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Research on Time of Arrival Estimation Algorithm Based on ESPRIT

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ABSTRACT

The ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) algorithm is a high-resolution method used for estimating the Direction of Arrival (DOA) of signals. Due to its effectiveness and stability, many improvements have been proposed based on the ESPRIT algorithm, making it a prominent technique among high-resolution algorithms. In this paper, we delve into the theoretical foundations of the ESPRIT algorithm and conduct a comparative analysis with conventional Time of Arrival (TOA) estimation algorithms. Through extensive simulations, we demonstrate that the ESPRIT algorithm outperforms traditional TOA estimation methods, particularly under low signal-to-noise ratio (SNR) conditions. However, it should be noted that the ESPRIT algorithm entails a higher computational complexity. Our findings underscore the algorithm's superior performance in challenging environments, albeit with an increased computational burden.

KEYWORDS

ESPRIT Algorithm; TOA; Super-Resolution

1. INTRODUCTION

With the advancement of fields such as communications, radar, and acoustics, time delay measurement has emerged as a critical signal processing technique. It plays a vital role in accurately locating targets, precisely identifying signal sources, and improving signal reception quality. The accuracy of time delay measurement directly affects the overall performance of a system. Therefore, developing effective time delay measurement methods has become a key area of interest in the signal processing community.

The ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [1] algorithm is a classic method in spatial spectrum estimation. Like the MUSIC (Multiple Signal Classification) [2-4] algorithm, it requires eigenvalue decomposition of the covariance matrix of received data. However, there are notable differences between the two. The MUSIC algorithm exploits the orthogonality of the noise subspace of the covariance matrix, while the ESPRIT algorithm leverages the rotational invariance property of the signal subspace. Thus, ESPRIT and MUSIC can be seen as complementary approaches. Compared to the MUSIC algorithm, the ESPRIT algorithm has the advantage of lower computational complexity and can estimate the Time of Arrival (TOA) of signals without the need for spectral peak searching. Unlike the Root-MUSIC algorithm, which estimates signal arrival times by constructing a polynomial based on the orthogonality between the steering vector and the noise subspace, the ESPRIT algorithm directly utilizes the rotational invariance property of the signal subspace.

The ESPRIT algorithm, introduced by Roy et al. in 1986, has become a fundamental method in modern signal processing and is widely applied. As research into the ESPRIT algorithm has advanced, several improvements have been proposed, including the Toeplitz approximation algorithm, MI-ESPRIT [5] algorithm, and weighted ESPRIT algorithm. This paper aims to explore time delay measurement methods based on the ESPRIT algorithm and to investigate their performance and advantages. We will provide a comprehensive explanation of the principles and implementation steps of the ESPRIT algorithm. Through simulation experiments, we will evaluate the performance of this method in time delay measurement and conduct a comparative analysis with the TLS-ESPRIT [6, 7] algorithm.

2. BASIC METHOD OF TIME DELAY ESTIMATION USING ESPRIT

In the Continuous Time Domain:

$$x(t) \Leftrightarrow X(\omega) \tag{1}$$

$$x(t-\tau) \Leftrightarrow X(\omega)e^{-j\omega\tau} \tag{2}$$

If we denote the observed delayed signal with noise as y[n]y[n]y[n], then:

$$y[n] = x(t-\tau)+z[n] \tag{3}$$

Where τ is the time delay, z[n] represents the noise, and $t=n/F_s$ with F_s being the sampling rate of the ADC. The Discrete-Time Fourier Transform (DTFT) of the signal y[n] can be expressed as:

$$Y(e^{j\Omega})=Y(\omega) \tag{4}$$

Here, $\Omega\in(-\pi,\pi)$ and $\omega=\Omega\cdot F_s$. Without considering signal aliasing, the DTFT of the received signal can be represented as:

$$Y[k] =Y(e^{j\Omega})=Y(\omega) \tag{5}$$

In the above expression, k=0, 1, …, N−1, $\Omega=2\pi k/N$, and $\omega=2k\pi F_s/N$. Without considering signal aliasing, the Discrete Fourier Transform (DFT) of the received signal can be represented as:

$$Y[k]=X[k]e^{-j2k\pi Fs\tau/N}+Z[k] \tag{6}$$

Here, X[k] is the DFT of the sampled version of x(t), and Z[k] represents the noise. Therefore, we map the desired time delay parameter τ to a complex exponential parameter. The cross-correlation between the template signal x[n] and the received signal y[n] can be expressed as:

$$r_{xy}[n]= \sum_{m=-\infty}^{+\infty} x[m]y[m+n] \tag{7}$$

As defined, the cross-correlation operation is similar to the convolution operation in that both involve sliding multiplication of two sequences. However, the key difference is that, in cross-correlation, both sequences are directly multiplied without flipping either sequence, followed by summing the products. In convolution, one of the sequences is flipped before the sliding multiplication and summation. Therefore, the cross-correlation signal between x[n] and y[n] can be represented as:

$$r_{xy}[n] = x[-n]*y[n] \tag{8}$$

$$r_{xy}[n] = x[-n]*(x[n]+z[n]) \tag{9}$$

In the frequency domain, the cross-correlation of the two signals can be represented as:

$$R[k] = X^{*}[k]Y[k] \tag{10}$$

$$R[k]=X^{*}[k](X[k]e^{-j2k\pi Fs\tau/N}+Z[k]) \tag{11}$$

$$R[k]=|X[k]|^{2}e^{-j2k\pi Fs\tau/N}+X^{*}[k]Z[k] \tag{12}$$

In the above expression, we can see that for any kkk, the following holds:

$$R[k]/R[k-1] \propto e^{-j2\pi Fs\tau/N} \tag{13}$$

To obtain a more accurate time delay estimation, we use multiple estimates and calculate their average. Therefore, we construct two matrices, R1 and R2, as follows:

$$R1 = \left[R[0],R[1],\ldots R[k-1]\right] \tag{14}$$

$$R2 = [R[1],R[2],...R[k]] \tag{15}$$

It can be deduced that the matrices R1 and R2 satisfy the relationship R2=R1·Φ, where Φ represents a phase shift matrix:

$$\tau=\frac{N*\arg(\mathrm{diag}(\Phi))}{2\pi Fs} \tag{16}$$

From the above formula, it is evident that the resolution of the time delay τ is not dependent on the sampling rate Fs. The performance of the system improves with an increase in the Signal-to-Noise Ratio (SNR) and is independent of the sampling rate Fs. Because this frequency-domain method can resolve time intervals smaller than the sampling rate, it is considered a super-resolution method.

Based on this, the steps for the overall computation are as follows:

1) Compute the N-point Discrete Fourier Transform (DFT) of the transmitted signal Xn.

2) Receive the delayed signal and compute the cross-correlation r[n]=y[n]∗x[−n].

3) Identify the peak value of the absolute value of the cross-correlation r[n] and record n^{*}; this represents the overall delay estimate for the sample.

4) Compute the DFT of r[n] to obtain R[k] for $n=n^{*}, \ldots, n^{*}+N-1$.

5) Construct the signal subspace matrices R1 and R2.

6) Calculate Φ=R2/R1.

7) Estimate the arrival time τ using Φ.

3. TLS-ESPRIT ALGORITHM

The TLS-ESPRIT (Total Least Squares ESPRIT) algorithm is an advanced version of the ESPRIT algorithm that incorporates the Total Least Squares (TLS) method to enhance estimation accuracy. Here, we apply the TLS-ESPRIT algorithm for the estimation of signal time delays.

Assume the signal is:

$$x(n)=\sum_{i=1}^{p} s_i e^{jn\omega_i} +w(n) \tag{17}$$

In the above expression, s_i and ω_i represent the complex amplitude and frequency of the harmonic signals, respectively, while w(n) denotes Gaussian white noise with a mean of 0 and variance σ^2. We can then construct:

$$y(n)=x(n+1) \tag{18}$$

Introduce an M×1 dimensional vector, where M>p:

$$X(n)=[x(n),\ldots x(n+M-1)]^T \tag{19}$$

$$W(n)=[w(n),\ldots w(n+M-1)]^T \tag{20}$$

$$Y(n)=[y(n),\ldots y(n+M-1)]^T \tag{21}$$

$$Y(n)=[x(n+1),\cdots x(n+M)]^T \tag{22}$$

The expression for x(n) can be written in matrix form as:

$$X(n)=AS+W(n) \tag{23}$$

$$Y(n)=A\phi S+W(n) \tag{24}$$

In the above expression:

$$A=\begin{bmatrix} e^{jn\omega_1} & \cdots & e^{jn\omega_p} \\ \vdots & \ddots & \vdots \\ e^{j(n+M-1)\omega_1} & \cdots & e^{j(n+M-1)\omega_1} \end{bmatrix} \tag{25}$$

$$S = [s_1, \ldots, s_p]^T \tag{26}$$

$$\phi=\begin{bmatrix} e^{j\omega_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\omega_p} \end{bmatrix} \tag{27}$$

The role of Φ is to relate X(n) and Y(n). Φ is a unitary matrix, which we refer to as the rotation operator.

$$M=\begin{bmatrix} X \\ Y \end{bmatrix}=\begin{bmatrix} A \\ A\phi \end{bmatrix} S+N=\overline{A}S+N \tag{28}$$

In an ideal scenario, the covariance of the above expression can be obtained as:

$$R=E\left[MM^H\right] \tag{29}$$

Performing eigenvalue decomposition on the above expression yields:

$$R=U_S \textstyle\sum_S U_S^H + U_N \sum_N U_N^H = \sum_{i=1}^{2M} \lambda_i e_i e_i^H \tag{30}$$

Where U_S is the signal subspace spanned by the eigenvectors corresponding to the largest eigenvalues, and U_N is the noise subspace spanned by the eigenvectors corresponding to the smallest eigenvalues. In the above formula, it can be analyzed that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N > \lambda_{N+1} = \ldots = \lambda_{2m}$.

From this decomposition, the new signal subspace U_S can be obtained, where U_S includes two signal subspaces, U_{S1} and U_{S2}. Define:

$$U_{S12}=[U_{S1}|U_{S2}] \tag{31}$$

We can perform eigenvalue decomposition on the matrix $U_{S12}{}^H U_{S12}$. Through this eigenvalue decomposition, we obtain:

$$U_{S12}^H U_{S12}=E\Sigma E^H \tag{32}$$

Σ is a diagonal matrix composed of eigenvalues, and E is the matrix of eigenvectors corresponding to Σ. That is

$$E=\begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \tag{33}$$

Where $[\begin{smallmatrix} E_{11} \\ E_{12} \end{smallmatrix}]$ is the matrix consisting of eigenvectors corresponding to eigenvalues of 0, which belongs to the noise subspace.

Let $G=-(E_{12}*inv(E_{22}))$. Performing eigenvalue decomposition on G yields:

$$\tau_i=\frac{-\arg(\lambda_i)}{2*\pi*d} \tag{35}$$

Thus, the processing steps for the TLS-ESPRIT algorithm are as follows:

(1) Compute the Covariance Matrix:

Construct the covariance matrix of the received signal data.

(2) Perform Eigenvalue Decomposition:

Perform eigenvalue decomposition on the covariance matrix to obtain the signal subspace matrix U_S and the noise subspace matrix U_N.

(3) Form the Matrices US1 and US2:

Use the eigenvectors associated with the largest eigenvalues to form U_{S1} and U_{S2}, which represent the signal subspace.

(4) Construct Matrix G:

Compute $G=-(E_{12}\cdot E_{22}{}^{-1})$, where E_{12} and E_{22} are the matrices from the eigenvalue decomposition.

(5) Perform Eigenvalue Decomposition on G:

Perform eigenvalue decomposition on matrix G to obtain the eigenvalues and eigenvectors.

(6) Estimate Parameters:

Use the eigenvalues and eigenvectors from the decomposition of G to estimate the signal parameters.

4. SIMULATION AND RESULTS ANALYSIS

Under the same signal-to-noise ratio (SNR), we conducted simulations for both the ESPRIT and TLS-ESPRIT algorithms. The sampling rate used was 400 kHz, and the time delay error of the signal was 138 ns. The simulation setup is detailed in the table below:

Table 1. Parameter Value

Parameter	Value
Sampling Rate	400KHz
Signal Delay Error	138ns
SNR	-3dB, 0dB, 3dB, 10dB

The simulation results are shown in Table 2 and Figure 1. Table 2 shows the simulation result data.

Table 2. Error Metrics

SNR\ Error Metrics	ESPRIT(ns)	TLS-ESPRIT(ns)
10dB	1	0.6
3dB	6	6.5
0dB	23	15
-3dB	50	25

Figure 1 shows the errors produced by the two algorithms when estimating delay.

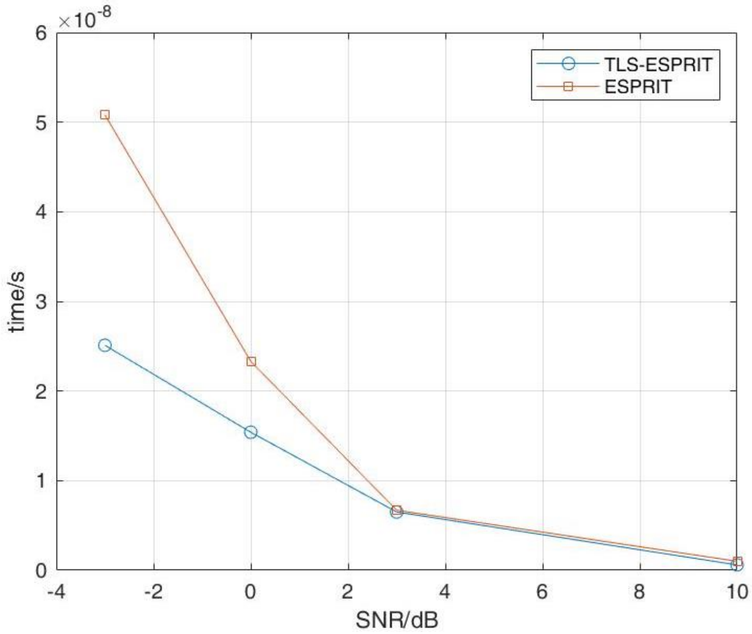

Figure 1. Result Comparison Curve

This paper derives the theoretical framework for the ESPRIT algorithm and compares its performance with that of the TLS-ESPRIT algorithm. Through simulations, the performance of these two algorithms was compared. As shown in Table 2, when the signal-to-noise ratio (SNR) is low, the TLS-ESPRIT algorithm demonstrates reduced bias in time delay estimation compared to the ESPRIT algorithm.

Figure 1 illustrates that under the same SNR conditions, smaller errors result in estimated values that are closer to the true values. Both algorithms are capable of accurately estimating delays within the code phase when the SNR is above zero. However, at lower SNR levels, the TLS-ESPRIT algorithm outperforms the ESPRIT algorithm. Theoretical analysis and simulation results indicate that the TLS-ESPRIT algorithm effectively suppresses noise and improves the resolution of time delay estimates under low SNR conditions.

Nevertheless, it should be noted that the computational complexity of the TLS-ESPRIT algorithm is higher than that of the ESPRIT algorithm.

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