

# Energy-optimal Speed Trajectories of Electric City Buses

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## ABSTRACT

Electric vehicles are a key technology in the shift towards sustainable transportation. However, significant improvements can be made in terms of their energy efficiency, and driving style plays a major role. Speed trajectories can be more easily implemented with city buses; therefore, this investigation aims to find energy-optimal speed trajectories between bus stops of a given distance. While many studies have been conducted on small passenger vehicles, research on the trip characteristics of buses is more lacking. Three-segment speed trajectories were evaluated in this investigation, using closed-form equations derived from integrating battery power. Results show that higher average velocities correlate with higher energy consumption, and speed trajectories with larger values of acceleration and deceleration have lower energy consumption and are more energy efficient.

## KEYWORDS

Electric Bus; Energy Consumption; Speed Trajectory; Driving Style; Urban Driving.

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## 1. INTRODUCTION

With the shift towards electric transportation, the energy efficiency of vehicles becomes a more critical question. One area where significant energy savings can be achieved is through driving style, specifically, by using energy-optimal speed trajectories between points. Other factors include environmental temperature, vehicle mass, and aerodynamics. City buses are ideal candidates for implementing speed trajectories, with known routes and timings, stop-to-stop movement, and dedicated bus lanes. The speed trajectories in this investigation consist of three phases: constant acceleration until a peak velocity, coasting, and constant deceleration until rest.

Recent investigations have shown that energy consumption is influenced by driving style, or the dynamics of acceleration and deceleration. Galvin (2017)[1] found that average velocity could not be used as a reliable predictor for energy consumption, and stated that acceleration played a significant role instead. Galvin's investigation showed that 'erratic driving', characterised by larger magnitudes of acceleration and deceleration, increased energy consumption significantly. Galvin concluded that EVs may not be more efficient in urban driving than highway driving when the amount and intensity of the acceleration and deceleration are too high. Galvin also points out that the amount of charge that the battery can accept through regenerative braking is limited. Wu et al. (2015)[2] also found that while power flowing out of the battery scaled proportionally with acceleration (up to a certain point), regenerated power quickly reached a constant value. They agree that the limit was due to the battery's ability to accept charge. On the other hand, they found that EVs performed better in urban driving conditions. Liu et al. (2024)[3] investigated the energy efficiency of "convex acceleration curves" of three acceleration values. They found that increasing the acceleration decreased the energy consumption per km. Liu et al. also found that larger values of acceleration increased the "battery capacity attenuation", or the long-term loss of battery capacity. This raises the issue of balancing

energy efficiency with preserving battery capacity, and introduces the need for battery management systems. While these investigations provide a valuable base for discussion, they are focused on small passenger vehicles, which differ in important characteristics from city buses. Furthermore, they consider aerodynamic losses and different trajectory shapes. Therefore, it remains unclear what the optimal speed trajectory for city buses is, under normal urban driving conditions.

## 2. BACKGROUND

### 2.1. Battery Power

The battery power use,  $P_{bat}$ , of an electric vehicle (EV) is composed of power used for the vehicle's motion,  $P_{wheel}$ , and power used for auxiliary systems such as air conditioning and entertainment (called parasitic power,  $P_{para}$ ).

$$P_{bat} = \frac{P_{wheel}}{\eta_{fwd}} + P_{para} \quad (1)$$

An efficiency factor,  $\eta_{fwd}$ , is applied to account for losses in the power train from the battery to the wheels. Since  $P_{para}$  is a small fraction of the power consumption of an electric bus, it is neglected in this investigation.

$P_{wheel}$  has four components: rolling resistance, drag, accelerating power, and hill climbing.

$$P_{wheel} = mgf_r v + \frac{1}{2} C_d A \rho v^3 + mav + mg \tan(\theta) \quad \text{For small angles of } \theta. \quad (2)$$

Where  $m$  is the mass,  $g$  is the acceleration due to gravity,  $f_r$  is the rolling resistance coefficient,  $v$  is the velocity,  $C_d$  is the drag coefficient,  $\rho$  is the fluid density,  $a$  is the acceleration, and  $\theta$  is the angle of inclination.

Since driving speeds are low in cities, drag is low and therefore neglected in this investigation. It also assumed that no elevation changes occur, i.e. that hill climbing power is zero.

$P_{wheel}$  is therefore:

$$P_{wheel} = mgf_r v + mav \quad (3)$$

EVs can regenerate energy during braking.  $P_{wheel}$  will then be negative, and a separate regenerative efficiency,  $\eta_{regen}$ , is applied to find the power recovered at the battery.

Combining the equations for power use and regenerative braking gives:

$$P_{bat} = \begin{cases} \frac{P_{wheel}}{\eta_{fwd}} & : P_{wheel} > 0 \\ P_{wheel} \eta_{regen} & : P_{wheel} < 0 \end{cases} \quad (4)$$

## 2.2. Calculating the Phases of the Speed Trajectory

To calculate the time spent coasting,  $t_{coast}$ :

$$a_{fr} = \frac{F_{fr}}{m} = \frac{mg_{fr}}{m} = g_{fr} \quad (5)$$

$$v_{peak} = a_{acc} t_{acc} \quad (6)$$

$$v_{coast} = v_{peak} - a_{fr} t_{coast} \quad (7)$$

$$t_{dec} = \frac{v_{coast}}{a_{dec}} \quad (8)$$

$$t_{coast} = T - t_{acc} - t_{dec} \quad (9)$$

are combined to give:

$$t_{coast} = \frac{a_{dec}(T - t_{acc}) - v_{peak}}{a_{dec} - a_{fr}} \quad (10)$$

To calculate the time spent accelerating,  $t_{acc}$ :

$$\frac{1}{2} a_{acc} t_{acc}^2 + \frac{v_{peak} + v_{coast}}{2} t_{coast} + v_{coast} t_{dec} - \frac{1}{2} a_{dec} t_{dec}^2 = d \quad (11)$$

is solved, and the smaller root is taken.

Once  $t_{acc}$  is found, peak velocity,  $v_{peak}$ , time spent coasting,  $t_{coast}$ , velocity at the end of coasting,  $v_{coast}$ , and time spent decelerating,  $t_{dec}$ , are found using Equations (5) - (8) and (10). Then the accelerating distance,  $s_{acc}$ , and decelerating (stopping) distance,  $s_{dec}$ , are found by  $\frac{1}{2} a t^2$ , and the coasting distance,  $s_{coast}$ , is found by  $(\frac{v_{peak} + v_{coast}}{2})t$ .

## 2.3. Energy Consumption

The energy consumption,  $E_{bat}$ , can be found by integrating  $P_{bat}$  over the duration of the trip:

$$\begin{aligned} E_{bat} &= \int P_{bat} dt \\ &= \frac{1}{\eta_{fwd}} \int_0^{t_{acc}} mg_{fr} v(t) + ma_{acc} v(t) dt + \eta_{regen} \int_{t_{acc}}^{t_{tot}} mg_{fr} v(t) + ma_{dec} v(t) dt \\ &= \frac{1}{\eta_{fwd}} [mg_{fr} s_{acc} + KE] + \eta_{regen} [mg_{fr} (s_{coast} + s_{dec}) - KE] \end{aligned} \quad (12)$$

Where KE is the kinetic energy found by  $\frac{1}{2}mv^2$ .

### 3. EXPERIMENT

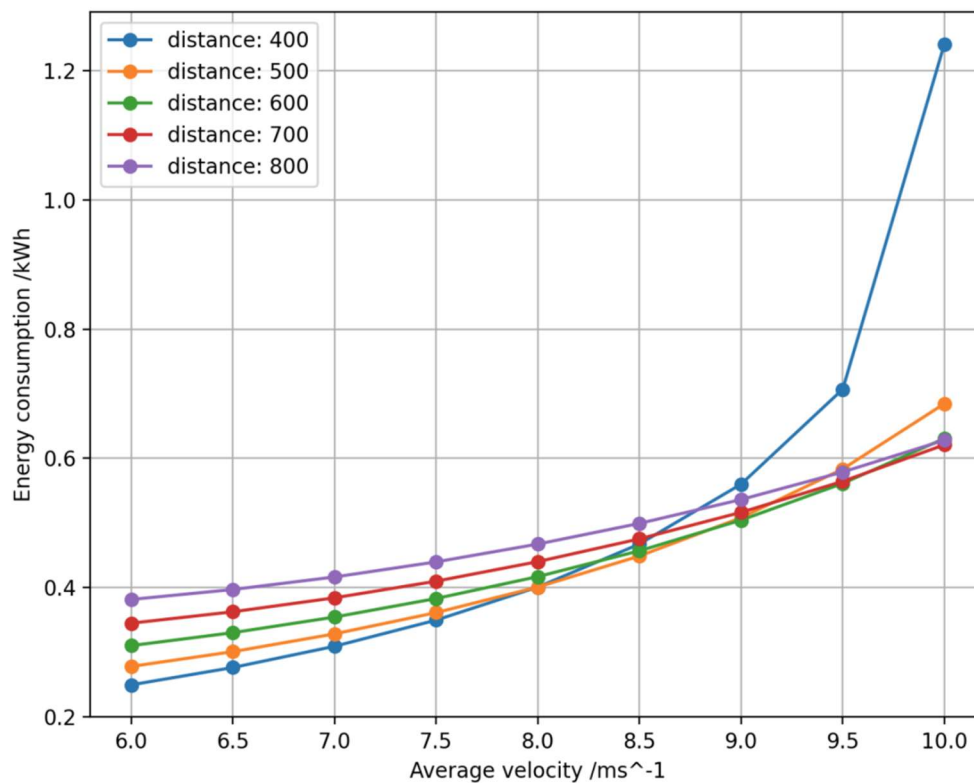
#### 3.1. Investigating the Effect of Average Velocity on Energy Consumption at Different Distances

Before testing different speed trajectories, it is useful to understand the effects of broad parameters, namely average velocity. From equation (12), a positive quadratic correlation is hypothesised, since KE depends on  $v^2$ .

Vavg was tested in the range from 6.0 ms<sup>-1</sup> - 10.0 ms<sup>-1</sup> with intervals of 0.5 ms<sup>-1</sup>. The energy consumption in kWh was calculated. The parameters are given in Table 1. Acceleration and deceleration were held constant at  $\pm 1.0$  ms<sup>-2</sup>.

**Table 1.** - parameter characteristics of an electric bus

m	25 000 kg
fr	0.006
$\eta_{fwd}$	0.9
$\eta_{regen}$	0.3



**Figure 1.** - Graph of energy consumption vs average velocity at different distances. The table of results can be found in Appendix A.

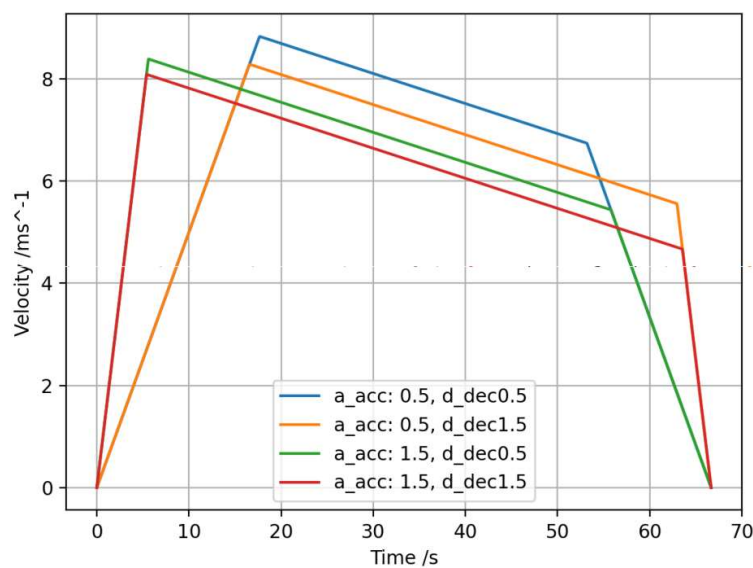
A positive non-linear relationship between energy consumption and average velocity can be seen in Figure 1, as expected from equation (12). However, the effect of average velocity on energy consumption is much greater at shorter distances. This may be due to acceleration accounting for a larger proportion of the trip duration and total energy expended. If this investigation also accounted for air resistance, the change might be even larger as air resistance has a larger effect at higher speeds.

### 3.2. Comparing Energy Consumption of Different Combinations of Acceleration and Deceleration

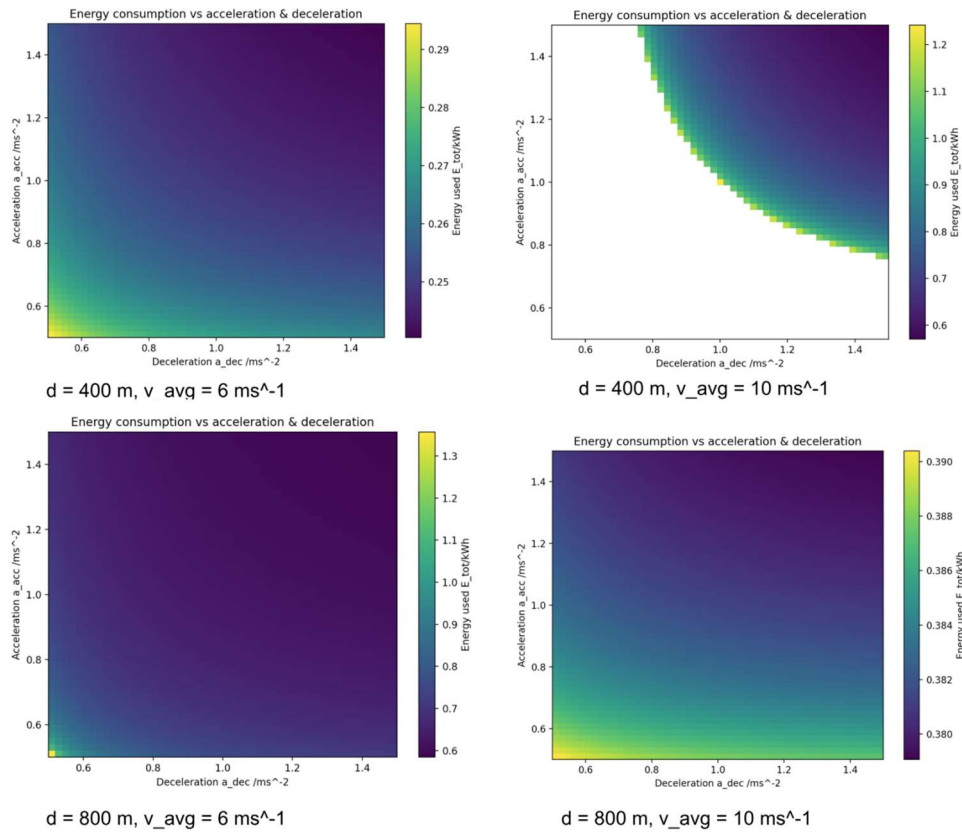
To find the energy-optimal speed trajectory, different combinations of acceleration and deceleration are tested. The magnitudes of acceleration and deceleration tested are in the range  $0.5\text{ms}^{-2}$  -  $1.5\text{ms}^{-2}$  with intervals of  $0.02\text{ms}^{-2}$ . The parameters are the same as in 3.1, and are in Table 1. The distances and average velocities tested are 400 m and 800m, and 6  $\text{ms}^{-1}$  and 10  $\text{ms}^{-1}$ .

For each trial, a magnitude for the acceleration and for the deceleration is chosen. The method for generating the speed trajectory from those two values is shown in Section 2. The value of the peak velocity is adjusted to achieve the given average velocity, and the acceleration and deceleration times are calculated to match. A sample of the tested speed trajectories is given in Figure 2. The resulting energy consumption for each speed trajectory is shown on a heatmap in Figure 3.

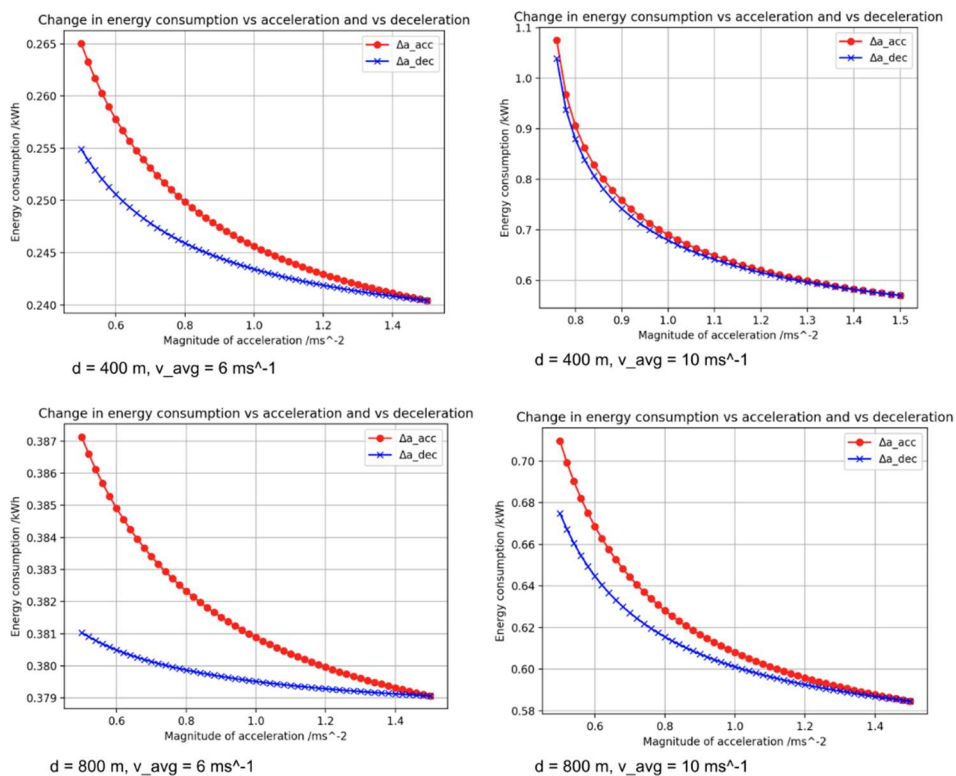
From Figure 2, it can be seen that the energy consumption decreases with increasing acceleration and deceleration. Higher magnitudes of acceleration mean that the acceleration phase constitutes a smaller proportion of the trip duration relative to coasting. Higher magnitudes of deceleration allow for longer coasting at lower velocities, which reduces the energy expended for acceleration at the start. The average velocity limits what magnitudes of acceleration and deceleration are possible, but it cannot be concluded whether it affects the relationship between the energy consumption, and acceleration and deceleration. The distance reduces the effects of acceleration and deceleration on energy consumption, likely because the coasting time is proportionally longer. Thus, the energy efficiency follows the same trend. At 400 m and 6  $\text{ms}^{-1}$ , the worst energy efficiency was 0.74 kWh/km at  $0.5\text{ms}^{-2}$  acceleration and deceleration, and the best efficiency was 0.60 kWh/km at  $1.5\text{ms}^{-2}$  acceleration and deceleration. The change in efficiency was 18.9%. At 800 m and 6  $\text{ms}^{-1}$ , the best and worst efficiencies occurred at the same magnitudes of acceleration and deceleration, and were 0.49 kWh/km and 0.47 kWh/km - a change of 4.3%.



**Figure 2.** - Sample of 4 speed trajectories from combinations of two magnitudes of acceleration and of deceleration



**Figure 3.** - Heatmaps of energy consumption of different combinations of magnitudes of acceleration and deceleration, at two distances and two average velocities



**Figure 4.** - Graphs of energy consumption vs acceleration (and constant deceleration) and energy consumption vs deceleration (and constant acceleration), for two distances and two average velocities. The results table for  $d = 400 \text{ m}$  and  $v_{\text{avg}} = 6 \text{ ms}^{-1}$  can be found in Appendix B.

To compare the effect of only acceleration and only deceleration, slices were taken from the graphs in Figure 3 and plotted as line graphs in Figure 4. The deceleration was held constant at  $-1.5 \text{ ms}^{-2}$  for changing acceleration, and vice versa.

The acceleration has a greater effect on energy consumption than deceleration, although the magnitude of this effect varies with distance and velocity. The efficiency of the power train during acceleration relative to the regenerative efficiency of the EV might explain the higher significance of acceleration, compared to deceleration. The curves of both changing acceleration and changing deceleration flatten as they grow, meaning that the rate of change of energy consumption with acceleration and deceleration decreases at higher values of acceleration and deceleration.

## 4. DISCUSSION

Section 3.1 found that energy consumption increased with higher average velocities. The effect was greater for smaller distances, and it was proposed that this is due to acceleration accounting for a larger proportion of the total trip. The energy efficiency of the bus improves when the distance increases with the same average velocity. This supports that acceleration has a major effect on the energy consumption, as at larger distances with the same average velocity, acceleration takes a smaller proportion relative to coasting. Galvin (2017)[1] also points out that acceleration plays a significant role in the higher energy consumption of higher velocities.

Section 3.2 found that energy consumption decreased with increased acceleration and deceleration. Acceleration appeared to have more significance over the energy consumption compared to the deceleration, though both curves flattened as their values became larger.

Similarly, Liu et al. (2024)[3] found that increasing the acceleration decreased the energy consumption per km. However, the improved energy efficiency may not have been entirely due to the magnitude of the accelerations, and the effects of the acceleration curves used in their investigation may have played a role. According to Li and Liu (2018), the energy consumption per km is lower when a vehicle accelerates with a convex acceleration curve, as opposed to a single, constant acceleration (as referenced in Liu et al., 2024)[3]. On the other hand, Liu et al.'s investigation did account for air resistance, while this investigation did not. The air resistance may have mitigated some of the acceleration curve's benefits.

Galvin's investigation of three-segment speed trajectories, similar to this investigation, found that the driving style with the largest acceleration and peak velocity had twice the energy consumption as the driving style with low acceleration and peak velocity, over the same distance. Furthermore, an experiment with erratic acceleration and deceleration also showed a significant increase in energy consumption. Galvin concluded that EVs may not be more efficient when the frequency and intensity of the acceleration and deceleration are above a certain point. The discrepancy between this investigation's findings and Galvin's may be due to air resistance, as the small vehicle used in Galvin's investigation is much more affected than a large city bus. Furthermore, Galvin's investigation was carried out at much longer distances and higher average velocities. Alternatively, the acceleration values in this investigation may not have reached the threshold of decreasing energy efficiency.

## 5. CONCLUSION

In this investigation, 2500 speed trajectories generated from combinations of magnitudes of acceleration and deceleration were tested. The speed trajectories consisted of three segments; therefore, the battery power could be integrated to give closed-form equations.

Through those equations, it was found that energy consumption increased non-linearly with average velocity. Furthermore, speed trajectories with higher magnitudes of acceleration and deceleration had lower energy consumption.

Possible extensions to this investigation include integrating air resistance and hill climbing, accounting for engine rpm and torque efficiency, and creating a simulation to test more complex trajectories.

## REFERENCES

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## APPENDICES

**Appendix A** - Table of results for energy consumption vs average velocity at different distances (Section 3.1)

**Table 2.** - Energy consumption vs average velocity at different distances

$V_{avg} / \text{ms}^{-1}$	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
d /m									
400	0.24908	0.27592	0.30879	0.34939	0.40053	0.46725	0.55992	0.70701	1.24178
500	0.27762	0.30048	0.32802	0.36099	0.40060	0.44863	0.50796	0.58349	0.68465
600	0.30986	0.32990	0.35409	0.38283	0.41674	0.45674	0.50419	0.56113	0.63071
700	0.34461	0.36225	0.38389	0.40967	0.43994	0.47524	0.51634	0.56438	0.62095
800	0.38139	0.39677	0.41617	0.43958	0.46714	0.49916	0.53615	0.57879	0.62808

**Appendix B** - Tables of results for energy consumption vs acceleration and vs deceleration, for  $d = 400 \text{ m}$  and  $v_{avg} = 6 \text{ ms}^{-1}$  (Section 3.2)

**Table 3a.** - Energy consumption of different values of acceleration (columns 1 - 17), with constant deceleration at  $1.5 \text{ ms}^{-2}$

Acceler ation / $\text{ms}^{-2}$	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82
Energy consum ption /kWh	0.26 504	0.26 328	0.26 170	0.26 027	0.25 897	0.25 779	0.25 670	0.25 570	0.25 477	0.25 392	0.25 312	0.25 238	0.25 169	0.25 104	0.25 044	0.24 987	0.24 933

**Table 3b.** - Energy consumption of different values of acceleration (columns 18 - 34), with constant deceleration at  $1.5 \text{ ms}^{-2}$

Acceler ation / $\text{ms}^{-2}$	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82
Energy consum ption /kWh	0.24 882	0.24 834	0.24 789	0.24 746	0.24 705	0.24 667	0.24 630	0.24 595	0.24 561	0.24 529	0.24 499	0.24 470	0.24 442	0.24 415	0.24 389	0.24 364	0.24 341

**Table 3c.** - Energy consumption of different values of acceleration (columns 35 - 51), with constant deceleration at 1.5 ms-2

Acceler ation /ms <sup>-2</sup>	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82
Energy consum ption /kWh	0.24 318	0.24 296	0.24 275	0.24 254	0.24 234	0.24 215	0.24 197	0.24 179	0.24 162	0.24 146	0.24 130	0.24 114	0.24 099	0.24 084	0.24 070	0.24 057	0.24 043

**Table 4a.** - Energy consumption of different values of deceleration (columns 1 - 17), with constant acceleration at 1.5 ms-2

Decele ration /ms <sup>-2</sup>	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82
Energ y consu mption /kWh	0.25 492	0.25 386	0.25 292	0.25 207	0.25 129	0.25 059	0.24 994	0.24 935	0.24 881	0.24 830	0.24 783	0.24 740	0.24 699	0.24 661	0.24 625	0.24 592	0.24 560

**Table 4b.** - Energy consumption of different values of deceleration (columns 18 - 34), with constant acceleration at 1.5 ms-2

Decele ration /ms <sup>-2</sup>	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98	1	1.02	1.04	1.06	1.08	1.1	1.12	1.14	1.16
Energ y consu mption /kWh	0.24 531	0.24 503	0.24 476	0.24 451	0.24 427	0.24 405	0.24 383	0.24 363	0.24 343	0.24 325	0.24 307	0.24 290	0.24 274	0.24 258	0.24 243	0.24 229	0.24 215

**Table 4c.** - Energy consumption of different values of deceleration (columns 35 - 51), with constant acceleration at 1.5 ms-2

Decele ration /ms <sup>-2</sup>	1.18	1.2	1.22	1.24	1.26	1.28	1.3	1.32	1.34	1.36	1.38	1.4	1.42	1.44	1.46	1.48	1.5
Energ y consu mption /kWh	0.24 202	0.24 189	0.24 177	0.24 165	0.24 154	0.24 143	0.24 132	0.24 122	0.24 112	0.24 102	0.24 093	0.24 084	0.24 075	0.24 067	0.24 059	0.24 051	0.24 043