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# Comparison of Two Under-sampling Algorithms, Non-uniform Random Sampling and Variable Density Sampling in CS-MRI

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## ABSTRACT

This paper is to compare the two undersampling modes NURS and VDRS in Compressed Sensing MRI. In this paper, brain images are reconstructed using Matlab using NURS and VDRS respectively. After analyzing the sampling pattern and probability distribution function of the two undersampling modes and outputting different sampled images to select the appropriate soft threshold, the soft thresholded sampled images are reconstructed by the PCOS algorithm. After evaluating the quality of image reconstruction by utilizing several indexes such as eye observation discrimination, PSNR and SSIM, it is found that VDRS mode is better than NURS mode in this brain image reconstruction application. The experiments show that VDRS mode has less artifacts and higher image quality in brain image reconstruction, while NURS mode has a lot of artifacts, even though most of the noise is removed after noise reduction, but it also reduces the original appearance of the image and the image quality will be degraded as well. This study can provide an idea for future scholars on how to choose the undersampling pattern in CS-MRI applications, and also promote the research on different undersampling algorithms.

## KEYWORDS

Undersampling; Compressed Sensing; MRI; Non-uniform Random Sampling; Variable Density Sampling

## 1. INTRODUCTION

Aneurysms are a type of vascular disease characterized by an abnormal bulging or ballooning in the wall of a blood vessel. They can occur in any blood vessel but are most common in arteries, especially the aorta and brain arteries. Although aneurysms can affect people of any age, they are more prevalent in older adults [1]. For early diagnosis, we need clearer imaging methods, while MRI is one of the most accurate solutions that prevents severe complications. MRI uses strong magnetic fields and radio waves to generate detailed images of the body's internal structures. When placed in a magnetic field, hydrogen atoms in the body align with the field, and radiofrequency pulses then disrupt this alignment, emitting signals that are captured and converted into images by the MRI scanner. However, current limitations restrict its application, such as MRI suffers from long scan times and high risk of motion artifacts.

Traditional researchers employed various methods to maximize the utility of MRI while grappling with its inherent limitations. For instance, researchers like Klaas attempted to accelerate MRI scans by developing parallel imaging techniques such as SENSE (Sensitivity Encoding) [2]. This method used multiple receiver coils to simultaneously capture different sections of the imaging data. However, despite these advancements, the technique was limited by the need for extensive calibration data and the complexity of coil sensitivity maps, which often led to reconstruction artifacts and reduced image quality. Additionally, Jingyi Xie [3] experimented with adaptive acquisition strategies that adjusted the imaging parameters in real-time based on initial scan data. This adaptive approach improved the efficiency of data acquisition and partially mitigated some limitations associated with scan duration. Nevertheless, these methods were still constrained by the computational power available at the time and the complexity of implementing real-time adjustments, which limited their widespread adoption and effectiveness.

To address current pain points, compressed sensing is becoming a new paradigm. Compressed Sensing MRI reconstructs high-quality images from fewer data samples by exploiting the sparsity of medical images in a specific transform domain, such as wavelet or Fourier domains [4]. By acquiring only, a small, random subset of data and applying sophisticated mathematical algorithms, CS-MRI can accurately reconstruct the full image, thus significantly reducing scan times while maintaining image quality. This technique relies on the principles of sparsity, incoherence, and nonlinear reconstruction algorithms to achieve efficient and accurate image reconstruction. By acquiring far fewer data samples than traditional methods, CS-MRI drastically shortens scan times, enhancing patient comfort and reducing the risk of motion artifacts. Techniques like Non-Uniform Random Sampling and Random Sampled k-Space Data enable the collection of minimal random samples to reconstruct high-quality images. Nonetheless, the effectiveness of CS-MRI depends on finely-tuned parameters and optimization processes, such as sparse representation methods and regularization parameters. Selecting these parameters can be complex and may need careful adjustment for different scanning conditions and targets.

However, not many scholars have compared and analyzed the different algorithms in CS-MRI. The differences and advantages and disadvantages between these different algorithms should also be considered as elements in the application. Thus, this paper, will compare the two under-sampling patterns, Non-Uniform under-sampling pattern and Variable Density Under-sampling pattern, in Matlab.

Overall, although its practical application still faces challenges related to computational complexity and optimization of sampling strategies, CS-MRI is still a powerful tool to relieve the burden of improving diagnostic accuracy.

# 2. METHODS

## 2.1. Under-sampling Patterns for Comparing

### 2.1.1. Non-uniform Random Sampling

Non-uniform Random Sampling (NURS) is a sophisticated technique employed across various fields such as signal processing and medical imaging, characterized by selecting samples in a manner that is both random and non-uniformly distributed throughout the data domain [5]. Unlike uniform random sampling, where each sample point has an equal probability of being chosen, NURS varies the probability of selecting a sample point based on a predefined probability distribution function (PDF). This strategic approach ensures that some regions are sampled more densely while others are sampled more sparsely, depending on the specific requirements of the application.

The essence of NURS lies in its dual attributes of randomness and non-uniformity. The randomness inherent in NURS helps to mitigate aliasing artifacts and provides a more comprehensive representation of the data. Simultaneously, the non-uniform aspect allows for targeted sampling, where regions of higher interest or information density receive more sampling attention. This is particularly beneficial in scenarios like medical imaging, where certain parts of the data may hold more critical information for accurate reconstruction. Its random sampling pattern and probability distribution function are shown in Fig. 1.

One of the primary advantages of NURS is its ability to improve data reconstruction quality. By concentrating sampling efforts on more informative regions of the data domain, NURS enhances the fidelity of the reconstructed data. Moreover, this method can reduce the total number of samples required to achieve a desired level of accuracy, which is especially advantageous in contexts where data acquisition is resource-intensive, such as in MRI scans. In MRI, NURS is utilized to sample the k-space data more effectively. Perceptual recognition at much lower levels than required by the Nyquist sampling theorem [6]. This non-uniform sampling strategy optimizes the balance between image quality and acquisition time, enabling rapid imaging.

### 2.1.2. Variable-Density Random Sampling

Variable-Density Random Sampling (VDRS) is an advanced sampling technique widely utilized in fields such as signal processing and medical imaging. Unlike uniform sampling methods, VDRS employs a non-uniform sampling strategy where the sampling density varies across the data domain based on a predefined probability distribution function (PDF). This method allows for denser sampling in regions with higher information content and sparser sampling in less critical regions (see Fig. 2), thereby optimizing the overall sampling process for specific application needs.

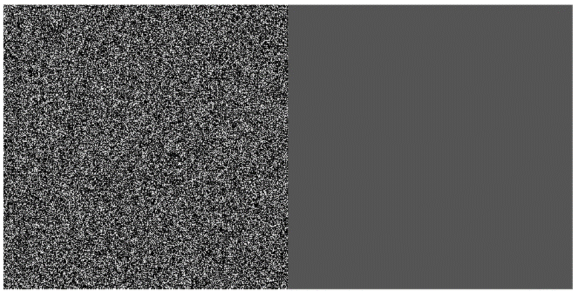

**Figure 1.** Non-uniform Random Sampling pattern and its PDF

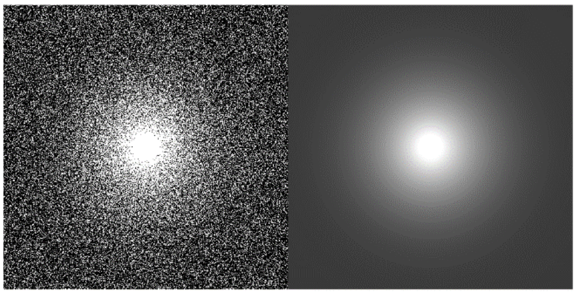

**Figure 2.** Variable-Density Random Sampling pattern and its PDF

The fundamental principle of VDRS is its ability to strategically vary the sampling density to capture the most significant data features more effectively. This approach contrasts with non-uniform random sampling by incorporating a deterministic aspect into the probability distribution, thus enabling more controlled and efficient sampling. The variable density aspect ensures that areas of high interest or complexity are sampled more frequently, improving the accuracy and quality of data reconstruction.

One of the key advantages of VDRS is its potential to enhance the quality of reconstructed data while reducing the total number of samples required. By concentrating sampling efforts in regions with higher informational value, VDRS achieves superior reconstruction fidelity compared to uniform sampling methods. This is particularly beneficial in applications such as MRI, where it is crucial to balance image quality with acquisition speed and resource constraints.

In MRI applications, VDRS is used to sample k-space data with variable density, focusing more sampling points in the central regions of k-space where most of the image's energy and critical low-frequency information reside. The peripheral regions, containing finer high-frequency details, are sampled less densely. This approach ensures that the essential components of the image are captured with higher accuracy, leading to improved image quality and reduced scan times.

Studies such as those by Lustig et al. [7] highlight the effectiveness of VDRS in enhancing image reconstruction in compressed sensing frameworks. These works demonstrate how shows the reconstructed image after under-sampling by Variable density sampling pattern, and the image on the right is the difference image between the two image matrices VDRS leverages the inherent sparsity of the data to achieve high-quality reconstructions with fewer samples, making it a powerful tool in the optimization of data acquisition processes. The ability to tailor the sampling density according to the specific characteristics of the data domain makes VDRS a significant advancement in the field of efficient data sampling and reconstruction.

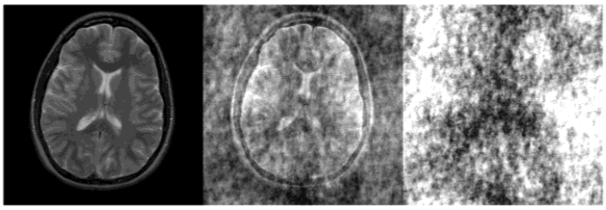

**Figure 3.** The first image on the left is the original brain image, the middle image shows the reconstructed image after under-sampling by Non-Uniform sampling pattern, and the image on the right is the difference image between the two image matrices

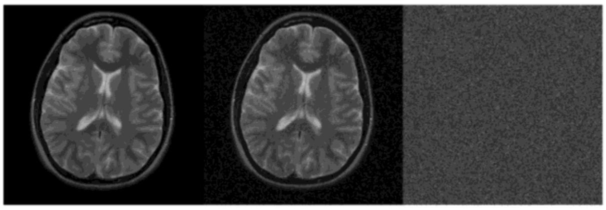

**Figure 4.** The first image on the left is the original brain image, the middle image

## 2.2. Evaluation

In this section, MRI data (containing a picture of the brain and masks and PDFs for the two under-sampling patterns) from a source provided by Michael Lustig is used [8].

### 2.2.1. Random sampling process simulation

First, after loading the provided brain image, a 2D Fourier transform of the image is performed to convert the image from the spatial domain to the frequency domain, as Compressed Sensing utilizes the sparsity of the image in the frequency domain to achieve reconstruction [6].

The image is then multiplied by a mask. The masks used are matrices of corresponding sizes consisting of zeros and ones in the two random under-sampling patterns, which simulates the situation in which only specific locations are sampled during actual data acquisition in medical imaging, i.e., only the frequency components at the sampling locations are retained, and the other locations are set to zero.

The resulting new image matrix is later divided by the corresponding PDF to correct for the effects of non-uniform sampling and to ensure the fairness and accuracy of the reconstruction process. In Compressed Sensing, different frequency components may be sampled with different probabilities, and the PDF describes this sampling probability distribution. Since some frequency components may be sampled with high probability while others are less sampled, direct use of these sampling data may result in inconsistent weighting of different frequency components in the reconstructed image.

Dividing by PDF can normalize the frequency components to correct this inconsistency and make the reconstructed image more accurate. The images sampled with Non-uniform random sampling and Variable Density Sampling counterparts are shown in Fig. 3 and Fig. 4, respectively.

### 2.2.2. Reconstruction from Random Sampled k-Space Data

(1) Wavelet transforms

In order to ensure the sparsity of the image, we introduce here the method of wavelet transform [9]. The wavelet transform can analyze signals simultaneously in both the time and frequency domains, having a multiresolution characteristic. This means it can capture details and the overall structure of the signal at different scales. Meanwhile, the wavelet transform has localization properties in both the time and frequency domains, allowing it to effectively separate different parts of the signal. The effect presented after transforming the Phantom image provided by Matlab with Wavelet transform [10] is shown in Fig. 5.

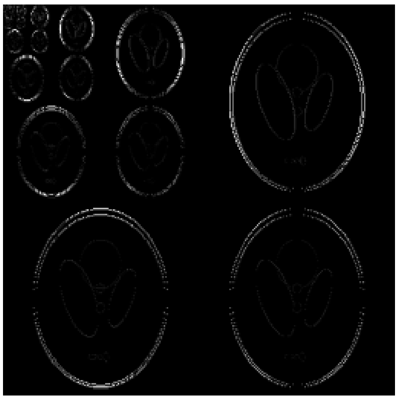

**Figure 5.** The image after wavelet transform processing.

Each set of wavelet coefficients corresponds to a specific scale, representing different frequency bands within the image. The spatial location of each wavelet coefficient within a band indicates its corresponding position in the image space. These coefficients reveal the edges of the image at various resolutions and directions.

The wavelet transform facilitates the sparsification of brain images by concentrating significant image energy into a select subset of coefficients. This process enhances image denoising by allowing for the thresholding of coefficients that predominantly contain noise.

(2) Soft-thresholding

When dealing with sparse signals, we will use the penalized solution of the $\ell^1$ norm ( $||x_1|| = \sum|x_i|$) instead of the Tikhonov regularization of the penalized $\ell^2$norm ($||x_2|| = \sqrt{\sum|x_i|^2}$) [11]. Similarly, when processing sparse images, what is solved is

$$argmin\ \frac{1}{2}||F_u\hat{x} - y||_2^2 + \lambda|\hat{x}|_1$$

$\hat{x}$ is the estimated signal, $F_u\hat{x}$ is the under-sampled Fourier transform of the estimate, and $y$ are the samples of the Fourier transform that we have acquired. When dealing with complex signals, soft thresholding can be written as

$$S(x,\lambda) = \begin{cases} 0 & if\ |x| \leq \lambda \\ \frac{(|x| - \lambda)}{|x|}x & if |x| > \lambda \end{cases}$$

This process enforces data consistency in the frequency domain during image reconstruction of randomly sampled frequency domain data. By enforcing data consistency in the frequency domain, it can ensure that the reconstructed image accurately reflects the real situation, reduces artifacts, and improves image quality.

(3) Reconstruction

Employ a Projection Onto Convex Sets (POCS) algorithm, incorporating an additional step of computing the wavelet transform prior to applying soft-thresholding, followed by performing the inverse wavelet transform subsequent to the soft-thresholding process.

First, a reasonable value of $\lambda$ is known by checking the threshold value. In this step, it is necessary to identify a significant number of coefficients that are below $\lambda$, but not all of them are below $\lambda$. The actual signal in the wavelet transform domain, especially in the low-frequency components (i.e., large-scale wavelet coefficients), concentrates more energy. These coefficients are usually larger because the main structure and features of the signal are captured. Due to its high-frequency nature, noise usually has small coefficients and is widely distributed in the wavelet transform domain. The energy of the noise is spread across multiple wavelet coefficients, making each coefficient's value smaller. Set coefficients below a certain threshold to zero and only keep those above the threshold. Fig. 6 and Fig. 7 show the case of Non-Uniform random sampling and Variable Density sampling for different values of $\lambda$, both from %2.5 to 20%, respectively.

In this experiment, the image reconstruction levels of two different under-sampling modes with different thresholds from 2.5% to 20% were calculated in steps of 2.5%. PSNR and SSIM metrics were used as a criterion for evaluation. Although there are some differences between the two metrics PSNR and SSIM in evaluating the level of image reconstruction [12], the differences are not significant here.

The results obtained during image reconstruction using Variable Density sampling pattern are shown in Fig. 8 and Table 1.

In the process of image reconstruction using Non-Uniform random sampling pattern, in order to speed up the convergence, so a relatively large $\lambda$ is first used for iteration. After every 15 iterations, the value of $\lambda$ is allowed to decrease by half. Use 90 iterations. The results obtained are shown in Fig. 9 and Table 2.

## 3. RESULTS AND DISCUSSION

In Fig. 3, we can see that the image reconstructed directly by Fourier transform and inverse transform when Non-Uniform random sampling is used has serious aliasing. But aliasing is not the same as white random noise. This typically happens when the sampling rate is below the Nyquist rate, which is twice the highest frequency present in the signal. In imaging, aliasing can cause artifacts such as jagged edges [13] and incorrect reconstructions. However image reconstruction utilizing Variable Density sampling is much smoother. This can be concluded by comparing Fig. 4 with Fig. 3.

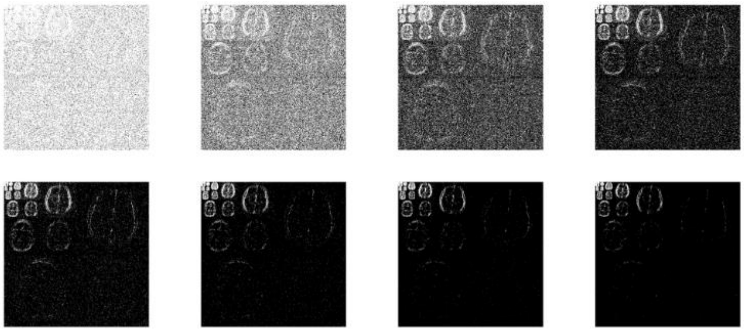

**Figure 6.** From the top left to the bottom right are the wavelet transforms at the soft threshold settings of λ = 2.5%, 5%, 7.5%, 10%, 12.5%, 15%, 17.5%, and 20% in the Non-Uniform random sampling case.

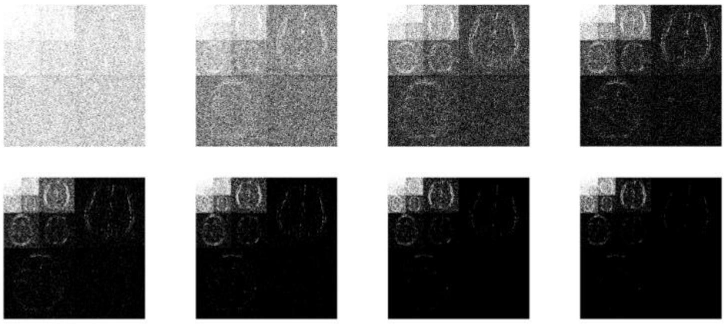

**Figure 7.** From the top left to the bottom right are the wavelet transforms at the soft threshold settings of λ = 2.5%, 5%, 7.5%, 10%, 12.5%, 15%, 17.5%, and 20% in the Variable density sampling case.

In selecting a suitable value of λ, according to the above mentioned with Fig. 6 and Fig. 7, the value of λ should be determined to be more appropriate in the neighborhood of 10%, which is the suitable threshold.

After different thresholding and iteration cases, based on the resulting reconstructed images (Fig. 8 and Fig. 9), it is found that the results of adopting Variable Density sampling pattern are much better than those of adopting Non-Uniform random sampling pattern. This conclusion can also be confirmed based on the PSNR and SSIM (Table 1 and Table 2) data. The very obvious comparison is that although the artifacts of non-uniform random sampling are reduced a lot, the brightness of the reconstructed image is also reduced, and it is very difficult to recognize the details of the original image clearly. The average PSNR value is 17.96, while the average SSIM value is only 0.44, and the best threshold is at λ = 20%, but the effect is not ideal; on the contrary, the reconstructed image by variable density sampling has a high degree of recognition, with an average PSNR value of 32.88, and an average SSIM value of 0.78, and the best threshold is at λ = 5%. It indicates that the latter is a more desirable result in the under sampling mode.

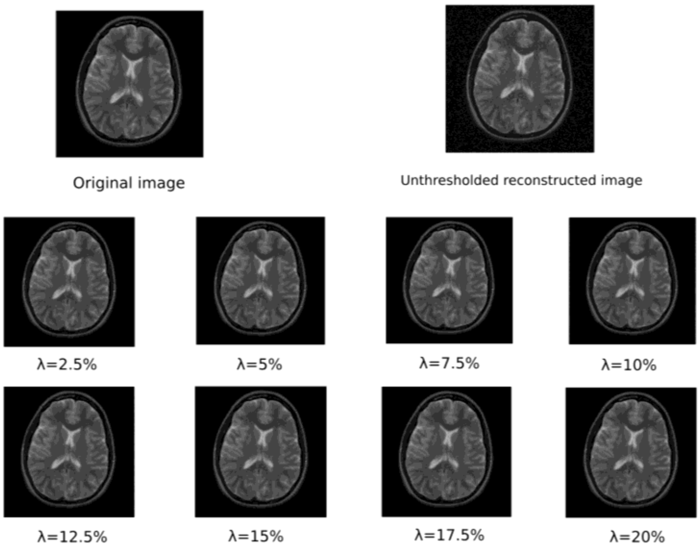


**Figure 8.** Image reconstruction effect of variable density sampling pattern under different thresholds.

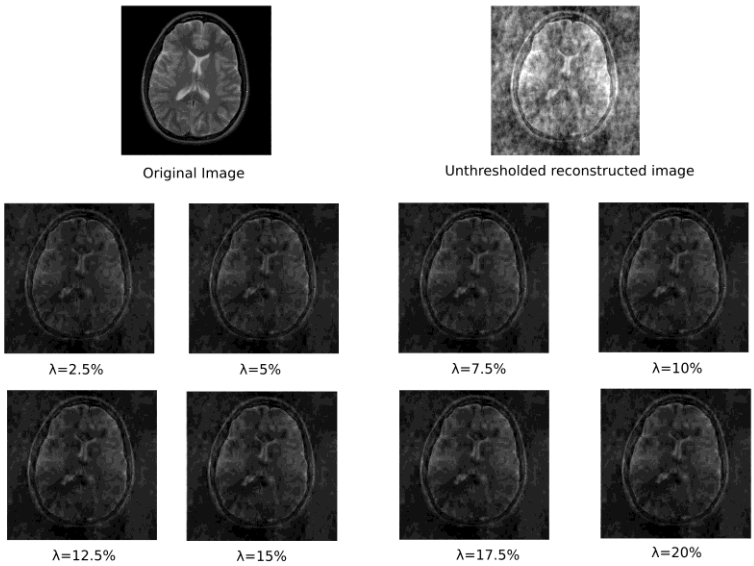


**Figure 9.** Image reconstruction effect of Non-Uniform sampling pattern under different thresholds.

In the application of MRI, it is easy to see through the results of this experiment that the results of variable density sampling are more excellent. This may be due to the difference in the undersampling Fourier transform operators [14] of the two modes. In MRI imaging, the center part of the k-space (low-frequency region) contains most of the energy and basic structural information of the image, while the peripheral part (high-frequency region) contains detail information. Variable density sampling can choose to sample more densely in the center of k-space and reduce the sampling density in the peripheral part according to this characteristic. That is to say variable density sampling is more likely to satisfy the requirements of compressed sensing because it takes into account the sparse characteristics of the image when sampling. With a properly designed sampling strategy, high quality reconstruction can be achieved at a lower sampling rate. This strategy ensures that the most important information is adequately captured, thus improving the quality of the image reconstruction. And since variable density sampling follows a predefined PDF during the sampling process, it can provide higher reconstruction accuracy and fewer image artifacts [15]. However, for non-uniform random sampling, there is no fixed pattern in selecting the sampling points, and its randomness may lead to under-sampling of some important regions, which may affect the reconstruction quality of the image.

Thus it may generate more artifacts and noise in the reconstruction process, which requires more complex reconstruction algorithms and more computational resources to compensate for the uncertainty in the sampling process.

**Table 1.** PSNR and SSIM of VDRS

| Non-Uniform Random Sampling | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| λ | 2.5 | 5 | 7.5 | 10 | 12.5 | 15 | 17.5 | 20 |
| PSNR | 17.82 | 17.90 | 17.95 | 17.97 | 17.99 | 18.01 | 18.02 | 18.03 |
| SSIM | 0.433 | 0.437 | 0.439 | 0.440 | 0.441 | 0.441 | 0.442 | 0.442 |

**Table 2.** PSNR and SSIM of NURS

| Variable Density Sampling | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| λ | 2.5 | 5 | 7.5 | 10 | 12.5 | 15 | 17.5 | 20 |
| PSNR | 33.08 | 33.14 | 33.02 | 32.90 | 32.81 | 32.74 | 32.68 | 32.64 |
| SSIM | 0.778 | 0.786 | 0.785 | 0.783 | 0.781 | 0.780 | 0.779 | 0.778 |

## 4. CONCLUSION

Through this research, by comparing two sampling methods, researchers can optimize data acquisition efficiency for specific imaging scenarios, potentially reducing scan times while maintaining or improving image quality. This study can reveal the optimal parameters for each method under various conditions, enabling customized approaches for different anatomical regions or imaging requirements and enhancing diagnostic accuracy. Additionally, the insights gained from this comparative study may guide future research toward hybrid sampling strategies that combine the strengths of both methods, advancing the capabilities of CS-MRI.

However, this paper also has some limitations, that is, the experimental sample is too small. Although two under sampling modes as well as different thresholds are used to reconstruct the image for the same brain image, it cannot be generalized to the whole case. In real practical applications, the situation is variable and complex. What happens in other fields with these two under-sampling modes cannot be generalized from this paper either. According to other scholars, non-uniform random sampling also has its own area of strength. In multidimensional NMR spectra [16], especially those without significant dynamic range issues can be efficiently recorded and accurately reconstructed using non-uniform sampling.

With the continuous development of CS-MRI technology, other algorithms have also been developed, such as the Randomized RF Pulse Encoding Method [17] and Edge Adaptive Enhancement Method [18], etc., to further improve the imaging speed and quality of CS-MRI, which still has a great potential and development prospect.

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