

# Topology Aware EEG Classification Using Persistent Images and Color Equivariant CNNs

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## ABSTRACT

Electroencephalography owing to its high temporal resolution and non-invasive nature, holds significant potential for auxiliary diagnosis in neuropsychiatric disorders such as schizophrenia. However, conventional EEG analysis methods often fall short in capturing high-order structural features of complex brain functional connectivity and in achieving robust generalization across subjects. To address these limitations, this study proposes a novel image classification framework, CEResNet, which integrates topological data analysis with color-equivariant convolutional neural networks. Specifically, the proposed method first employs persistent homology to extract multi-scale high-order topological features from multi-band EEG signals. These features are then transformed into persistence images via Gaussian kernel mapping, serving as inputs for the deep learning model. A color-equivariant convolutional module is subsequently embedded into the ResNet-18 architecture to enhance the model's robustness against color distribution shifts. Experimental results demonstrate that the proposed approach achieves a classification accuracy of 0.9231 in distinguishing schizophrenia patients from healthy controls, significantly outperforming various traditional machine learning baselines and existing comparative models. These findings validate the effectiveness of encoding topological structures into images combined with color-equivariant modeling strategies, offering a novel perspective for intelligent identification of neuropsychiatric disorders.

## KEYWORDS

Schizophrenia; Electroencephalography; Persistent homology; Persistent image; ResNet

## 1. INTRODUCTION

Electroencephalography (EEG), as a non-invasive neural signal acquisition technique, has been widely applied in the early detection, auxiliary diagnosis, and functional brain research of neuropsychiatric disorders, owing to its high temporal resolution, low cost, and ease of operation [1]. In recent years, EEG has demonstrated unique physiological value in revealing abnormal patterns of brain functional connectivity in complex psychiatric disorders such as schizophrenia (SZ), and is gradually emerging as an important biomarker for characterizing latent neuropathological changes [2].

Traditional EEG analysis methods typically rely on spectral features [3], time-frequency representations [4], phase synchrony indices, or the construction of brain functional connectivity networks based on static graph theory [5]. Although these methods can capture local or low-order connectivity to some extent, they fall short in representing higher-order structural information, especially given the intrinsic complexity of EEG signals—characterized by nonlinearity, high noise levels, multi-scale variability, and inter-subject differences. These limitations become particularly

evident in cross-subject, cross-frequency, or small-sample learning scenarios, where classification stability and generalization performance are often inadequate.

To address the shortcomings of conventional methods in modeling complex structures, topological data analysis (TDA) [6] has been increasingly introduced into the domain of brain network modeling. In particular, persistent homology (PH) [7] enables the extraction of high-order topological features from EEG signals and their induced networks across multiple scales. It records the birth and death of these features using a persistence diagram (PD), effectively characterizing the stability and persistence of brain network structures as scale changes. Previous studies have demonstrated the discriminative power and robustness of PH in brain network analysis. For example, Spaziani et al. [8] utilized PH to describe the evolution of brain networks across different sleep stages; Wang et al. [9] dynamically tracked changes in functional connectivity in stroke patients using persistence landscapes; Liu et al. [10] applied persistent entropy to characterize the survival distribution of topological features in intermediate brain regions. Kang et al. [11] further integrated persistence landscapes, persistent entropy, Betti curves, and heat kernel signatures, and adopted a multi-classifier strategy, achieving significant performance gains in schizophrenia classification. These studies collectively validate the effectiveness of PH in modeling complex brain networks.

Despite the notable advantages of PH in extracting stable topological features, its core representation—PDs [12]—are inherently non-Euclidean, making them incompatible with most mainstream deep learning models. To overcome this challenge, the persistence image (PI) [13] was proposed as a vectorized representation of PDs. Through Gaussian kernel density mapping, persistent points are projected onto a two-dimensional pixel space, allowing topological structures to be perceived by deep models in image form. In a PI, pixel intensities typically reflect the persistence and density distribution of topological features, thus embedding rich structural information within the visual space.

However, the color distribution in PIs is highly sensitive to kernel function parameters and individual structural differences, often resulting in significant color-space shifts across samples. These variations may be further amplified by convolutional neural networks (CNNs) [14], making models highly susceptible to color perturbations. This is particularly problematic when there is a distribution mismatch between training and testing data, leading to noticeable declines in generalization performance. While existing approaches—such as grayscale conversion or color normalization—can partially mitigate this issue, they often come at the cost of losing key discriminative topological information, thereby limiting model expressiveness and performance potential.

In the field of computer vision, studies have shown that CNNs are highly sensitive to shifts in color distribution, particularly when training samples are unevenly distributed in color space or under varying lighting conditions. This can lead to a "color bias" phenomenon [15]. To enhance model robustness, some research efforts have focused on constructing color-invariant features, albeit often at the expense of color discriminability. To address this challenge, Lengyel et al. [16] proposed the Color Equivariant Convolution (CEConv) module at NeurIPS. By introducing a discrete rotation group  $H_n$  in the hue component of the HSV color space and leveraging weight-sharing mechanisms, CEConv significantly enhances robustness to color perturbations while preserving color-discriminative information. This method has been validated on several benchmark datasets (e.g., CIFAR-100, Flowers102), and is particularly effective in tasks where color carries structured semantic meaning.

Building upon the aforementioned insights, this study proposes an EEG image classification framework named CEResNet, which integrates topological data analysis with color-equivariant modeling. The framework first extracts high-order topological features from EEG signals using persistent homology, and constructs persistence images through Gaussian kernel mapping as model input. Subsequently, the CEConv module is embedded into a ResNet-18 architecture [17], enhancing

both the model’s robustness to color perturbations and its capacity to express topological structural information.

The main contributions of this paper can be summarized as follows:

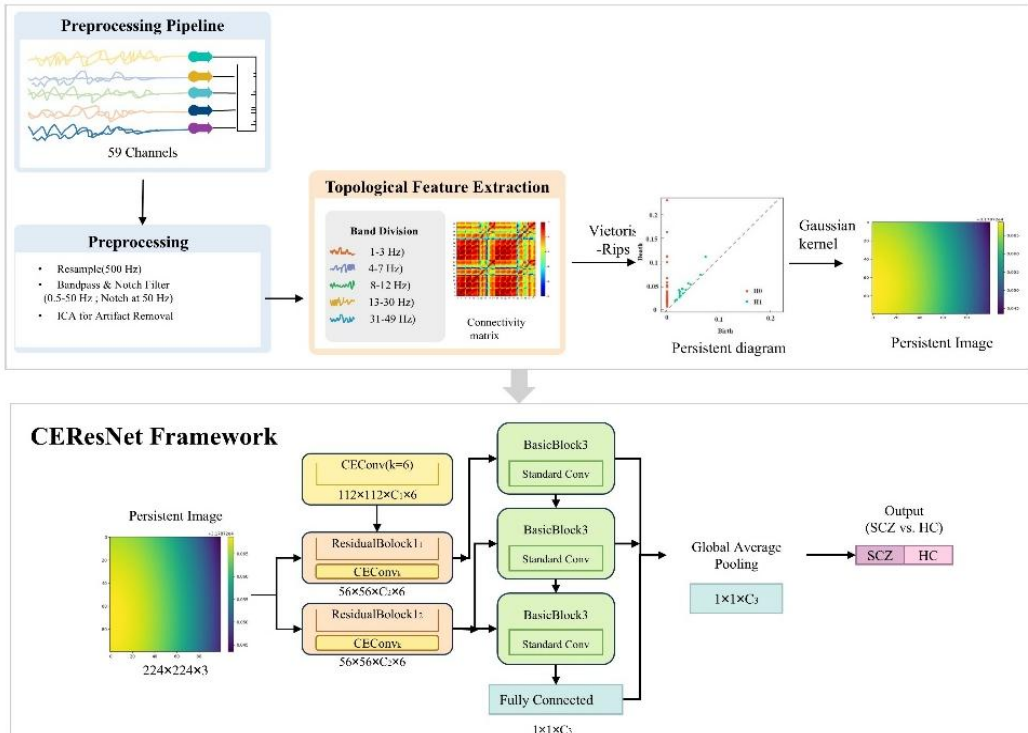
We propose a vectorized representation method for EEG topological features based on persistence images, which transforms high-order topological structures into two-dimensional visual representations via Gaussian kernel mapping. This facilitates the effective modeling of topological information within a deep learning framework, particularly in the context of schizophrenia recognition.

We construct a novel classification framework that combines topological data analysis with color-equivariant modeling. To the best of our knowledge, this is the first work to integrate persistence images with CEConv modules, significantly enhancing robustness to color disturbances and improving generalization performance in cross-subject scenarios.

## 2. MATERIALS AND METHODS

### 2.1. Overview of the Proposed Method

The overall framework for EEG-based schizophrenia classification proposed in this study is illustrated in Figure 1. First, the raw EEG signals undergo standardized preprocessing and are segmented into multiple frequency bands to construct multi-band brain functional connectivity networks. Then, based on the inter-channel dissimilarity matrix, multi-scale high-order topological features are extracted using PH, and the resulting PDs are vectorized into two-dimensional PIs via Gaussian kernel density mapping. Finally, the PIs are fed into a deep neural network, CEResNet, which integrates CEConv modules for end-to-end training to automatically classify schizophrenia patients and healthy controls.



**Figure 1.** Overall architecture of the proposed EEG-based schizophrenia classification framework

## 2.2. Dataset

The EEG dataset used in this study was collected at Beijing Huilongguan Hospital and includes a total of 195 participants, comprising 103 first-episode schizophrenia patients (SCZ group) and 92 healthy controls (HC group). All subjects were aged between 18 and 45 years. To ensure statistical comparability between groups, no significant differences were observed in demographic variables such as age, gender, or years of education ( $p > 0.05$ ), thereby reducing potential confounding effects on experimental results. Detailed demographic and clinical information is summarized in Table 1.

**Table 1.** Clinical Data Statistical Tables

| Features            | SCZ (n = 103) | HC (n = 92) | statistical values                 |
|---------------------|---------------|-------------|------------------------------------|
| Average age         | 30.55±8.002   | 30.55±7.287 | F1,195< 1                          |
| Length of education | 14.17±2.494   | 14.79±2.557 | F 1,195 = 3.022, p = 0.084         |
| Sex (male/female)   | 51/52         | 48/44       | $\chi^2_{12} = 0.182$ , pr = 0.670 |
| PANSS score         | 74.31±10.87   | /           | /                                  |
| positive score      | 20.53±4.72    | /           | /                                  |
| negative score      | 17.18±5.63    | /           | /                                  |

Schizophrenia diagnoses were made by experienced psychiatrists according to the DSM-5 diagnostic criteria. Symptom severity was assessed using the Positive and Negative Syndrome Scale (PANSS). EEG signals were recorded using a NeuroScan 64-channel electrode system at a sampling rate of 500 Hz, with electrode impedance maintained below 5 k $\Omega$ . The reference electrodes were placed on the bilateral mastoids, and the ground electrode at AFz. Vertical and horizontal electrooculograms were recorded concurrently to aid in the identification and removal of ocular artifacts. All recordings were conducted in an eyes-closed resting state under a quiet environment, with participants instructed to relax and minimize head and body movements.

## 2.3. EEG Preprocessing

To improve EEG signal quality and ensure the stability of subsequent topological feature extraction, standardized preprocessing was performed using the MNE-Python toolbox. First, electrode positions were normalized, and missing channels (e.g., F11, F12, FT11, FT12) were reconstructed via spatial interpolation based on a standard electrode layout. All EEG signals were then resampled to 500 Hz for consistency and optimized time-frequency analysis.

After removing auxiliary channels, bad channels were manually identified and repaired using spatial interpolation. Signals were referenced to M1/M2 and further re-referenced using average referencing to reduce common-mode noise. To eliminate eye movement and muscle artifacts, Independent Component Analysis (ICA) was performed, and components strongly correlated with artifacts were manually removed [18]. The preprocessed EEG signals were finally segmented into non-overlapping 20-second epochs for subsequent feature extraction and modeling. For frequency-domain analysis, five canonical frequency bands were extracted based on neurophysiological relevance: Delta (1–3 Hz), Theta (4–7 Hz), Alpha (8–12 Hz), Beta (13–30 Hz), and Gamma (31–49 Hz). Comparative experiments were primarily conducted in the Theta band, in accordance with findings from prior literature [19].

## 2.4. Topological Feature Extraction and Persistence Image Construction

For each frequency band, functional connectivity graphs were constructed from the 64-channel EEG time series. To quantify pairwise dissimilarity between EEG channels, we adopted the Pearson Dissimilarity measure proposed by Schober et al. [20], defined as:

$$D_{ij} = 1 - \left| \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}} \right| \quad (1)$$

Where  $C_{ij}$  denotes the covariance between channels  $i$  and  $j$ , and  $C_{ii}, C_{jj}$  are the variances of each channel. The distance  $d_{ij} \in [0,1]$ , with higher values indicating weaker connectivity. Given the resulting distance matrix  $D \in \mathbb{R}^{N \times N}$ , a Vietoris–Rips simplicial complex was constructed to model high-order interactions among nodes. A simplex was formed when the distance between any two nodes was less than a threshold  $\varepsilon$ . As  $\varepsilon$  increases, the topological structure evolves, generating a PD for each frequency band:

$$PD = \{(b_k, d_k) \mid k = 1, 2, \dots, K\} \quad (2)$$

Where  $b_k$  and  $d_k$  denote the birth and death scales of the  $k$ th topological feature, respectively. To visualize and vectorize the extracted topological features, each PD was transformed into a PI. A 2D Gaussian kernel was used to estimate the density of each persistence point:

$$PI(x, y) = \sum_{k=1}^K \exp \left( -\frac{(x-b_k)^2 + (y-d_k)^2 \cdot w_k}{2\sigma^2} \right) \quad (3)$$

Where  $\sigma$  is the kernel bandwidth and  $w_k = d_k - b_k$  is the importance weight of each topological feature. The resulting PI for each frequency band was used as input to the deep learning model.

## 2.5. CEResNet Model Architecture

To effectively capture high-order topological features in EEG signals and improve robustness to color-related distribution shifts, we propose CEResNet, a persistence image classification framework based on Color Equivariant Convolutional Neural Networks. The model adopts ResNet-18 as the backbone and incorporates CEConv modules in the initial layers to enhance sensitivity to structural color variations in topological images.

Each PI image, with dimensions  $224 \times 224 \times 3$ , encodes both the persistence magnitude and the spatial density distribution of topological features. Retaining full color information is therefore critical for maintaining discriminative capacity. To model the equivariance of PI structures under hue perturbation, we employ CEConv for initial feature extraction. CEConv introduces a discrete rotation group  $H_n$  in the HSV hue space to enable equivariant modeling of color changes. The hue transformation matrix  $H_n(k)$  is defined as:

$$H_n(k) = \begin{bmatrix} \cos\left(\frac{2k\pi}{n}\right) + a & a - b & a + b \\ a + b & \cos\left(\frac{2k\pi}{n}\right) + a & a - b \\ a - b & a + b & \cos\left(\frac{2k\pi}{n}\right) + a \end{bmatrix} \quad (4)$$

Here,  $n$  denotes the number of discrete steps in the hue rotation group, and  $k \in \{0, 1, \dots, n-1\}$  represents the  $k$ -th rotation. The parameters of the hue transformation matrix are defined as:  $a = \frac{1}{3}(1 - \cos\left(\frac{2k\pi}{n}\right))$ , and  $b = \frac{1}{\sqrt{3}}\sin\left(\frac{2k\pi}{n}\right)$ . The matrix  $H_n(k)$  operates in the RGB pixel space and

effectively performs a rotation of the color channels around the axis  $[1, 1, 1]$ , enabling the model to learn representations that are consistent under hue transformations. To formalize this operation, a composite group is defined as  $G = \mathbb{Z}^2 \times H_n$ , where  $\mathbb{Z}^2$  represents the translation group and  $H_n$  the discrete hue rotation group. At the input layer, the operation of CEConv is expressed as follows:

$$[f \circ \psi^i](x, k) = \sum_{y \in \mathbb{Z}^2} \sum_{c=1}^{C_l} f_c(y) \cdot H_n(k) \psi_c^i(y - x) \quad (5)$$

Here,  $f_c(y)$  denotes the input image value at position  $y$  in the  $c$ -th channel,  $\psi_c^i$  represents the  $c$ -th channel of the  $i$ -th convolutional kernel, and  $H_n(k)$  is the hue rotation matrix corresponding to the  $k$ -th discrete rotation. This operation enables parameter sharing of convolutional kernels across different hue spaces, thereby enhancing the model's ability to recognize topological structures under color perturbations.

In the proposed CEResNet architecture, the CEConv modules are embedded into the first two layers of the ResNet-18 backbone. The input PI has a size of  $224 \times 224 \times 3$ , and after processing by the first CEConv layer, the feature map expands to  $112 \times 112 \times (C_1 \times n)$  due to the introduction of hue-channel dimensions, where  $C_1$  denotes the number of output channels and  $n$  is the number of discrete hue rotations. Subsequently, CEConv is further applied within the residual modules BasicBlock1 and BasicBlock2 to preserve the network's ability to model color-structured representations. In the deeper layers of the network (BasicBlock3 and BasicBlock4), traditional convolutional operations are employed to reduce model complexity and computational overhead. The final classification of SCZ versus HC is achieved through a global average pooling layer followed by a fully connected layer.

In the hidden layers, the CEConv operation is extended to the group space  $\mathbb{Z}^2 \times H_n$ , and is defined as follows:

$$[f \circ \psi^i](x, k) = \sum_{y \in \mathbb{Z}^2} \sum_{r=1}^n \sum_{c=1}^{C_l} f_c(y, r) \cdot \psi_c^i(y - x, (r - k) \% n) \quad (6)$$

Here, both the feature map  $f_c(y, r)$  and the convolution kernel  $\psi_c^i$  are defined over the group space  $\mathbb{Z}^2 \times H_n$ , where  $(r - k) \% n$  denotes the modulo- $n$  operation. Each value of  $k$  corresponds to one hue channel in the CEConv output, yielding  $n$  equivariant responses. To further alleviate the risk of parameter explosion, a filter factorization strategy is employed. Specifically, the convolutional kernels in CEConv are decomposed into the product of a spatial convolution kernel  $S$  and a hue rotation channel-wise weight matrix  $P$ , expressed as:

$$F_{c',c,g,u,v}^l = S_{c',c,l,u,v} \cdot P_{c',g,c,g,l,l} \quad (7)$$

This design effectively controls the growth of model parameters while improving training efficiency and scalability.

### 3. RESULTS AND ANALYSIS

#### 3.1. Dataset Partitioning and Experimental Settings

This study aims to develop a robust classification framework to distinguish resting-state EEG signals of first-episode SCZ from HC. To objectively assess the model's generalization performance on unseen individuals, all data were partitioned at the subject level—ensuring that no data from the same

individual appeared in both the training and test sets. This strict separation effectively prevents information leakage.

Specifically, the dataset was randomly divided into training and independent test sets in an 8:2 ratio, with complete subject-level separation. During model training, the batch size was set to 32, and the maximum number of epochs was 250. The Adam optimizer was employed with an initial learning rate of  $1 \times 10^{-5}$ . To mitigate overfitting and improve training efficiency, an early stopping strategy was implemented: training was terminated if classification accuracy on the validation set did not improve for 100 consecutive epochs.

All experiments were conducted on a high-performance computing platform. The software and hardware environment included: Python 3.8, the PyTorch deep learning framework, an NVIDIA RTX 3090 GPU, 64 GB system memory, and a 32-core CPU, ensuring stability and reproducibility of both training and inference processes.

To comprehensively evaluate classification performance, four widely-used metrics were adopted: Accuracy, Precision, Recall, and F1-score, defined as follows:

$$Accuracy = \frac{TP + TN}{FP + FN + TP + TN} \quad (8)$$

$$Precision = \frac{TP}{FP + TP} \quad (9)$$

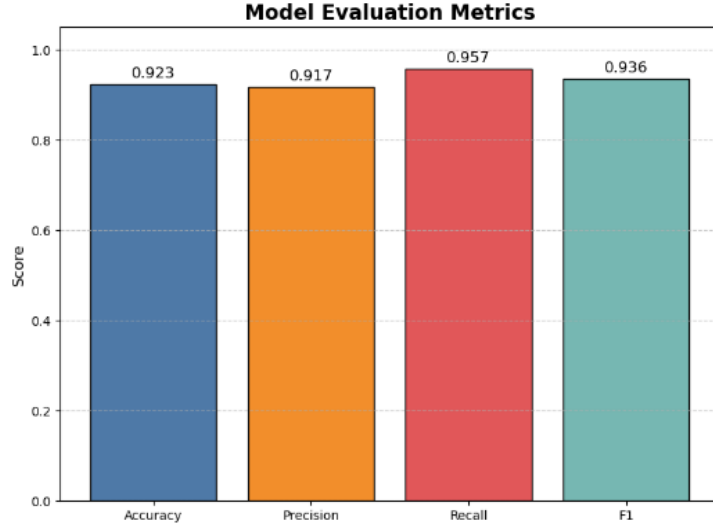
$$Recall = \frac{TP}{FN + TP} \quad (10)$$

$$F1 = \frac{2 \times Precision \times Recall}{Precision + Recall} \quad (11)$$

Here, TP denotes the number of correctly identified schizophrenia cases, TN the correctly identified healthy controls, FP the healthy controls misclassified as schizophrenia, and FN the schizophrenia cases misclassified as healthy. Notably, F1-score, as the harmonic mean of Precision and Recall, is particularly valuable in scenarios with potential class imbalance.

### 3.2. Evaluation of CEResNet Performance

The performance of the proposed CEResNet framework in schizophrenia classification was first evaluated on the independent test set. As shown in Figure 2, the model achieved strong classification results. The experimental findings demonstrate that CEResNet, equipped with CEConv, effectively captures high-order topological structures and color information from PIs, exhibiting superior discriminative capability and generalization performance.



**Figure 2.** Classification results of CEResNet on the test set

To further validate the effectiveness of CEResNet, we conducted a systematic comparison of five commonly used vectorization methods for persistent homology features: Diagram Properties, Betti Curves (BC), Persistent Entropy (PE), Persistence Landscapes (PL), Heat Kernel Maps (HM). These feature sets were evaluated using four traditional machine learning classifiers: Logistic Regression [21], Support Vector Machine (SVM) [22], Random Forest (RF) [23], and LightGBM [24]. Additionally, we introduced a “PI + CNN (features)” strategy, where shallow CNNs were used to extract features from PIs before feeding them into classical classifiers, to assess the representational capacity of image-based topological features. The classification results are summarized in Table 2.

**Table 2.** Classification results for different topological feature representations using traditional classifiers

| Features            | Logistic Regression | SVM   | RF    | LightGBM |
|---------------------|---------------------|-------|-------|----------|
| Diagram properties  | 0.703               | 0.678 | 0.722 | 0.739    |
| BC                  | 0.738               | 0.741 | 0.752 | 0.755    |
| PE                  | 0.697               | 0.721 | 0.749 | 0.744    |
| PL                  | 0.655               | 0.706 | 0.721 | 0.734    |
| HM                  | 0.823               | 0.811 | 0.817 | 0.841    |
| BC + PE + PL + HK   | 0.879               | 0.854 | 0.866 | 0.897    |
| PI + CNN (features) | 0.885               | 0.893 | 0.907 | 0.910    |

From Table 2, it can be observed that HM features yielded relatively stable performance across classifiers, with the LightGBM model achieving an accuracy of 0.841. By integrating multiple topological features (BC + PE + PL + HM), classification accuracy further improved to 0.897, highlighting the complementary nature of different topological descriptors.

Building on this, applying CNN-based feature extraction to PI images (PI + CNN) further boosted accuracy to 0.910 with LightGBM. However, the proposed CEResNet model outperformed all these methods by directly performing end-to-end modeling on PI images without relying on handcrafted features, achieving the highest classification accuracy of 0.9231. This result demonstrates the effectiveness of incorporating color equivariant modeling strategies in topological image analysis.

### 3.3. Performance Comparison Across Topological Representations

To more comprehensively assess the discriminative capacity of different topological feature representations in schizophrenia EEG classification, we further evaluated their performance across the four metrics—Accuracy, Precision, Recall, and F1-score—as summarized in Table 3.

**Table 3.** Comparison of classification performance across different topological representations

| Features           | Method   | Accuracy | Precision | Recall | F1    |
|--------------------|----------|----------|-----------|--------|-------|
| Diagram properties | LightGBM | 0.739    | 0.720     | 0.825  | 0.769 |
| BC                 | LightGBM | 0.755    | 0.731     | 0.844  | 0.783 |
| PE                 | LightGBM | 0.744    | 0.767     | 0.737  | 0.752 |
| PL                 | LightGBM | 0.734    | 0.831     | 0.621  | 0.711 |
| HM                 | LightGBM | 0.841    | 0.839     | 0.864  | 0.851 |
| BC + PE + PL + HK  | LightGBM | 0.897    | 0.887     | 0.922  | 0.904 |
| PI                 | CEResNet | 0.923    | 0.917     | 0.957  | 0.936 |

Among the traditional machine learning models, HM features achieved the best performance when used individually, with accuracy of 0.841 and F1-score of 0.851, suggesting that HM is effective in capturing stable topological structures in brain networks. Feature fusion (BC + PE + PL + HM) led to further improvements, reaching accuracy of 0.897 and F1-score of 0.904, validating the complementarity of multi-source topological descriptors.

In contrast, the proposed CEResNet model, by integrating Color Equivariant Convolution into the ResNet-18 architecture, directly and effectively models PI representations under color perturbations. CEResNet achieved accuracy of 0.923, recall of 0.957, and an F1-score of 0.936, outperforming all comparison methods. These results highlight CEResNet’s superior sensitivity and robustness in capturing complex EEG topological features and suggest its strong potential for real-world applications requiring high generalization, such as cross-subject classification and image-invariant brain decoding.

## 4. DISCUSSION

### 4.1. Analysis of Classification Performance and Validation of Model Effectiveness

In the binary classification task distinguishing SCZ from HC using EEG data, the proposed CEResNet model achieved outstanding performance on the independent test set (Accuracy = 0.9231, F1-score = 0.9362), significantly outperforming approaches that combine traditional topological features (e.g., Betti curves, persistent entropy, heat kernel maps) with classical classifiers such as SVM, Random Forest, and LightGBM. Compared with the best-performing traditional method, CEResNet improved the F1-score by approximately 3.2%, highlighting its superior precision and generalization capability. The persistence image (PI) preserves high-order topological information comprehensively, while the CEConv module alleviates performance degradation caused by color perturbations through parameter sharing in hue space, jointly enhancing the model’s robustness to distributional shifts across samples.

### 4.2. Methodological Advantages and Innovation in Topology–Color Joint Modeling

Persistent homology constructs Vietoris–Rips simplicial complexes to characterize the evolution of brain network topology across multiple scales, avoiding the limitations of static threshold selection in conventional graph-theoretic methods. Through PI representations, these topological features can be visualized and further processed using deep learning. In this work, we innovatively introduce the CEConv module into the field of EEG image analysis, achieving the dual goals of preserving discriminative color information and enhancing model robustness to hue variations. This effectively

improves generalization performance in cross-subject scenarios and offers a generalizable and transferable modeling strategy for topological image recognition tasks.

### **4.3. Limitations**

Despite the promising results, this study has several limitations. First, the sample size is relatively limited, comprising only 195 participants. Although strict subject-level partitioning was employed to mitigate overfitting, the robustness of the model in larger or more heterogeneous populations remains to be further validated. Second, the generation of PIs is sensitive to the choice of Gaussian kernel parameters, and currently lacks a unified and automatic optimization mechanism, which may affect the reproducibility and adaptability of the approach.

Additionally, topological information across frequency bands is not explicitly modeled, potentially limiting the representation of cross-frequency coupling features. Lastly, the interpretability of the model remains limited, and the critical decision regions within the topological space have not yet been clearly identified.

### **4.4. Future Research Directions**

Future work will focus on four main directions to further improve the proposed framework: 1) Incorporating larger-scale, multi-center clinical EEG datasets, covering different stages of disease progression, to enhance model robustness and generalizability; 2) Exploring frequency-band attention mechanisms and multi-band fusion strategies to strengthen the modeling of multi-scale and cross-frequency EEG information; 3) Introducing automated parameter tuning techniques, such as Bayesian optimization, to enable adaptive control of parameters in the PI construction process; 4) Leveraging model interpretability techniques, including Grad-CAM and Layer-wise Relevance Propagation, to increase clinical transparency and provide insights into decision-making. These methods will also be extended to the topological analysis of other neuropsychiatric conditions, such as Alzheimer's disease and ADHD, particularly in multi-modal neuroimaging scenarios.

## **5. CONCLUSION**

This study proposes an EEG classification framework, CEResNet, which integrates topological data analysis with color equivariant convolution. By combining PIs with a CEConv-ResNet architecture, the proposed method significantly enhances the accuracy of schizophrenia recognition and improves model robustness against color perturbations. Experimental results demonstrate that CEResNet outperforms traditional topological feature-based machine learning models across multiple evaluation metrics. These findings validate the effectiveness of jointly modeling topological structures and color information for brain network analysis and offer a promising new approach for the intelligent identification of neuropsychiatric disorders.

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Data Availability Statement.

The raw data of the present study will be made available by the authors.

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