

Preset Performance Reverse Step Attitude Controller for Four-Rotor Unmanned Aerial Vehicles

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ABSTRACT

Aiming at the attitude control problem in flight control of four-rotor UAV, a control scheme based on the preset performance backstepping controller (ESO-NPPCBSC) for attitude Angle of four-rotor UAV with extended state observer is proposed. A preset performance function with specified time convergence is designed to constrain the transient performance and steady-state performance of the tracking error. Compared with the traditional scheme, the preset performance function can make the tracking error of the controlled system converge to the preset precision range within the specified time, and the convergence rate can be adjusted flexibly and the error conversion function can be used to transform the tracking error without constraints. The attitude Angle tracking error satisfies the preset performance condition by controlling the conversion error. The control law of attitude Angle of four-rotor UAV is designed based on reverse step method, which solves the problem of low control accuracy of four-rotor UAV under the condition of uncertain interference.

KEYWORDS

UAV Attitude Angle control; Extended state observer; Preset performance control; Backstepping control

1. INTRODUCTION

In recent years, the design, development and application of four-rotor Unmanned Aerial Vehicles (UAVs) have attracted the attention of scholars around the world [1]. Compared with fixed wing UAVs, four-rotor uAVs have advantages such as simple structure, low manufacturing cost, easy maintenance, light fuselage and easy manipulation. In addition, four-rotor UAVs also have excellent maneuverability, environmental durability, hover, vertical takeoff and landing [2], which makes four-rotor uAVs widely used in military, agricultural, commercial, civil and scientific research and other related fields [3].

Four-rotor UAV is a control system with multi-input multi-output (MIMO), high nonlinear and underdriven internal parameter coupling degree [4]. At the same time, the dynamics modeling of quadrotor UAV contains many unknown interference factors, such as model parameter change, model mismatch, external environmental interference, etc. [5]. Therefore, it is of high practical value and practical significance to design a reliable flight control algorithm to solve flight control problems related to poor anti-interference ability and low control accuracy of quadrotor UAV in actual operation [6].

Sun Zidong [7] et al. designed an improved backstepping control algorithm combining fuzzy control and dynamic surface control, which not only approached the unknown function appearing in the system, but solved the complexity explosion problem existing in the backstepping design process. By introducing a performance function, the tracking error of the controlled system converges in a finite

time. Dalwadi Nihal [8] et al. designed a backstepping controller based on nonlinear observer. The backstepping controller can achieve good trajectory tracking when the nominal altitude drops, and improve the control quality of the quadrotor UAV during the transition from hovering state to horizontal flight state. Ma TieNan [9] et al. designed a single nonlinear extended state observer by using the feedback linearization technique to reduce the complexity of observer parameter adjustment. A preset performance control method of the four-rotor UAV under the attitude and input saturation constraints is designed to improve the stability of the controlled system. Davood Allahverd [10] et al. designed an inverse integral sliding mode control (BISMC) based on iterative learning control, which improved the problems existing in the nonlinear translational and rotational dynamics of the four-rotor UAV and enabled the controlled system to have high transient and steady-state performance. RodriguezAbreo Omar [11] and others through a based on genetic algorithm is designed to step of parameters self-tuning control algorithm, the algorithm is based on four rotor uav autonomous trajectory tracking to backstepping controller gain, implements the gain of different magnitude error self-tuning. Chen Yanjie [12] et al. designed an adaptive sliding mode interference observer (ASMDO) and designed a finite time preset performance control method on the basis of ASMDO to realize effective control of UAV manipulator with uncertainty and external interference. The system converges in finite time and has specified transient and steady-state properties.

Based on the inspiration of the above literature, this paper aims to design a preset performance control scheme whose convergence time can be artificially set for small quadrotor UAV as the research object, and verifies the control performance of this scheme in the presence of incident interference through numerical simulation. The main contributions are as follows:

An extended observer is designed to dynamically compensate the complex interference. A preset performance function whose convergence time can be artificially specified is proposed. Based on this, a preset performance control law based on backward step method is designed, which can make the tracking error of the controlled system converge within the specified time and the convergence speed can be adjusted according to the requirements.

2. SYSTEM MODELING AND PROBLEM DESCRIPTION

Driven by four propellers, four-rotor UAV can realize six degrees of freedom movement in three-dimensional space. Therefore, the mathematical model of four-rotor UAV is a typical multi-input, multi-output and strongly coupled underactuated controlled system. In this paper, Newton-Euler formula is adopted to deduce the attitude dynamics of the four-rotor UAV [13]. The following hypothesis is given:

- (1) Quadrotor UAV is a uniform and symmetrical rigid body;
- (2) The mass and moment of inertia of the four-rotor UAV do not change;
- (3) The geometric center of the quadrotor UAV coincides with its own center;
- (4) Quadrotor UAV only accepts gravity and propeller pull.

According to Newton-Euler formula, the attitude dynamics equation of the four-rotor UAV is established as follows:

$$\begin{cases} \ddot{\phi} = \frac{I_y - I_z}{I_x} \dot{\theta} \dot{\psi} + \frac{U_\phi}{I_x} + d_\phi \\ \ddot{\theta} = \frac{I_z - I_x}{I_y} \dot{\phi} \dot{\psi} + \frac{U_\theta}{I_y} + d_\theta \\ \ddot{\psi} = \frac{I_x - I_y}{I_z} \dot{\phi} \dot{\theta} + \frac{U_\psi}{I_z} + d_\psi \end{cases} \quad (1)$$

Where, ϕ , θ and ψ represent the roll Angle, pitch Angle and yaw Angle of the attitude Angle system of the four-rotor UAV respectively. d_ϕ , d_θ and d_ψ respectively represents the interference of roll Angle system, the interference of pitch Angle system and Interference of yaw Angle system. I_x , I_y and I_z respectively represent the moment of inertia of the four-rotor UAV with respect to the x axis, the moment of inertia with respect to the y axis and the moment of inertia with respect to the z axis. U_ϕ , U_θ and U_ψ respectively represent the control input of the three attitude angles of the four-rotor UAV.

In order to facilitate the subsequent controller design, the original four-rotor UAV attitude Angle system dynamics model needs to be converted into n order integral series dynamics model with strict feedback form. In order to simplify the design process, a symbol representation method as shown in Equation (2) is proposed before model transformation:

$$\begin{aligned}
X_1 &= [\phi \quad \theta \quad \psi]^T \\
X_2 &= [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T \\
d_2 &= [d_\phi \quad d_\theta \quad d_\psi]^T \\
\Theta_\phi &= I_x^{-1}(I_y - I_z)\dot{\theta}\dot{\psi} \\
\Theta_\theta &= I_y^{-1}(I_z - I_x)\dot{\phi}\dot{\psi} \\
\Theta_\psi &= I_z^{-1}(I_x - I_y)\dot{\phi}\dot{\theta} \\
f(X_2) &= [\Theta_\phi \quad \Theta_\theta \quad \Theta_\psi]^T \\
K_2 U_2 &= [I_x^{-1}U_\phi \quad I_y^{-1}U_\theta \quad I_z^{-1}U_\psi]^T
\end{aligned} \tag{2}$$

Where, X_1 , X_2 and $K_2 U_2$ represents the control input of attitude Angle, attitude angular velocity and yaw Angle of the four-rotor UAV respectively. Based on Formula (1) and combined with Formula (2), the state space equation of the integral series attitude Angle system of the four-rotor UAV can be described as:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = f(X_2) + K_2 U_2 + d_2 \end{cases} \tag{3}$$

Where, d_2 represents the sum of external airflow disturbance, unmodeled system dynamics and model uncertainty influence disturbances received by the four-rotor UAV.

Due to the underactuation characteristic of the four-rotor UAV, the designed controller cannot track the 6 degrees of freedom at the same time. The controller designed in this paper can ensure no attitude control of UAV. The human-machine attitude Angle stably tracks the expected input $(\phi_d \quad \theta_d \quad \psi_d)^T$.

3. DEFAULT PERFORMANCE FUNCTION AND ERROR TRANSFORM

Presetting performance means that the transient and steady-state performance of the system is limited within the range of the pre-designed performance function by designing a conversion function, so as to improve the transient and steady-state performance [14]. The default performance method can be expressed as the following inequality:

$$\begin{cases} -M_1 \rho(t) < e(t) < \rho(t), & (e(0) \geq 0) \\ -\rho(t) < e(t) < M_2 \rho(t), & (e(0) < 0) \end{cases} \tag{4}$$

Where, $0 \leq M \leq 1$ is design parameter; $e(t)$ is a constrained error signal; $\rho(t)$ is a predesigned performance function that meets the following definitions:

- (1) $\rho(t)$ Is a smooth bounded function, $\rho(t)$ is decreasing monotone and $\rho(t) \in R^+ \rightarrow R^+$;
- (2) For $\varepsilon(t)$, have $|\varepsilon(t)| < \rho(t)$ and satisfy $\lim_{t \rightarrow \infty} \rho(t) = \rho(\infty) > 0, t \in [0, \infty)$.

Attitude Angle system for quadrotor UAV, The performance function $\rho(t)$ that can converge in a finite time is selected as follows:

$$\rho(t) = \begin{cases} [(T-t)/t]^{\frac{1}{1-r}} (\rho(0) - \rho(\infty)) + \rho(\infty), & 0 \leq t \leq T \\ \rho(\infty), & t > T \end{cases} \quad (5)$$

Where, $r \in (0,1)$, $\rho(0)$, $\rho(\infty)$ and T is the adjustment parameter of the performance profile at the specified time, By adjusting the value T of the time, you can make the time specified by the performance function $\rho(t)$ converge.

In order to make tracking errors converge within the preset performance, inequality constraints should be converted into equality constraints, and constrained errors into unconstrained errors [15]. This paper introduces the error transform shown in Equation (6):

$$e(t) = \rho(t)s(z(t)) \quad (6)$$

Where, $s(z(t))$ is the error conversion function of the tracking error, $z(t)$ is conversion error. Select the error conversion function shown in Equation (7):

$$z(\varepsilon(t)) = \frac{e^{\varepsilon(t)} - e^{\varepsilon(t)}}{e^{\varepsilon(t)} + e^{\varepsilon(t)}} \quad (7)$$

By inverting equation (7), the expression of conversion error $\varepsilon(t)$ can be obtained as follows:

$$\varepsilon(t) = z^{-1}\left(\frac{e(t)}{\rho(t)}\right) = \frac{1}{2} \ln \left(\frac{M_1 + e^{\varepsilon(t)}/\rho(t)}{M_2 - e^{\varepsilon(t)}/\rho(t)} \right) \quad (8)$$

According to different control requirements, reasonable error conversion function $s(z(t))$ and error conversion function $z(t)$ are designed, the transient performance and steady-state performance of the UAV attitude Angle system can be set in advance so that the tracking error of the system is always limited within the error $\varepsilon(t)$ boundary of the preset performance function. It's important to note that, when $e(0)=0$, the value of M_1, M_2 cannot be 0, Otherwise, the value $z(t)$ is infinity. Therefore, the design parameters M_1, M_2 need to be satisfied $0 \leq M_1, M_2 \leq 1$.

By differentiating equation (8), the derivative of conversion error $\dot{\varepsilon}(t)$ can be expressed as follows:

$$\dot{\varepsilon}(t) = \frac{\partial z^{-1}}{\partial (e(t)/\rho(t))} \frac{1}{\rho(t)} \left(\dot{e}(t) - e(t) \frac{\dot{\rho}(t)}{\rho(t)} \right) \quad (9)$$

For equation (9), make $r(t) = \frac{\partial z^{-1}}{\partial(e(t)/\rho(t))} \frac{1}{\rho(t)}$, $v(t) = \frac{\dot{\rho}(t)}{\rho(t)}$, Then formula (9) can be rewritten as:

$$\dot{\varepsilon}(t) = r(\dot{e}(t) - e(t)v(t)) \quad (10)$$

Therefore, as long as the conversion error $\varepsilon(t)$ is bounded, the tracking error $e(t)$ can meet the requirements of the preset performance.

4. PRESET PERFORMANCE BACKSTEPPING CONTROLLER DESIGN

On the basis of given virtual control quantity, attitude closed-loop control can be regarded as a linear control. By using the idea of error conversion, the tracking error is guaranteed to be in the preset boundary function, and the extended state observer is designed to estimate the sum of the uncertain influence interference, and then the adaptive backstepping controller is designed.

Step 1: Conversion error e_1 is introduced into the attitude Angle tracking error $z_1(t)$ of the four-rotor UAV, then the tracking error equation of the system can be described as:

$$e_1(t) = z_1(t) = p - p_r \quad (11)$$

Where, p and p_r represents the actual attitude Angle and the expected attitude Angle respectively. Based on the rolling Angle channel tracking error equation of Equation (11), the Lyapunov function is constructed as follows:

$$V_1(e_1) = \frac{1}{2} e_1^2 \quad (12)$$

The derivative of equation (12) is obtained:

$$\dot{V}_1(e_1) = e_1 \dot{e}_1 = e_1(\dot{p} - \dot{p}_r) = e_1 r_1(\bar{U}_2 - \dot{p}_r - e_1 v) \quad (13)$$

Where, $\bar{U}_2$ represents the virtual control input of the attitude Angle system of the four-rotor UAV. In order to ensure the tracking error of attitude Angle is stable, A must be satisfied $\dot{V}_1(e_1) \leq 0$. Based on Equation (13), the virtual control law of the stable attitude Angle system satisfying Lyapunov's sense is designed as follows:

$$\bar{U}_2(t) = -k_1 r_1^{-1}(t) e_1(t) + \dot{p}_r(t) + e_1(t) v_1(t) \quad (14)$$

If equation (14) is substituted into Equation (13), equation (13) can be rewritten as:

$$\dot{V}_1(e_1) = -k_1 e_1^2 \quad (15)$$

Step 2: The error equation for defining the tracking speed of the attitude Angle system of the four-rotor UAV is as follows:

$$e_2 = \dot{p} - \dot{p}_r = \dot{p} - \bar{U}_2 \quad (16)$$

Where, $\dot{p}$, $\dot{p}_r$ Respectively represent the actual attitude Angle tracking speed. The degree and desired attitude Angle track the velocity. Aiming at the error equation of attitude Angle tracking velocity, the Lyapunov function is constructed as follows:

$$V_2(e_2) = \frac{1}{2}e_2^2 \quad (17)$$

The derivative of Equation (17) is obtained:

$$\begin{aligned} \dot{V}_2(e_2) &= e_2 \dot{e}_2 \\ &= e_2(\ddot{p} - \dot{U}_2) \\ &= e_2(K_2 U_2 + d_2 - \dot{U}_2) \end{aligned} \quad (18)$$

Where, U_2 represents the actual control input of the attitude Angle system. In order to ensure that the tracking velocity error of attitude Angle e_2 is stable, it needs to meet $\dot{V}_2(e_2) \leq 0$. Then the actual control law of the stable attitude Angle system satisfying Lyapunov's sense is designed as follow

$$U_2 = K_2^{-1} \left(-k_2(\dot{p} - \dot{p}_r) - d_2 + \dot{U}_2 \right) \quad (19)$$

If equation (19) is substituted into Equation (18), equation (18) can be rewritten as:

$$\dot{V}_2(e_2) = -k_2 e_2^2 \quad (20)$$

For Equation (19), it is necessary to design an extended state observer for the velocity term $\dot{p}$ and interference term d_2 in it for online observation. Parameter uncertainties and unknowns in the system state of the controlled system. The perturbation is merged into lumped interference X_d , where, $X_d = f(X_2) + d_2$ and the lumped interference X_d is extended into a new state variable X_3 of the system (3). $\dot{X}_3 = h$ is defined as the derivative of lumped interference, assuming X_3 that it is continuous and differentiable and satisfied $\|X_3\| \leq M$, where $M > 0$. Finally, The augmented state-space equation of the attitude Angle system of a four-rotor UAV can be described as:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = K_2 U_2 + X_3 \\ \dot{X}_3 = h \end{cases} \quad (21)$$

$X^T = [X_1 \ X_2 \ X_3]^T$ is used to represent formula (4), then formula (4) can be rewritten as:

$$\dot{X}^T = AX^T + U + \Delta \quad (22)$$

The expression of A and U is shown in Equation (23)

$$A = \begin{bmatrix} 0 & I_3 & 0 \\ 0 & 0 & I_3 \\ 0 & 0 & 0 \end{bmatrix}, U = \begin{bmatrix} 0 \\ K_2 U_2 \\ 0 \end{bmatrix} \quad (23)$$

For Equation (23), where the identity matrix I_3 is represented 3×3 , 0 represents the zero matrix of appropriate dimension. Based on the design method of the extended state observer, the extended state observer specific to the attitude Angle system of the four-rotor UAV is constructed to provide the state estimation, angular rate state estimation and lumped interference estimation of the attitude Angle system.

$$\dot{\hat{X}}_h = A_h \hat{X}_h + K_2 U_2 + H(X_1 - \hat{X}_1) \quad (24)$$

Where, the expression of A_h , $\hat{X}_h$ and H is shown in Equation (25):

$$\begin{aligned} A_h &= \begin{bmatrix} 0 & I_3 & 0 \\ 0 & 0 & I_3 \\ 0 & 0 & 0 \end{bmatrix} \\ \hat{X}_h &= [\hat{X}_1^T \quad \hat{X}_2^T \quad \hat{X}_3^T]^T \\ H &= [3w \cdot I_3 \quad 3w^2 \cdot I_3 \quad w^3 \cdot I_3]^T \end{aligned} \quad (25)$$

For equation (25), where, $\hat{X}_1$, $\hat{X}_2$ and $\hat{X}_3$ respectively represent four the estimated value of attitude Angle, attitude Angle rate and lumped interference of the rotor UAV I_3 are represented by the unit matrix 3×3 , where 0 represents the zero matrix of appropriate dimension, H is the observer gain matrix, w is the bandwidth of the linear expanded state observer and satisfies $w > 0$. The bandwidth of the linear w expanded state observer is the only adjustment parameter.

For Equation (19), due to the control system of attitude Angle system

An extended state observer is introduced to observe the state variables X_2 and X_3 lumped interference respectively. Therefore, based on Equation (19) and combined with Equation (24), the actual control law of the attitude Angle system can be rewritten as:

$$U_2 = K_1^{-1}(-k_2(\hat{X}_2 + \bar{U}_2) - \hat{X}_3 + \dot{\bar{U}}_2) \quad (26)$$

Where, $\hat{X}_2$ and $\hat{X}_3$ respectively represent the estimated values of system state variables X_2 and lumped interference $\hat{X}_3$.

5. SIMULATION RESULTS AND ANALYSIS

In order to verify the effectiveness of the controller designed in this paper, the ESO-NPPCBSC controller was simulated based on Simulink environment and compared with the proportional Integral-differential controller (PID) and the reverse step controller (ESO-BSC) with no default performance and extended state observer.

After repeated debugging, parameters of the preset performance function of the attitude Angle system of the four-rotor UAV are shown in Table 1:

Table 1. Preset performance parameter

Parameter	$r \in (0,1)$	$\rho(0)$	$\rho(\infty)$		T
Numerical value	0.6	2	0.02		0.2

For the state space model of the attitude Angle system of the four-rotor UAV, considering the output limitation of the attitude Angle of the four-rotor UAV in the trim state, the maximum change amplitude of the attitude Angle is defined as $\pi/18rad$. The reference control inputs of attitude Angle, ϕ_d , θ_d , ψ_d and ψ_d are selected as the step signals of amplitude, $0.1rad$, $0.12rad$ and $0.15rad$ respectively, and the external interference of the attitude Angle system is selected as $d_2 = \sin t$. The comparison of attitude Angle tracking response curves of ESO-NPPCBSC controller and PID controller is shown in Figure 1 to Figure 3:

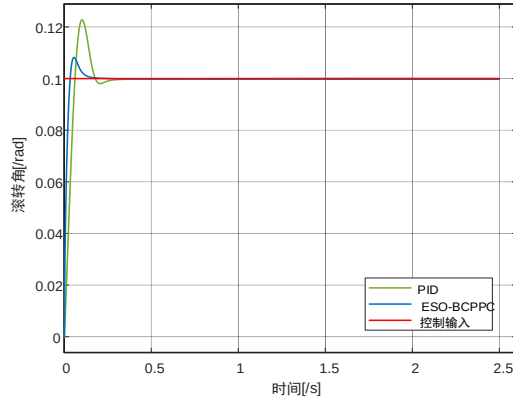


Figure 1. Roll channel tracks response curve

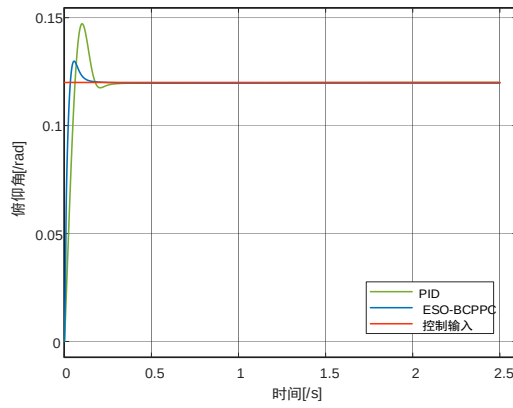


Figure 2. Pitch channel tracks response curve

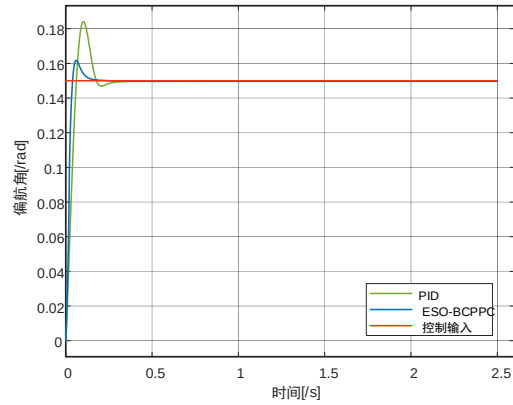


Figure 3. Yaw channel tracks response curve

The comparison of attitude Angle tracking response curves between the ESO-NPPCBSC controller and the ESO-BSC controller is shown in Figure 4 to Figure 6:

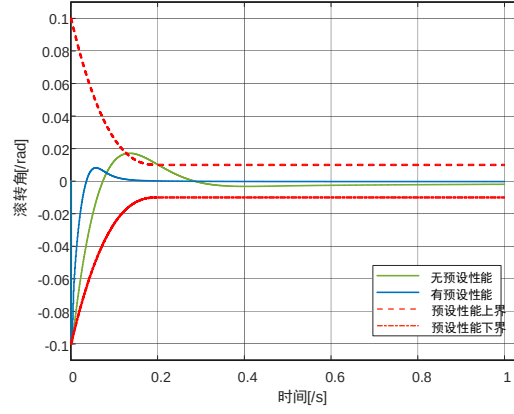


Figure 4. Comparison of roll Angle ESO-NPPCBSC and ESO-BSC

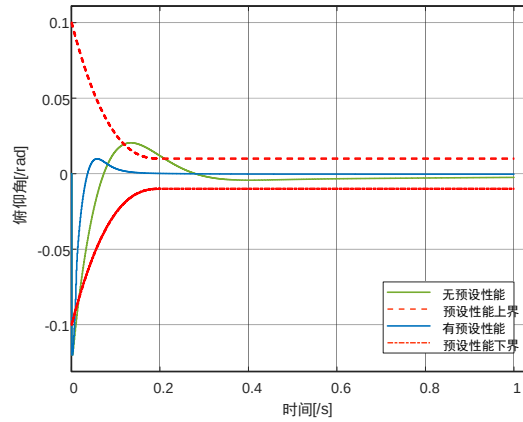


Figure 5. Comparison of pitch Angle ESO-NPPCBSC and ESO-BSC

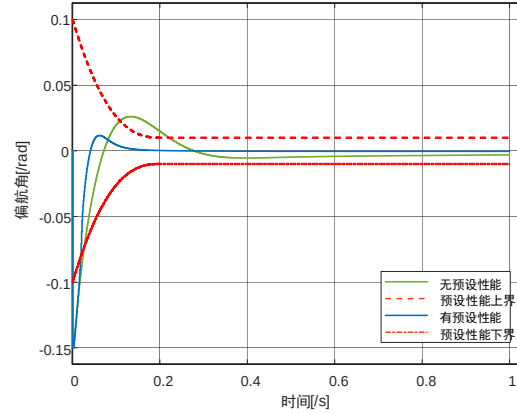


Figure 6. Comparison of yaw Angle ESO-NPPCBSC and ESO-BSC

The experimental results show that, compared with PID controller, the attitude Angle tracking response curve of the four-rotor UAV based on ESO-NPPCBSC controller has faster response time, smaller overshoot and shorter adjustment time, and the convergence time can be manually specified by adjusting preset performance parameters. Compared with the ESO-BSC without preset performance, the attitude Angle tracking response curve of the four-rotor UAV using the ESO-NPPCBSC controller can be limited within the envelope of preset performance, and the static difference after reaching the steady state is relatively small.

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