most powerful policies in transportation demand management, and a large number of studies [1-3] have been conducted to confirm the effectiveness of parking pricing. In the research related to parking charges, pricing is one of the key links, a reasonable pricing scheme can play the role of price leverage, guide on-street parking vehicles to stay in the appropriate zone, and regulate the utilization rate of off-street parking spaces, reduce the number of illegal parking, and have an impact on the dynamic flow of urban cars, residents' parking decisions, and the choice of travel modes, so as to regulate the relationship between the parking supply and demand, and to coordinate land and Harmonious development of land and transportation.

In 1954, American economist William Vickrey proposed that cities should set the price of on-street parking to keep the number of parking spaces low enough so that those who are willing to pay for parking almost always have space available. Some studies have focused on the impact of on-street parking on road traffic, with the hope of controlling on-street parking capacity at an appropriate price as a pricing objective. Arnott and Inci [4] developed a "bathtub model" model of downtown parking.

Fosgerau and Palma [5] explored the dynamics of traffic congestion and parking prices, and Qian et al. [6] investigated the impact of dynamic parking pricing on the parking demand of travelers, and showed that the parking price based on the parking time and parking space occupancy rate can achieve the optimal traffic pattern.

Some studies investigate the pricing method of on-street parking from the perspective of economics, taking into account the attributes of on-street parking as a public resource and a commodity. Calthrop and Proost's study [3] proves that the optimal on-street parking cost is equal to the marginal cost of off-street parking under the optimal quantity. Inci et al [8] point out that it is possible to charge a differentiated parking price for the on-street parking by setting up the simulated competition model of on-street and off-street parking. differentiated on-street parking fees to achieve social optimality. Chen Peng et al [9] established a joint parking fee pricing model by considering the relationship between the total cost of travel and the capacity of on-street and off-street parking spaces and parking prices. Li Pan-dao [10] explored the pass-through of parking charges and the ripple effect of parking prices, and used the input-output price model to study the ripple effect of parking charges.

The above literature reviews the current research status at home and abroad on the relationship between on-street parking supply and demand, differentiated grading of urban parking, and on-street parking pricing.

Throughout the current state of research at home and abroad, on-street parking pricing models have a relatively single expression of price. In previous on-street parking pricing models, almost only the rate is solved specifically, and other parts of the price strategy are less mentioned. And in a complete price strategy, it also contains such components as billable hour unit, free parking time, and maximum charge ceiling. Therefore, it is necessary to establish a complete price strategy structure for the requirements of different scenarios, and set multiple decision variables in the model for research and solution in order to derive a specific pricing scheme.

2. THEORY OF ON-STREET PARKING PRICES

2.1. Price strategy components

In essence, the supply and demand of on-street parking determines the price of on-street parking. Developing a pricing strategy for on-street parking involves not only setting the base rate for on-street parking, but also includes a variety of aspects such as the charging period, the length of the billing unit, the progressive billing rate, and the price cap.

- a. Charge Periods: The specific timeframe in which on-street parking is charged, such as daytime hours on weekdays, nighttime hours, or weekends.
- b. Free Parking Hours: The hours when parking charges do not begin, such as 15 minutes of free parking.
- c. Billing Unit Length: The smallest unit of time for which parking is billed, e.g., 30 minutes for a single billing.
- d. Base Rate: The base parking rate before progressive billing begins, e.g., \$4/30 minutes.
- e. Progressive Billing Length Threshold: The length of parking time at which progressive billing begins, e.g., the rate increases after 1 hour.
- f. Progressive Rate: The rate at which the cost of parking increases as the length of parking increases, e.g., after 1 hour the cost increases to \$5/30 minutes.

On the basis of on-street parking grading, based on different PDI and PDT, the charging strategy needs to reflect the relative differences between different levels of parking lots, and develop corresponding on-street parking charging strategies for different levels of parking lots.

2.2. Price Strategy Principles

When formulating the on-street parking price strategy, the following principles should be followed to ensure the effectiveness and rationality of the strategy.

- a. Principle of rationalization of rate structure: on-street parking charges should be differentiated according to the purpose of parking, in which the rate for travel parking should be higher than that for residential parking in order to give priority to short-term parking demand. At the same time, the rates for on-street parking should be higher than those for off-street parking, as a way to guide and encourage the transfer of vehicles to off-street parking and to increase the turnover rate of on-street parking spaces. In addition, parking rates should be increased accordingly during periods of high parking demand, and on the contrary, rates should be reduced during periods of low demand, so as to reflect changes in the relationship between supply and demand and to reasonably regulate parking demand.
- b. Principle of Dynamic Adjustment: Considering that the supply of urban parking resources and its cost will continue to change with the development of the city, and the travel structure of the city will continue to evolve, the on-street parking charges should be adjusted dynamically on a regular basis, based on the continuous monitoring and analysis of the current demand for parking, the situation of off-street parking resources, and the changes in the urban travel pattern to ensure that the charging strategy reflects the actual situation in a timely manner and manages the parking resources efficiently. management of parking resources.
- c. Principle of Operability: In order to ensure the effectiveness of the parking fee policy and the convenience of management, the same type of parking rate program should be consistent. This principle helps to keep the rate structure simple and clear, so that parkers and managers can better adapt to the policy and minimize confusion and misunderstanding. It also helps to enhance public acceptance and satisfaction, and improve the efficiency and effectiveness of policy implementation.

3. ON-STREET PARKING PRICE GAME MODEL

3.1. Problem description

This paper takes game theory as a framework to analyze the behavior of all players under the price change of on-street parking. The government, as the price-setting main body of on-street parking, although some on-street parking lot operation rights will be transferred to other charging units, the basis of its charges still needs to be implemented in accordance with the pricing standards set by the government price authorities. Therefore, when discussing the pricing problem of on-street parking, it can be simplified as a game problem between the pricing subject (the government) and the users of parking facilities (parkers) by dividing the users of parking facilities into long-time parkers and short-time parkers, and then further establishing a tripartite game model of the government, the long-time parkers, and the short-time parkers.

3.2. Basic Setting

According to the basic theory of the game, in a complete game process, the players of the game, the order of action, information, strategy space, utility function, etc. are the basic elements of the game.

3.2.1. Players

In this paper, three participants are set up, i.e. the government, the long-time parkers and the short-time parkers.

- a. Government: the government has a dominant position in setting the overall objectives and pricing strategies. It is able to flexibly utilize various components of price in different gaming scenarios,

including setting free parking hours, base rate, billing unit hours, etc. It is also able to choose charging strategies such as progressive billing strategy and long-time preferential strategy according to specific scenarios to form a guiding incentive mechanism.

b. Short-time parkers: Short-time parkers are passive participants in the parking game, whose parking duration is less than 3 hours, and whose main parking purpose is non-commuting during the daytime on weekdays. During the game, short-time parkers seek to maximize their individual utility while satisfying their short-time travel needs.

c. Long-time parkers: Long-time parkers are passive players in the parking game, whose parking duration ranges from 3 to 13 hours, and whose main purpose of parking during the daytime on weekdays is commuting. In the game process, long-time parkers seek to maximize their individual utility while satisfying their long-time parking needs, and they are more concerned about the parking price factor than short-time parkers due to the higher cost of parking brought by long-time parking.

In addition, the group of long-time parkers and short-time parkers each contains multiple individual parkers, and each individual makes choices based on its own utility, but here the players are regarded as a collection of individuals, i.e., the sum of utilities formed by the results of each individual's decision-making is taken as the total utility of the game for this type of parkers, and is involved in the solution of the game model.

3.2.2. Action order

The order of action is the sequence of the participants in the game process to choose the strategy. The government first formulates the parking price strategy, including determining the parking rate, billing unit, and progressive billing or long-time discount strategy. According to the government's parking price strategy, short-time and long-time parkers choose the optimal parking behaviors based on their own utility, including on-street parking and off-street parking.

3.2.3. Information

Information is the information that participants know about each other during the game, including each other's strategy set and benefits. In this model, the government makes a pricing decision before the parker makes a decision, and the parker reacts with knowledge of the government's pricing decision. In this case, the government is able to take advantage of its prior decision to influence the behavior of the parkers and thus maximize its own benefits. Therefore, the game is a case of asymmetric and incomplete information.

3.3. Premise Assumptions

The game model established in this paper is based on the following premises:

a. The government sets a uniform charging strategy for on-street parking lots, parking lot operators only have the right to operate parking lots without pricing rights, and parking lot operators do not join the game players in the game scenario with pricing as the goal.

b. On-street parking prices are not lower than off-street parking prices.

c. The parker's trip purpose, parking duration, and trip location do not change with the on-street parking pricing strategy.

d. Parkers' parking choices are simplified to on-street parking and off-street parking in this model. Although parkers may change their travel mode under the influence of price, under this model, it is not possible to zone the parking mode or travel mode chosen by parkers when they do not choose on-street parking because the price of off-street parking is not specified. Therefore, in this model, other travel modes or parking modes are categorized as off-street parking and the shift to on-street parking due to insufficient off-street parking facilities is not considered.

e. All players are absolutely rational and make decisions in accordance with maximum utility.

3.4. Game Analysis

In establishing the three-party game relationship, based on the consideration of the actual parking demand and the adjustment of the government strategy, it is necessary to distinguish the type of parking demand of the target parking lot, i.e., to distinguish the main type of parking demand under this scenario as long-time parking or short-time parking. According to the grading method in Chapter 3, the long-time and short-time parking lot scenarios represent the main PDT under this category. In order to optimize the allocation of parking resources and enhance its own utility under different grading scenarios, the government needs to formulate a charging strategy that is compatible with the type of PDT. Based on this, the gaming behaviors of each player under the long-time and short-time parking demand scenarios are specifically analyzed.

3.4.1. Government

In the high-demand scenario, due to insufficient parking supply, on-street parking lots should maximize the role of meeting short-term parking demand and control the occupancy rate of on-street parking spaces so as not to generate traffic congestion due to cruising.

In the low demand scenario, where the overall parking supply-demand conflict is relatively insignificant, the government's main objectives are to promote the allocation of on-street parking spaces to short-term parkers, to increase the turnover rate of on-street parking resources, and to reduce negative effects such as cruising with a reasonable occupancy rate. The government's main strategies in this scenario are as follows.

- a. Set reasonable parking fees to fully utilize parking resources and promote balanced resource allocation.
- b. Adopt progressive billing, with the cost rising as the length of parking increases, to ensure that short-time parkers have enough parking spaces.

3.4.2. Long-time parkers

When the cost of on-street parking is high, the government adopts the strategy of progressive billing to further increase the cost of long-time parking, and the probability of choosing on-street parking among long-time parkers will be lower, and they will be more inclined to look for off-street parking lots of relatively lower cost or other more affordable transportation modes.

When the cost of on-street parking is low, and the government sets cumulative billing rules to encourage short-term parking, to a certain extent, it will shift the demand for long-term parking, and some of the individuals who park for a longer period of time among the long-term parkers may be more inclined to choose off-street parking lots or look for other more affordable modes of transportation. However, although the cost is relatively high, some individuals who are long-time parkers will also consider the convenience of the parking lot and choose the parking or travel mode with the greatest benefit to themselves.

3.4.3. Short-Time Parkers

When the cost of on-street parking is higher, on the one hand, due to the progressive billing strategy, for short-time parkers who do not park for a long period of time, the higher parking cost has little impact, and some of the parkers may choose on-street parking. On the other hand, some individual short-term parkers may also consider shifting their parking mode to find relatively cheaper parking options, such as choosing off-street parking.

When the cost of on-street parking is low, short-term parkers are more likely to choose on-street parking for a less expensive and more convenient parking option.

By analyzing the game process of each party in detail, there are the following conclusions:

- a. The progressive billing strategy, as the government's main guiding incentive mechanism, directly regulates different types of parking scenes (long-time and short-time).
- b. For high demand and low demand scenarios, the government's strategy mainly lies in regulating parking fees and billing hours, etc., in order to influence the behavior of long-time and short-time parkers. These two factors are reflected in all scenarios, so the introduction of corresponding variables in the total model is considered in the modeling process.
- c. In the group of long-time and short-time parkers, individuals consider the degree of convenience as well as the cost of parking and make parking choices based on their own utility, forming a choice set.

3.5. Model Building

3.5.1. Model Description

According to the previous analysis, the government, as the maker of parking policy, has the right to decide the parking fee strategy and fee price, while all the long-time parkers and short-time parkers make their own parking choices according to the strategy set by the government, and this decision-making structure is in line with the characteristics of the Stackelberg game, so this paper chooses the Stackelberg game model. The model introduces the order of decision makers' choice of strategies, the leader chooses the strategy first, the followers decide their own choices according to the leader's strategy, the leader adjusts his own strategy according to the followers' choices, and under the rational condition, the participants will make the optimal response to any changes, and so on and so forth, and ultimately achieve the maximization of the benefits of the participants' mixed strategies. The purpose of this is to maximize the benefits of the participant's mixed strategy.

3.5.2. Strategy Space

This model is a tripartite model of the government, long-time parkers, and short-time parkers, which can be described as follows: for an individual parking lot i , the government acts as a leader and gives a pricing strategy $p_i(t)$, a function that expresses the price as a function of the length of the parking time t . The other parameters include the base rate, the length of the billing unit, and the progressive billing strategy. And the individual long-time parkers ilk and short-time parkers isk , based on the set of price strategies given by the government, choose the parking method that maximizes their utility function, forming a utility set. Ultimately, the model aims to seek a Nash equilibrium state where the government's pricing strategy and parkers' parking choices are mutually adaptive, and where no party can unilaterally change its strategy to increase its utility. The government pricing strategy in the Nash equilibrium is selected as the final price strategy for parking lot i .

The progressive billing strategy function for parking lot i is expressed as follows.

$$P_{prog,i}(t_{ijk}) = \begin{cases} 0 & \text{if } t_{ijk} \leq F_i \\ \left\lceil \frac{t_{ijk}}{U_i} \right\rceil \cdot B_i \cdot U_i & \text{if } F_i < t_{ijk} \leq T_i \\ \left\lceil \frac{t_{ijk}}{U_i} \right\rceil \cdot B_i \cdot U_i + \left\lceil \frac{t_{ijk} - T_i}{U_i} \right\rceil \cdot A_i \cdot U_i & \text{if } t_{ijk} > T_i \end{cases} \quad (1)$$

Eq: $\lceil \cdot \rceil$ denotes upward rounding; i is the parking lot index; j is the index of the type of parker, short (s) or long (l); k is the index of the individual parker; t_{ijk} is the length of parking time (h) of the parker ijk in parking lot i ; F_i is the length of the parking free period (h); U_i is the base billing unit length (h); B_i is the base rate (h/\$); T_i is the length threshold to start progressive billing (h); A_i is the progressive rate (h), which is used to calculate the additional cost of parking beyond a specific length of time.

In this model, the strategy space of parkers is set as on-street parking and off-street parking because only on-street parking pricing is studied and no off-street parking rates are developed.

The strategy space function for an individual long-time parker ilk , short-time parker isk parking near parking lot i can be expressed as follows.

$$S_{ilk} = \{(x_{ilk}, t_{ilk}) \mid x_{ilk} \in \{0,1\}, t_{ilk} \in (3, 13]\} \quad (2)$$

$$S_{isk} = \{(x_{isk}, t_{isk}) \mid x_{isk} \in \{0,1\}, t_{isk} \in (0, 3]\} \quad (3)$$

Eq: x_{ijk} represents the parker's choice, where 0 represents off-street parking and 1 represents on-street parking; t_{ijk} is the duration of parking, with a range of values determined by the definition of long and short-term parking, up to a maximum of weekday daytime hours (7:30 - 20:30), i.e., 13 hours.

3.5.3. Utility function

Within the service area of parking lot i , there are n_{is} short-time parkers is_j , and n_{il} long-time parkers il_j , whose parking durations are t_{ijs} and t_{ilj} , respectively.

Firstly, T_{ijk} is set as the set of parking time for parkers who choose on-street parking, and T_{ijk} , else is the set of parking time for parkers who choose off-street parking, which is mathematically expressed as follows.

$$E_i = (E_{i1}, E_{i2}, E_{i3}) \quad (4)$$

Eq: E_{i1} denotes the utility component associated with the saturation target deviation; E_{i2} denotes the portion of the on-street parking turnover rate, which is captured in terms of average parking duration; E_{i3} denotes the portion of utility associated with government revenue.

Parkers, as followers of the model, seek to maximize their individual utility in meeting their travel needs, specifically minimizing the price cost of parking and maximizing convenience, which in this case is expressed as minimizing the cost of walking time. The utility functions of both short-time parkers and long-time parkers contain these two components, and therefore are combined here, and the utility function of a parker ijk who parks within the service area of an on-street parking lot i is expressed as follows.

$$Z_{ijk} = (Z_{ijk1}, Z_{ijk2}) \quad (5)$$

Eq: Z_{ijk1} denotes the parking cost function; Z_{ijk2} denotes the walking time value function.

4. SUMMARY

This paper describes the on-street parking price game problem, sets up the basic model, and analyzes the three-party game of the government, long-time parkers and short-time parkers under specific scenarios. Based on the analysis results, the on-street parking price game model is constructed, which includes the strategy space of the three parties as well as the utility function, etc. This chapter completely establishes a three-party Stackelberg game model under multiple scenarios. This paper establishes a complete three-party Stackelberg game model under multiple scenarios, and through parameter assignment and further solving, it can obtain the progressive pricing strategy of a single on-street parking lot, which provides the basis for the formulation of specific on-street parking price strategy.

REFERENCES

- [1] Verhoef, Erik, Nijkamp, Peter, Rietveld, Piet. "The economics of regulatory parking policies: The (IM)possibilities of parking policies in traffic regulation", *Transportation Research Part A: Policy and Practice*, Vol. 29, No. 2, pp. 141-156, March 1995. [https://doi.org/10.1016/0965-8564\(94\)E0014-Z](https://doi.org/10.1016/0965-8564(94)E0014-Z).
- [2] Lindsey, Charles R., West, Douglas S. "Spatial Price Discrimination: The Use of Parking Coupons by Downtown Retailers".
- [3] Calthrop, Edward, Proost, Stef. "Regulating on-street parking", *Regional Science and Urban Economics*, Vol. 36, No. 1, pp. 29-48, January 2006. <https://doi.org/10.1016/j.regsciurbeco.2005.04.002>.
- [4] Arnott, Richard, Inci, Eren. "An integrated model of downtown parking and traffic congestion", *Journal of Urban Economics*, Vol. 60, No. 3, pp. 418-442, November 2006. <https://doi.org/10.1016/j.jue.2006.04.004>.
- [5] Fosgerau, Mogens, de Palma, André. "The dynamics of urban traffic congestion and the price of parking", *Journal of Public Economics*, Vol. 105, pp. 106-115, September 2013. <https://doi.org/10.1016/j.jpubeco.2013.06.008>.
- [6] Qian, Zhen (Sean), Rajagopal, Ram. "Optimal occupancy-driven parking pricing under demand uncertainties and traveler heterogeneity: A stochastic control approach", *Transportation Research Part B: Methodological*, Vol. 67, pp. 144-165, September 2014. <https://doi.org/10.1016/j.trb.2014.03.002>.
- [7] Ding, Hongwei, Qian, Ying, Zheng, Xiang, Bai, Hong, Wang, Shuai, Zhou, Jian. "Dynamic parking charge–perimeter control coupled method for a congested road network based on the aggregation degree characteristics of parking generation distribution", *Physica A: Statistical Mechanics and its Applications*, Vol. 587, p. 126481, February 2022. <https://doi.org/10.1016/j.physa.2021.126481>.
- [8] Inci, Eren, Lindsey, Robin. "Garage and curbside parking competition with search congestion", *Regional Science and Urban Economics*, Vol. 54, pp. 49-59, September 2015. <https://doi.org/10.1016/j.regsciurbeco.2015.07.003>.
- [9] Chen, Peng, Wang, Gaofei. "Research on Optimal Parking Charge Pricing Model in Urban Centers", *Transportation Science and Technology and Economy*, Vol. 22, No. 4, pp. 77-80, 2020. <https://doi.org/10.19348/j.cnki.issn1008-5696.2020.04.015>.
- [10] Li, Panda. "Study on the Pass-Through and Price Spillover Effects of Parking Charges", *Journal of Xi'an University of Finance and Economics*, Vol. 32, No. 2, pp. 27-36, 2019. <https://doi.org/10.19331/j.cnki.jxufe.2019.02.005>.