

Study on the Correlation Between Fractal Dimension and Permeability in Shale Reservoir Fracture Networks

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ABSTRACT

Shale reservoirs usually show the characteristics of low porosity and low permeability, and the internal natural fractures are generally developed, and their reservoir space and fluid seepage capacity are mainly controlled by complex fracture networks. As the dominant channel for fluid migration in the reservoir, the distribution of natural fractures not only directly determines the accumulation and production of oil and gas, but also has a key impact on the expansion and final transformation of fracture network morphology in the subsequent hydraulic fracturing process. It should be pointed out that such fracture systems often show heterogeneous and self-similar fractal characteristics in terms of spatial distribution. Therefore, the degree of development of the fracture network can be characterized by fractal dimensions. In this paper, the relationship between fractal dimension and permeability of fracture networks is studied by simulating fracture networks of different complexity through the simulation method of discrete fracture networks. The results show that the fractal dimension of shale reservoir fracture network is linearly positively correlated with the number of fractures and the maximum length of fractures. There is also a linear positive correlation between the fractal dimension corresponding to the maximum length and number of fractures and the equivalent permeability of different fractures, and the correlation is good. This discrete fracture network simulation of the natural fracture system of shale reservoirs is not only an important way to understand its reservoir characteristics and seepage mechanism, but also a key link to improve the efficiency of shale oil and gas development and optimize the fracturing design of fracturing.

KEYWORDS

Shale reservoirs; Geometric parameters; Natural fractures; Discrete fracture network; Fractal geometry

1. INTRODUCTION

With the continuous growth of global energy demand and the increasing depletion of conventional oil and gas resources, unconventional oil and gas resources, especially shale oil, have become an important area of global energy strategy succession [1]. China's shale oil resources are rich and widely distributed, showing huge exploration and development potential. However, shale reservoirs usually have typical geological characteristics such as low porosity, ultra-low permeability, and strong heterogeneity, which poses serious challenges to their cost-effective development [2]. In recent years, breakthroughs in horizontal well and volumetric fracturing technology have successfully realized the transformation of such "artificial reservoirs" and made commercial shale oil exploitation possible. In this technical context, natural fracture systems in reservoirs play a crucial role. They are not only important reservoir spaces and dominant seepage channels in shale formations, but also profoundly affect the formation and expansion behavior of complex fracture networks during hydraulic fracturing, and ultimately have a decisive impact on the productivity and recovery of a single well.

The natural fracture system in shale reservoirs exhibits strong disorder, multi-scale and highly discrete characteristics in terms of spatial distribution. Traditional equivalent continuous media models are difficult to accurately characterize such complex heterogeneous structures, but discrete fracture network (DFN) models have emerged as an effective means to characterize such fracture systems [3-4]. The DFN model can construct a fracture network in three-dimensional space that approximates the real geological situation by treating each fracture as an independent geometric entity and assigning it geometric parameters such as length, opening, inclination, and azimuth and corresponding probability distribution, thus laying a solid foundation for in-depth study of the physical behavior of fracture systems [5].

Although the DFN model can finely describe the geometry of fracture systems, how to quantitatively describe the complexity of their overall structure is still a difficulty and frontier of current research. Fractal geometry theory, proposed by Mandelbrot, provides a powerful mathematical tool for describing complex structures with self-similarity that are widely present in nature [6]. A large number of cores, outcrops, and imaging logging data confirm that the distribution of natural fractures has good statistical fractal characteristics [7]. As the core parameter of fractal theory, fractal dimension can effectively quantify the development density, space filling capacity and structural complexity of fracture networks. Therefore, the introduction of fractal dimensions into DFN modeling provides a concise and universal metric for quantifying and comparing different fracture network structures at the system level.

The geological and engineering significance of fracture systems lies in their control of reservoir seepage capacity. Permeability is a key parameter that characterizes the ability of a fluid to permeate in porous media. For shale reservoirs with a large number of discrete fractures, the equivalent permeability is strongly dependent on the topology, connectivity and conductivity of each fracture network. Numerical simulation methods, such as flow simulations based on finite element or finite volume methods, have become standard practices for calculating equivalent permeability tensors for complex DFN models [8]. However, the current research focuses on the influence of fracture density, connectivity and other parameters on permeability, and the fractal dimension that can comprehensively reflect the complexity of the network is not sufficiently studied to systematically correlate with the equivalent permeability obtained through rigorous numerical simulation, and the quantitative relationship model between the two still needs to be revealed in depth.

Based on the above research status, this paper aims to systematically study the permeability characteristics of natural fracture networks in shale reservoirs by combining fractal geometry theory with discrete fracture network modeling methods. The study will first generate a series of two-dimensional DFN models with different fractal dimensions based on the Monte Carlo stochastic simulation method. Finally, through systematic parameter analysis, the intrinsic association between fractal dimension and equivalent permeability is revealed, and a quantitative relationship model between the two is attempted. This study hopes to provide a new theoretical basis and numerical simulation tools for the quantitative prediction of fracturing "sweet spots" in shale reservoirs, the optimal design of fracturing schemes, and the accurate evaluation of development effects.

2. THEORETICAL MODEL

2.1. Fractal Theory

According to Mandelbrot's fractal theory [6], when the distribution of objects exhibits fractal characteristics, the following laws are satisfied between their number and the measurement scale:

$$\lg(N_n) = \lg(C) - D \lg(r_n) \quad (1)$$

In the above equation, N_n is the number of feature objects, m is the linearity of the feature object, C is the proportional constant, and D is the fractal dimension.

There are many methods for calculating fractal dimensions, and the box dimension method has low requirements for the self-similarity of structures, which is suitable for statistical self-similar fracture networks (such as natural fractures). Therefore, the box dimension method is used to calculate the fractal dimension of natural fractures in continental shale reservoirs. The method covers the measured area with square boxes with different side lengths, and then counts the number of boxes covered by the fractures, and finally obtains the logarithmic relationship fitting curve of different side lengths and the number of boxes, and the absolute value of the curve slope is the fractal dimension of the measured fracture network

2.2. Discrete Fracture Network Model

For the random natural fracture system commonly developed in continental shale reservoirs, its accurate characterization needs to cover two levels: the geometric parameters of a single fracture (length, opening, dip, etc) and the spatial distribution characteristics of the fracture network, which together control the seepage characteristics of the reservoir. In this study, a stochastic modeling algorithm was used to generate a discrete fracture network in a two-dimensional region of $100 \text{ m} \times 100 \text{ m}$, and then its equivalent permeability was solved to quantify the influence of fracture structure on seepage.

2.2.1. Natural fracture length

The length and angle of natural fractures in heterogeneous tight reservoir rocks usually obey certain distribution characteristics. Previous studies have found that the length of the natural fracture network of shale reservoirs follows the power law distribution, and the probability density function is:

$$P(l) = \frac{1-\alpha}{l_{\max}^{1-\alpha} - l_{\min}^{1-\alpha}} \cdot l^{-\alpha} \quad (l_{\min} \leq l \leq l_{\max}) \quad (2)$$

In the above equation, the α is the power law exponent, and the larger the exponent, the fewer long fractures.

2.2.2. Location and direction of natural fractures

In this paper, when randomly generating natural fractures, the position of natural fractures is represented by the center point of the fracture (x, y), and the probability density function describing the fracture distribution is obtained through the Poisson distribution:

$$P(x = \eta) = \frac{\lambda^\eta}{\eta!} e^{-\lambda} \quad (3)$$

In the above equation, n is the number of fractures, λ which is the expected value, that is, the average number of fractures per unit area or time.

The directional distribution of fractures can be characterized by Fisher distribution and normal distribution, which are often used to simulate the spatial distribution of natural fracture occurrence (inclination and dip), and characterize the distribution of fracture direction according to the degree of dispersion from the main distribution direction, which is more in line with the distribution of natural fracture direction in shale reservoirs. Therefore, the random generation method of Fisher distribution is used to generate natural fracture azimuth data, and the distribution formula is:

$$f(\theta) = \frac{k \sin \theta e^{k \cos \theta}}{e^k - e^{-k}} \quad (4)$$

In the above equation, θ it is the angular deviation from the main direction; k is the diffusion coefficient ($k > 0$, the larger the value, the more concentrated the direction).

2.3. Seepage model

As shown in Fig. 1, the fluid flow of the model is based on Darcy's law of seepage, the upper and lower boundaries are closed boundaries, there is no fluid flow, and there is a pressure on the left and right sides of the model, so that the model is in a stable differential pressure state, and the fluid flows from left to right.

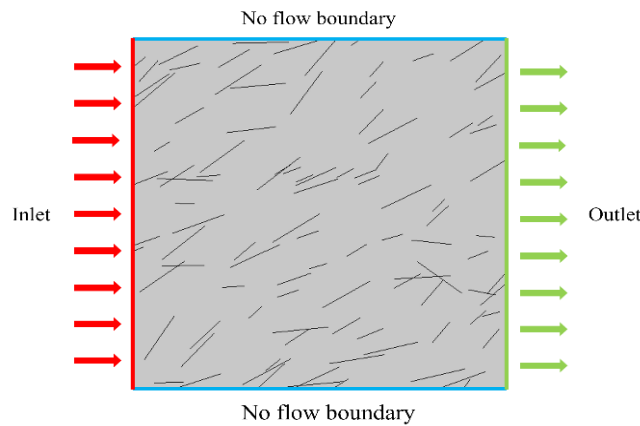


Figure 1. Numerical simulation model

Table 1. Numerical simulation parameters

parameter	symbol	numeric value
matrix porosity	ϕ	0.07
Fluid density kg/m^3	ρ_f	850
Fluid viscosity $\text{Pa} \cdot \text{s}$	M	1×10^{-2}
Inlet pressure Pa	P_1	2×10^6
Outlet pressure Pa	P_2	0
Matrix permeability m^2	km	1.5×10^{-17}
Initial pressure Pa	P_0	2.5×10^7
Roughness coefficient	f_f	1.6

3. ANALYSIS OF THE INFLUENCE OF FRACTAL DIMENSION ON THE PERMEABILITY OF SHALE RESERVOIRS

Fractal dimension is an important parameter to characterize the development degree of natural fracture network in shale reservoirs. The angle between the main direction angle of the fracture and the permeability direction is a (the permeability direction of the reservoir in this paper is the model horizontal from left to right). In this paper, the parameters $\alpha=45^\circ$, $k=10$, $l_{\min}=10\text{m}$, $\alpha=2$ are set as shown in Fig. 18, and Fig. 2(d)(e)(f) is the binarized image of the discrete fracture network corresponding to Fig. 2(a)(b)(c), which is convenient for calculating the fractal dimension D . As shown in Fig. 3(a)(b), when the number of fractures is taken within the interval $[100,200]$, the fractal dimension increases with the increase of the number of fractures, and the two are linearly positively

correlated. In addition, there is a linear positive correlation between the fractal dimension corresponding to different number of fractures and the equivalent permeability. As shown in Fig. 3(c)(d), when the maximum length of the fracture is taken within the interval [20,40]m, the fractal dimension increases with the increase of the maximum length of the fracture, and the two show a linear positive correlation. There is also a linear positive correlation between the fractal dimension corresponding to the maximum length of different fractures and the equivalent permeability. From the above, it can be concluded that the fractal dimension can well characterize the development degree of the fracture network, and has a good positive correlation with the fracture geometry parameters and the equivalent permeability of the reservoir.

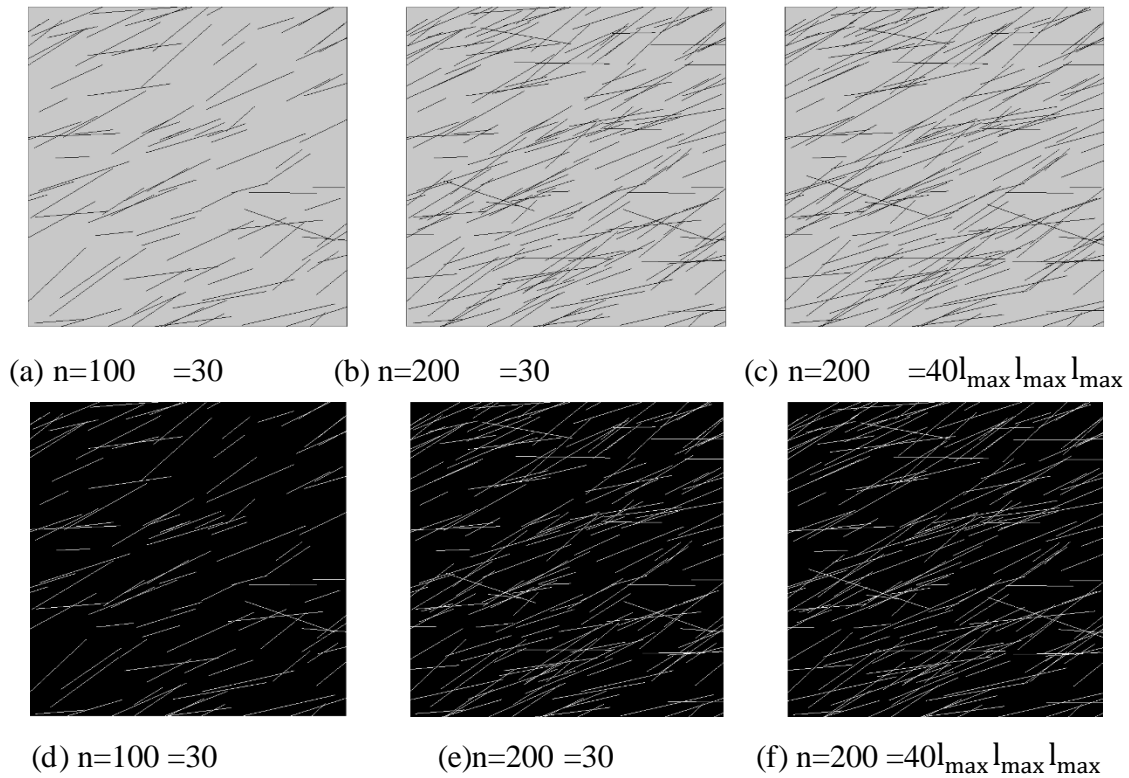
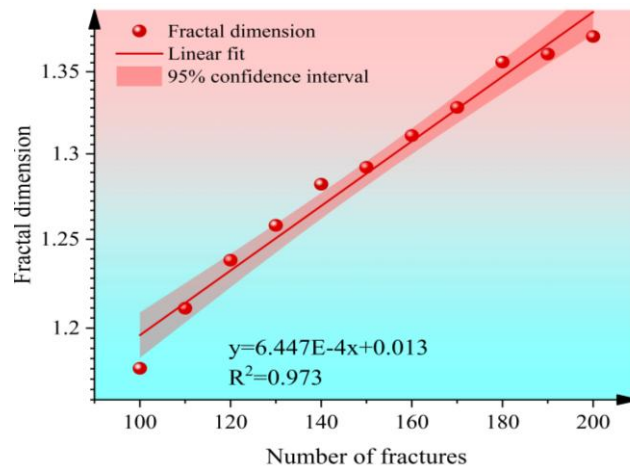
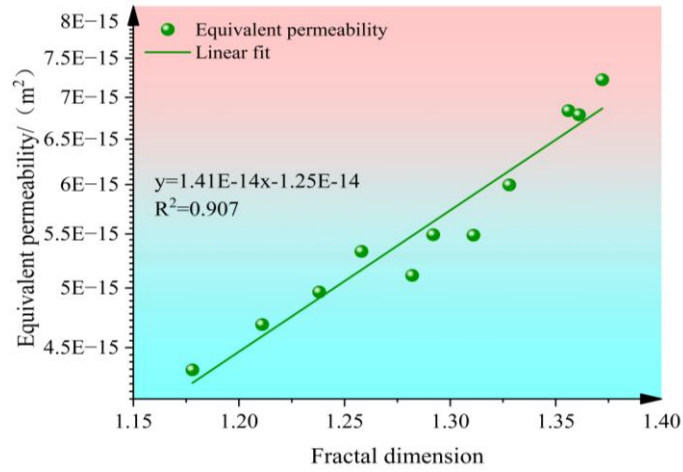


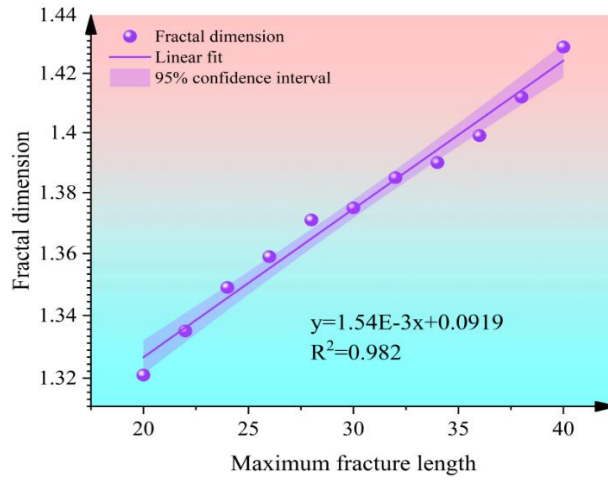
Figure 2. Discrete fracture network and binarized image



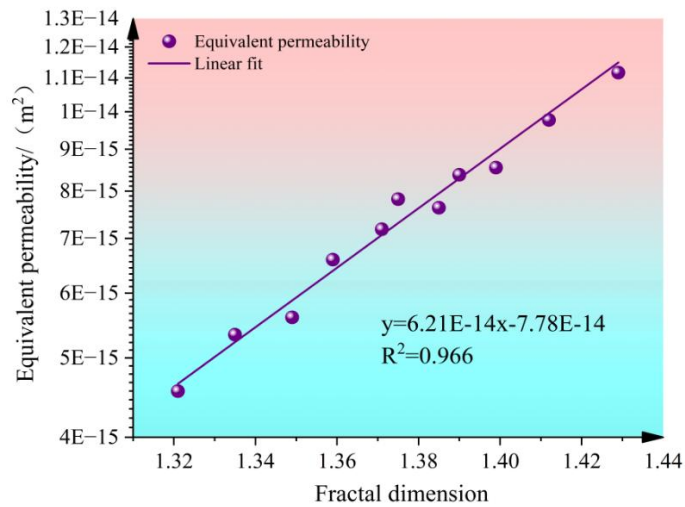
(1) The relationship between the number of fractures and the fractal dimension



(2) Relationship between fractal dimensions of different number of fractures and equivalent permeability



(3) The relationship between the maximum length of the fracture and the fractal dimension



(4) Relationship between fractal dimensions of maximum fracture lengths and equivalent permeability

Figure 3. Effect of fractal dimension on equivalent permeability

4. CONCLUSION

According to the power law distribution of natural fracture length of shale reservoirs, the Fisher distribution of fracture angle and the Poisson distribution of fracture position, a two-dimensional discrete fracture network is randomly generated by Monte Carlo. A two-dimensional discrete fracture network model that conforms to the geometric distribution characteristics of the natural fracture network of shale reservoirs is established. The seepage model of shale reservoir with mesoscale natural fracture distribution was simulated by finite element method, and the influence of fractal dimension on the equivalent permeability of shale reservoir was analyzed. The results show that the fractal dimension can well characterize the development degree of the fracture network, and has a good linear positive correlation with the fracture geometry parameters and the equivalent permeability of the reservoir. It has important guiding significance for the determination of on-site hydraulic fracturing schemes and parameters and the improvement of sustainable high yield of shale oil.

REFERENCES

- [1] Zhou Q, Jin Z, Yang G, et al. Shale oil exploration and production in the U. S.: Status and outlook [J]. *Oil & Gas Geology*, 2019, 40(3): 469-477.
- [2] Li G, Zhu R. Progress, challenges and key issues of unconventional oil and gas development of CNPC [J]. *China Petroleum Exploration*, 2020, 25(2): 1-13.
- [3] Hao Z, J. J. S. Complex fracture network simulation and optimization in naturally fractured shale reservoir based on modified neural network algorithm [J]. *Journal of Natural Gas Science and Engineering*, 2021, 95.
- [4] Xiaolin W, Liyuan Y, Hanqing Y. Correlations between Geometric Properties and Permeability of 2D Fracture Networks [J]. *Advances in Civil Engineering*, 2021, 2021.
- [5] Panton B, Elmo D, Stead D, et al. A discrete fracture network approach for the design of rock foundation anchorage [J]. *Mining Technology*, 2015, 124(3):150-162.
- [6] Mandelbrot B B. *The Fractal Geometry of Nature*. New York: W. H. Freeman, 1982.
- [7] Ling C, Liu B, Zhang C, et al. Fractal Characteristics of Overburden Rock Fractures and Their Impact on Ground Fissures in Longwall Coal Mining [J]. *Fractal and Fractional*, 2023, 7(10):
- [8] Lang P S, Paluszny A, Zimmerman R W. Permeability tensor of three-dimensional fractured porous rock and a comparison to trace map predictions. *Journal of Geophysical Research: Solid Earth*, 2014, 119(8): 6288-6307.