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Research on NURBS Curve Interpolation Algorithm Based on Mline-Gear

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ABSTRACT

Aiming at the problem of speed fluctuation caused by the large error between the ideal interpolation step and the actual interpolation step in the traditional non-uniform rational B-spline (NURBS) curve interpolation, a NURBS curve interpolation algorithm based on Mline-Gear is proposed. The differential and derivative definitions are derived to transform the Mline and Gear formulas respectively, and the prediction and correction formulas of the interpolation parameters are obtained. Then, it is judged whether the interpolation parameter values obtained by the correction formula meet the speed fluctuation rate requirements, and the interpolation can be continued. Otherwise, the Steffensen descent formula is used to iteratively correct the interpolation parameter values. In order to ensure the calculation accuracy and avoid solving high-order derivatives, the fourth-order Runge-Kutta method is used to solve the initial two interpolation parameters. Finally, the algorithm is simulated. The simulation results show that the algorithm reduces the feed speed fluctuation rate while ensuring a small number of iterations.

KEYWORDS

NURBS curve interpolation; Mline-Gear method; Speed fluctuation ratio

1. INTRODUCTION

Nowadays, the design shape of products is becoming more and more complex. In order to make the product profile curve more smooth and smooth, most products use NURBS curve to design the product profile. The traditional linear interpolation and arc interpolation are to discretize the NURBS curve into a large number of small line segments or arcs for interpolation. The disadvantage of this method is that there is an approximation error, which affects the contour accuracy; the motor can not give full play to its performance, and the overall processing speed is low; the feed speed fluctuates greatly[1]. One of the effective ways to solve the above problems is to use NURBS parametric curves for direct interpolation[2].

However, although NURBS curve interpolation has more advantages than traditional interpolation, it also faces more difficulties. For example, the truncation error when calculating the interpolation parameters can easily lead to large feed rate fluctuations[3]. At present, a lot of achievements have been made in the research of NURBS curve interpolation. Bedi[4]first proposed a uniform interpolation parameter algorithm, that is, the same interpolation parameter increment is used in each interpolation. This method is simple in logic. However, due to the nonlinear relationship between interpolation parameters and arc length, it is easy to cause the interpolation step length to be too long at the large curvature of the curve, resulting in excessive feed speed. After that, the method of calculating interpolation parameters based on first-order and second-order Taylor expansion[5-9]was proposed. However, the computational complexity of Taylor expansion and the accuracy of parameter calculation are contradictory. High-precision interpolation parameter calculation is also accompanied by complex high-order derivative calculation. Jiang et al.[10]gave the iterative formula of interpolation parameters based on the approximate linear relationship between the increment of two adjacent parameters and the interpolation step size, so that the interpolation step size is almost the same, but the influence of the change of curve curvature on the feed rate is not considered. Wu et al.[11]used the fourth-order Runge-Kutta method to calculate the interpolation parameters. The fourth-order Runge-Kutta method improves the calculation accuracy by increasing the number of calculation items. However, the order of derivative calculation in this method is first-order. In the example simulation, the number of iterations, velocity fluctuation rate, and bow height error are better than the first-order Taylor method. Luo[12]used the improved Milne-Hamming predictor-corrector method to calculate the interpolation point parameters. The algorithm uses the difference to simplify the Milne parameter prediction formula, and uses the local truncation error to correct the prediction value and the correction value, but the truncation error correction parameter value increases the computational complexity.

Combined with the above research methods, this paper proposes a NURBS curve interpolation algorithm based on Mline-Gear. Firstly, the algorithm obtains the ideal feed step by constraining the error of the bow height and limiting the normal acceleration. Then, the Mline and Gear methods are transformed by the definition of difference and derivative respectively, and the prediction and correction formulas of interpolation parameters are obtained. Then, it is judged whether the interpolation parameters obtained by the correction formula meet the requirements of velocity fluctuation rate. If they are satisfied, the interpolation can be continued. Otherwise, the Steffensen drop formula is used to iteratively correct the interpolation parameters. In order to ensure the calculation accuracy of the first two interpolation parameters, the fourth-order Runge-Kutta method based on NURBS vector calculation is adopted.

2. NURBS CURVE DEFINITION IMAGE PREPROCESSING

$$C_{(u)} = \frac{\sum_{i=0}^{n} N_{i,p}(u)\omega_i P_i}{\sum_{i=0}^{n} N_{i,p}(u)\omega_i} \tag{1}$$

In the formula: Pi is the control vertex, ω i is the weight factor corresponding to Pi ; $N_{i,p}(u)$ is an i p-order B-spline basis function, which is defined as

$$\begin{cases} N_{i,0}(u) = \begin{cases} 1, u_i \le u \le u_{i+1} \\ 0, \text{other} \end{cases} \\ N_{i,p}(u) = \dfrac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \dfrac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \\ \dfrac{0}{0}, \text{stipulations} \end{cases} \tag{2}$$

In the formula, U=(ui, ui+1, …, ui+p+1) is the node vector.

3. NURBS CURVE INTERPOLATION

3.1. The calculation of ideal interpolation step length

In NURBS curve interpolation, the ideal interpolation step size is affected by many factors, including the limitation of bow height error and normal acceleration on the ideal interpolation step size. The maximum feed speed allowed by the machine tool itself also restricts the ideal interpolation step size [13].

The maximum feed rate F constraint interpolation step length:

$$\triangle L_{i1} = F \cdot T \tag{3}$$

In the formula, T is the interpolation period.

The maximum bow height error $e_{\max}$ constraint interpolation step length:

$$\triangle L_{i2} \approx \sqrt{8\rho_i e_{\max}} \tag{4}$$

Where ρ_i is the radius of curvature at the parameter u = ui.

The maximum normal acceleration $a_{n,\max}$ constraint interpolation step length:

$$\triangle L_{i3} = T\sqrt{a_{n,\max}\rho_i} \tag{5}$$

It can be obtained that the ideal feed step $\triangle L_i$, $\triangle L_i$ is the minimum value of the above three interpolation steps, namely:

$$\triangle L_i = \min\left\{\triangle L_{i1}, \triangle L_{i2}, \triangle L_{i3}\right\} \tag{6}$$

3.2. Prediction and correction of interpolation parameters based on Mline-Gear

Among the common numerical algorithms for differential equations, the first-order accuracy Euler method is a low-order accuracy algorithm with simple calculation. The Runge-Kutta method is a linear single-step method that can achieve high-order calculation accuracy, but its computational complexity is larger than some linear multi-step methods to achieve high-order calculation accuracy. Adams explicit method, Milne method, Adams implicit method and Hamming method are multi-step methods, but the truncation error of the Hamming method is the smallest among the above multi-step differential equation numerical algorithms. In summary, in order to ensure the interpolation accuracy, this paper proposes a prediction correction method based on Mline-Gear.

The four-step fourth-order Mline formula is shown as follows:

$$\overline{y}_{i+1} = y_{i-3} + \frac{4}{3}h\left(2y_i' - y_{i-1}' + 2y_{i-2}'\right) \tag{7}$$

The hermit 's three-step fourth-order Gear formula is:

$$y_{i+1} = \frac{1}{25}y_i - \frac{36}{25}y_{i-2} + \frac{16}{25}y_{y-3} + \frac{12}{25}hy_{i+1} \tag{8}$$

Therefore, the Mline-Gear prediction correction formula is:

$$\begin{cases} \tilde{y}_{i+1} = y_{i-3} + \frac{4}{3}h\left(2y_i' - y_{i-1}' + 2y_{i-2}'\right) & \textit{prediction} \\ y_{i+1} = \frac{1}{25}y_i - \frac{36}{25}y_{i-2} + \frac{16}{25}y_{i-3} + \frac{12h}{25}\tilde{y}_{i+1}' & \textit{correction} \end{cases} \tag{9}$$

Substitute Eq. (9) : $\tilde{y}_{i+1}=\tilde{u}_{i+1}$, $y_i=u_i$, $y_{i-2}=u_{i-2}$, $y_{i-3}=u_{i-3}$, $h=T$, $y_i'=u_i'$, $y_{i-1}'=u_{i-1}'$, $y_{i-2}'=u_{i-2}'$, $y_{i+1}'=u_{i+1}'$, where u_{i+1} and $\tilde{u}_{i+1}$ represent the estimated and corrected values of interpolation parameters, respectively, and T is the interpolation period. After the above parameters are replaced, Formula (10) can be transformed into

$$\begin{cases}\tilde{u}_{i+1}=u_{i-3}+\dfrac{4}{3}T\left(2u_i'-u_{i-1}'+2u_{i-2}'\right)\\ u_{i+1}=\dfrac{1}{25}u_i-\dfrac{36}{25}u_{i-2}+\dfrac{16}{25}u_{i-3}+\dfrac{12T}{25}\tilde{u}_{i+1}'\end{cases} \tag{10}$$

There are two ways to deal with the derivation of time t by the parameter u in the formula:

1)In order to simplify the calculation, the forward and backward difference combinations can be used to approximate the derivative:

$$\begin{cases}u_i'=\dfrac{u_{i+1}-u_i}{T}\\ u_i'=\dfrac{u_i-u_{i-1}}{T}\end{cases} \tag{11}$$

2)Convert according to the definition of derivative:

$$\left.\frac{du}{dt}\right|_{t=t_i}=\frac{v(t_i)\cdot\Delta u}{\left\|C(u_{i-1}+\Delta u)-C(u_{i-1})\right\|} \tag{12}$$

In the formula, $v(t_i)$ is the velocity of the moment, which can be obtained from the velocity-time diagram; Δu is a small increment of the parameter u. The difference method can simplify the calculation of the Mline-Gear formula, but this calculation has a large error. Compared with the difference method, the approximation method defined by the derivative can better ensure the accuracy of the calculation. However, this is relatively large in computation. The Mline prediction formula is simplified by difference method, and the combination of different difference forms makes the formula have five forms, as shown in Eq.(13). The Gear correction formula is simplified by using the derivative definition approximation, and the simplified formula is shown in formula (14).

$$\tilde{u}_{i+1}=\frac{4}{3}u_i-\frac{4}{3}u_{i-1}+\frac{8}{3}u_{i-2}-\frac{5}{3}u_{i-3} \tag{13}$$

$$u_{i+1}=\frac{1}{8}(9u_i-u_{i-2})+\frac{3}{8}T\left(\frac{v(t_i+T)}{\frac{\left\|\mathbf{p}(\tilde{u}_{i+1}+\Delta u)-\mathbf{p}(\tilde{u}_{i+1})\right\|}{\Delta u}}+\frac{2v(t_i)}{\frac{\left\|\mathbf{p}(u_i+\Delta u)-\mathbf{p}(u_i)\right\|}{\Delta u}}-\frac{v(t_i-T)}{\frac{\left\|\mathbf{p}(u_{i-1}+\Delta u)-\mathbf{p}(u_{i-1})\right\|}{\Delta u}}\right) \tag{14}$$

Using the Mline-Gear prediction and correction formula to calculate the next interpolation point parameter, the first three interpolation parameters are substituted into the prediction formula to obtain the predicted value of the next interpolation parameter, and then the predicted value, the first three interpolation parameters, the velocity modulus related to them and the two-norm value of the guide vector are substituted into the correction formula to calculate the correction value of the next interpolation parameter.

Due to the truncation error of the linear multi-step numerical method, there is still a deviation between the interpolation parameter value calculated by the prediction correction formula and the actual value. As shown in Figure 1, the ideal interpolation point is $C(\tilde{u}_{i+1})$ and the ideal feed step is ΔL_i in theory, while the actual interpolation point calculated by the formula (13) is C(ui+1), and then the actual feed step $\Delta\tilde{L}_i = \|C(u_{i+1}) - C(u_i)\|$ is obtained. There is a difference between the actual interpolation step calculated by the interpolation algorithm and the ideal interpolation step, that is, the speed fluctuation will occur.

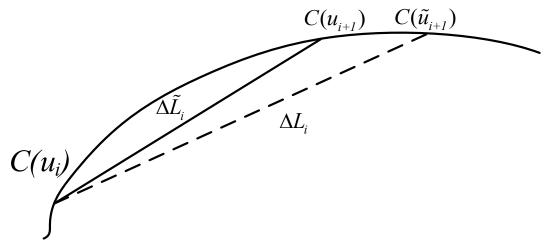


Fig.1: Principle diagram of feed speed fluctuation generation

The calculation accuracy of interpolation parameters is measured by velocity fluctuation rate. The calculation is as follows:

$$\delta = 1 - \frac{\|C(u_{i+1}) - C(u_i)\|}{\Delta L_i} \tag{15}$$

After each interpolation parameter calculation is completed, it is judged whether it meets the speed fluctuation rate requirements. If it does not meet the requirements, the iterative formula can be used to iteratively correct the interpolation parameter values until the requirements are met. When using the iterative algorithm, the nonlinear equation to be solved is:

$$F(u) = \Delta L_i - \|C(u_{i+1}) - C(u_i)\| \tag{16}$$

The Steffensen iteration method does not involve complex derivative calculation, and can convert the quadratic convergence speed of the function into the first-order convergence speed, thus accelerating the iteration process. Therefore, the method is selected to iteratively calculate the interpolation parameters. The formula for iterating the interpolation parameters using the Steffensen iteration method is as follows:

$$u_{i+1} = u^1_{i+1} - \frac{(F(u^1_{i+1}) - u^1_{i+1})^2}{F(F(u^1_{i+1})) - 2F(u^1_{i+1}) - u^1_{i+1}} \tag{17}$$

In the formula, u^1_{i+1} is the iterative value of the parameter of the next interpolation point, u_{i+1} is the calculated parameter of the next interpolation point, and F is the nonlinear equation represented by the formula (16).

3.3. A fourth-order Runge-Kutta method based on the derivation calculation

The aforementioned double Hamming interpolation algorithm requires the first three parameters to solve the next interpolation point parameter, so other methods are needed to solve the initial two interpolation point parameters except the starting point. Since the initial time only knows the starting point parameter u=0, the remaining parameters are still unknown, so the single-step method can be used to obtain the next interpolation parameter. In order to ensure the calculation accuracy and avoid solving the high-order derivative, the fourth-order Runge-Kutta method is used to solve the initial two interpolation parameters. The fourth-order Runge-Kutta formula is:

$$\begin{cases} y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\ K_1 = f(x_i, y_i) \\ K_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_1) \\ K_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_2) \\ K_4 = f(x_i + h, y_i + hK_3) \end{cases} \tag{18}$$

The calculation formula of initial interpolation parameters is:

$$u_{i+1} = u_i + \frac{T}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{19}$$

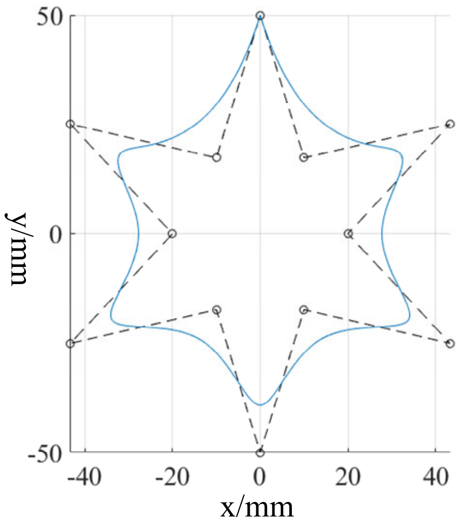
Let K_2, K_3 and K_4 in (19) be transformed by similar formula (12).

$$\begin{cases} K_2 = \dfrac{v\left(t_i + \frac{T}{2}\right)}{\left\| C'(u_i + \frac{T}{2}K_1) \right\|}, K_3 = \dfrac{v\left(t_i + \frac{T}{2}\right)}{\left\| C'(u_i + \frac{T}{2}K_2) \right\|} \\ K_4 = \dfrac{v\left(t_i + T\right)}{\left\| C'(u_i + TK_3) \right\|} \end{cases} \tag{20}$$

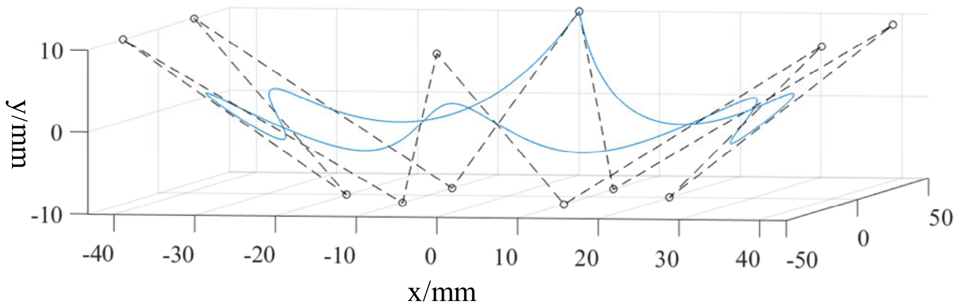
Because the Mline-Gear correction formula cannot be used to correct the interpolation parameters when calculating the initial interpolation parameters, the derivative calculation formula of NURBS curve is used to ensure the calculation accuracy of the initial interpolation parameters for the derivative calculation in the Runge-Kutta method.

4. SIMULATION EXPERIMENT

The test curves shown in Fig.3 are simulated by using the Mline-Gear interpolation algorithm, second-order Taylor expansion, fourth-order R-K algorithm and Adams implicit explicit prediction correction formula in this paper, and the velocity fluctuation rates of different interpolation algorithms in the interpolation process are compared.The speed planning adopts the improved S-type acceleration and deceleration control-quadratic polynomial acceleration and deceleration model [14].The acceleration curve and acceleration curve of the model are smooth transitions, and the stability of interpolation processing is better.The maximum feed acceleration is set to 4800mm/s2, and the maximum feed acceleration is 48000mm/s3.The remaining performance parameters are shown in Table 1.



(a)Curve top view



(b)Three-dimensional diagram of curve

Fig.2: NURBS curve test curve

Table 1: Performance parameter table

performance parameter	parameter value
interpolation period/ms	1
Maximum normal acceleration/mm·s-2	4800
Maximum bow height error/mm	0.001
Maximum normal acceleration/mm·s-3	48000
maximum feed rate/mm·s-2	240

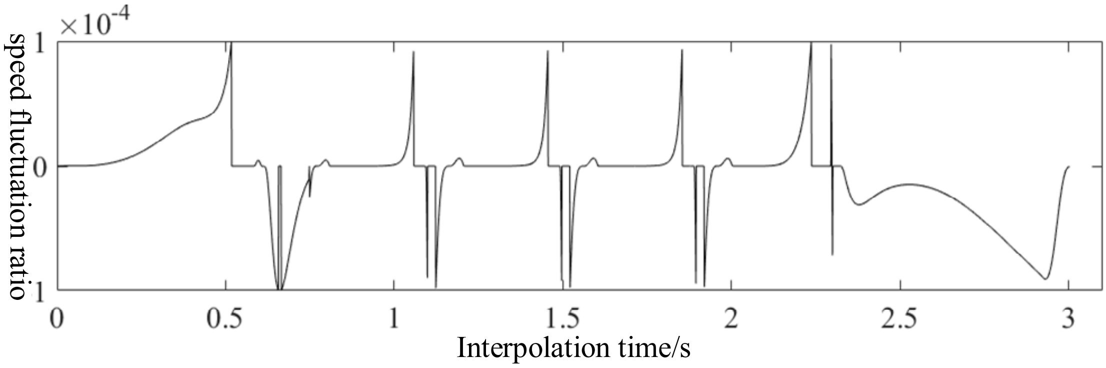
When using the four algorithms to calculate the interpolation parameters, the Steffensen iteration method is used as the interpolation parameter iteration algorithm, and the upper limit of the speed fluctuation rate is set to 0.0001.The speed fluctuation rate of the four algorithms is shown in Figure 3.The range of fluctuation of the speed fluctuation rate decreases in the order of second-order Taylor expansion, fourth-order R-K, Adams implicit-explicit algorithm, and Mline-Gear algorithm.In order to evaluate each algorithm more intuitively, the maximum speed fluctuation rate and other information in the speed fluctuation rate diagram of each algorithm are counted, as shown in Table 2.

Table: 2 Statistical table of interpolation algorithm information

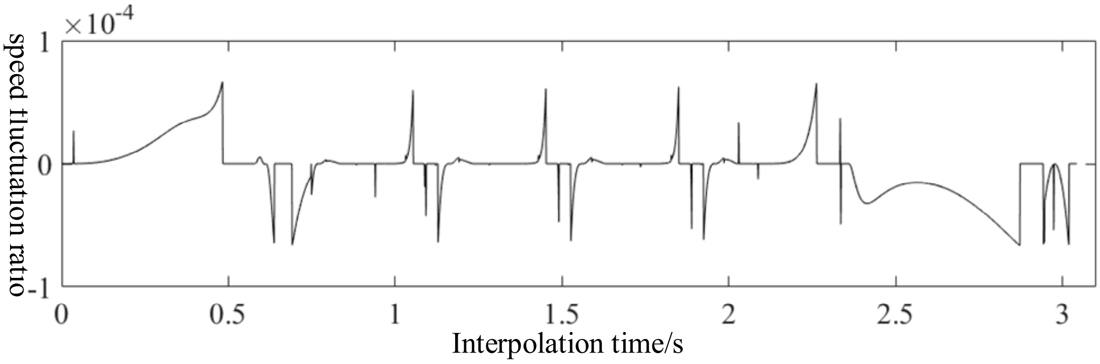
Algorithm name	Maximum speed	Average absolute value of velocity fluctuation rate	Number of interpolation cycles	Iteration times	average number of iterations

Second-order Taylor	9.92454×10-5	1.51642×10-5	3105	4397	1.41610
Fourth-order R-K	6.73675×10-5	8.54249×10-6	3105	4014	1.29275
Adams looms large	6.42547×10-5	8.12534×10-6	3105	3989	1.28470
Mline-Gear algorithm	4.76598×10-5	5.94216×10-6	3105	3552	1.14397

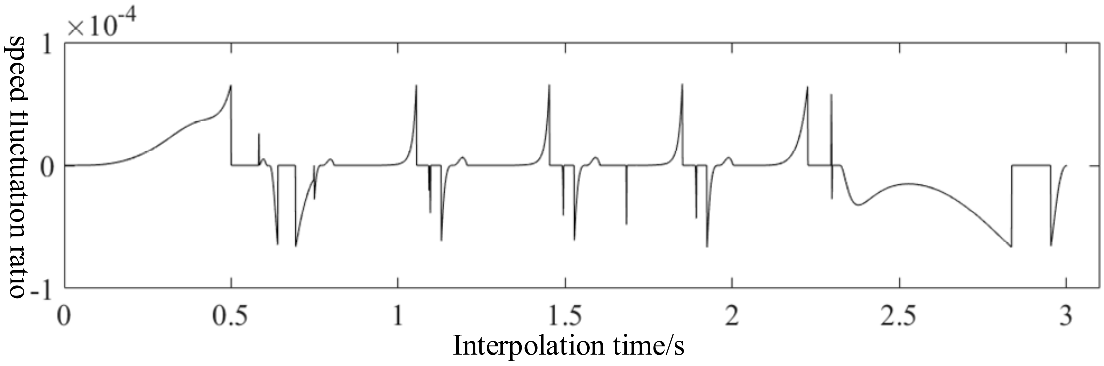
It can be seen from Table 2 that the second-order Taylor expansion is the largest in terms of the extreme value and average value of velocity volatility, followed by the fourth-order R-K algorithm and Adams implicit-explicit algorithm, and the Mline-Gear algorithm is the smallest. The number of iterations of each interpolation cycle of the four algorithms is shown in Figure 4. It can be seen that the number of iterations of the double Mline-Gear algorithm and the Adams algorithm is small. In order to more clearly compare the iterative information of each algorithm, the number of iterations and the average number of iterations of each algorithm are also summarized in Table 2. From Table 2, it can be seen that in terms of the average number of iterations, the second-order Taylor expansion is similar to the fourth-order R-K algorithm, and the Adams algorithm is similar to the double Hamming algorithm. The latter two algorithms are slightly smaller, but the Mline-Gear algorithm is smaller than the Adams algorithm. The above statistical information shows that compared with the other three algorithms, the interpolation parameters of the Mline-Gear algorithm have higher calculation accuracy and effectively reduce the fluctuation rate of the progress speed during interpolation.



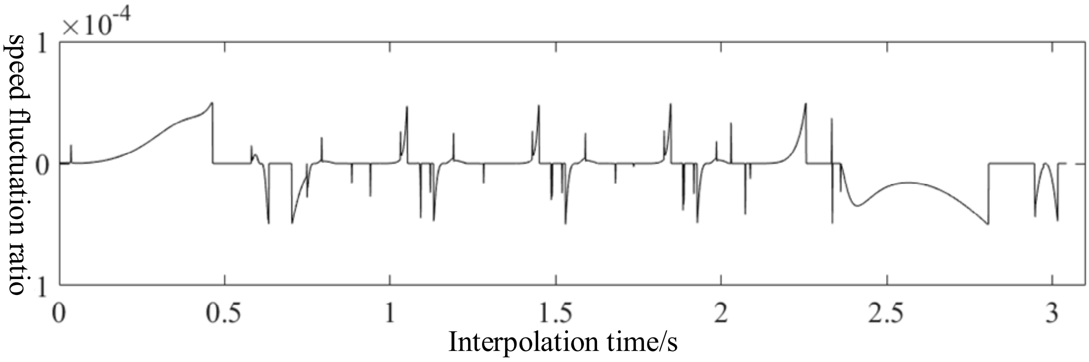
(a) Second-order Taylor expansion



(b)Fourth-order R-K method

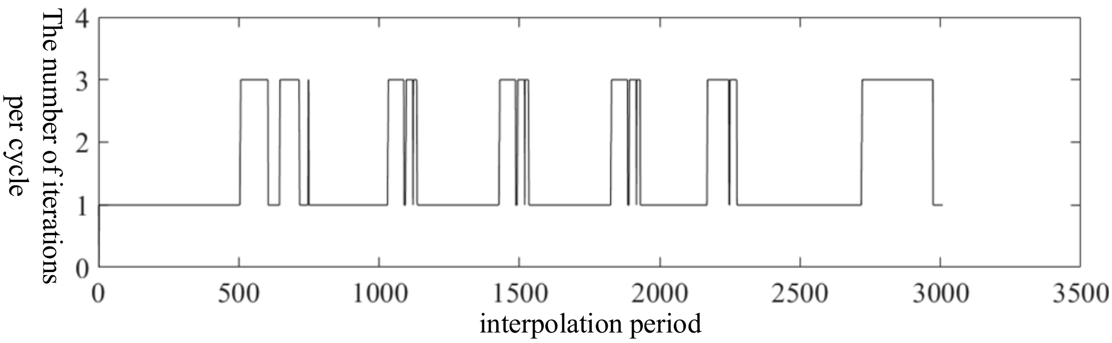


(c)Adams implicit-explicit prediction correction

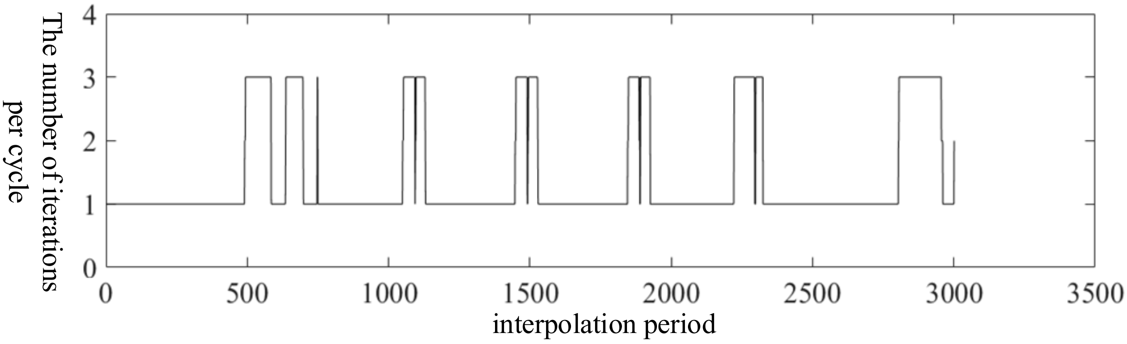


(d)Mline-Gear algorithm

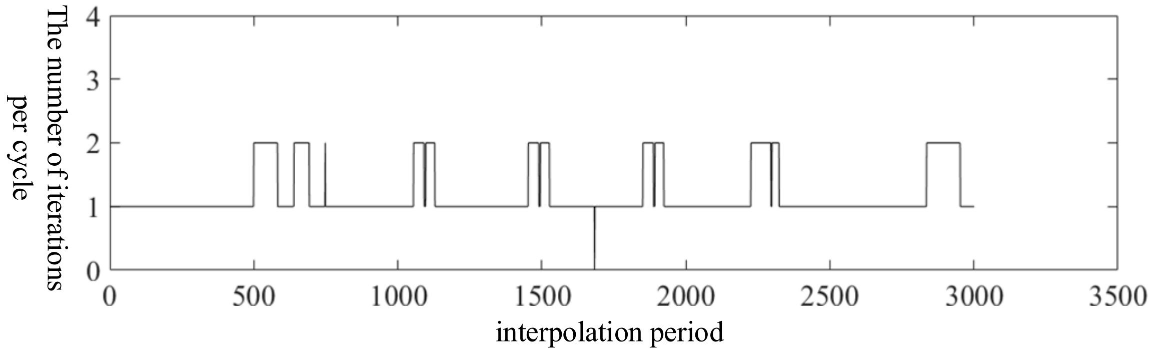
Fig. 3: Comparison chart of velocity volatility



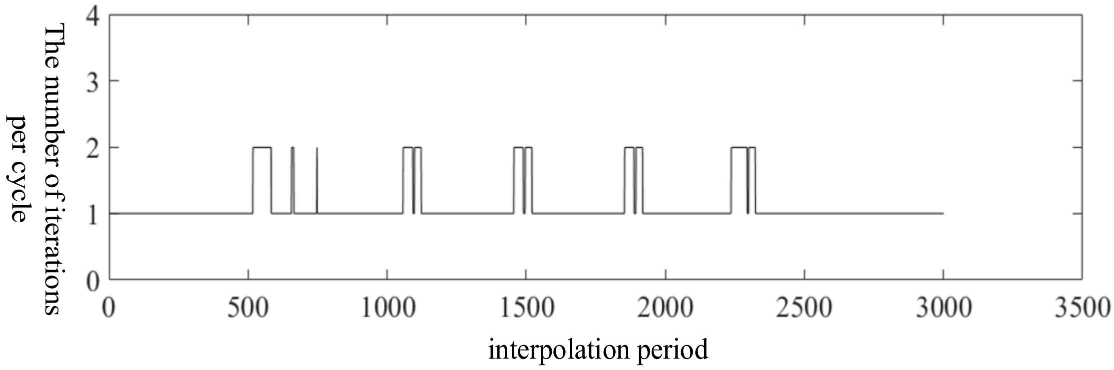
(a)Second-order Taylor expansion



(b)Fourth-order R-K method



(c)Adams implicit-explicit prediction correction



(d)Mline-Gear algorithm

Fig.4: Comparison of the number of iterations per cycle

5. CONCLUSION

In this paper, NURBS curve interpolation algorithm based on Mline-Gear is proposed. Firstly, the algorithm performs adaptive speed planning on the constraints of bow height error and normal acceleration to ensure the machining accuracy and obtain the ideal interpolation step size. After calculating the initial interpolation parameter value by R-K method, the Hamming method is transformed by the definition of difference and derivative respectively, and the correction value of the subsequent interpolation parameter is obtained. The transformation method reduces the calculation amount and ensures the calculation accuracy as much as possible. In order to reduce the feed speed fluctuation rate, the Steffensen iterative formula is used to correct the interpolation parameters that do not satisfy the speed fluctuation rate.

The simulation results show that compared with the second-order Taylor, fourth-order R-K and Adams implicit-explicit predictor-corrector algorithms, the Mline-Gear interpolation algorithm has smaller speed fluctuation rate and fewer iterations, and meets the accuracy requirements of processing.

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