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Iterative Learning Tracking Control for Nonlinear Systems Based on Data Driven Control

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ABSTRACT

This paper introduces a novel methodology for tracking control of general nonlinear systems. Unlike traditional methods, which rely on linearization or nonlinear cancellation. This article starts from the iterative domain and utilizes the concept of iterative learning control to improve the control performance of the system. It employs a data-driven approach for model-free adaptive control, addressing the challenges posed by strong system nonlinearity, difficulties in control, and even modeling issues caused by disturbances and other factors. It enables precise trajectory tracking without the need for complex computations, making it suitable for real-time control applications. The effectiveness of the methodology is demonstrated through examples.

KEYWORDS

Data driven, Iterative Learning, Nonlinear Systems

1. INTRODUCTION

Tracking control of nonlinear systems has emerged as a prominent research area in recent years. In [1] presents a new control mechanism for uncertain high-order, multiple-input-multiple-output nonlinear systems. The control strategy, based on limited assumptions, ensures semiglobal asymptotic tracking using a Lyapunov-based stability argument. Reference [2] presents state feedback controllers for tracking control of switched nonlinear systems with unknown functions, utilizing adaptive backstepping and the common Lyapunov function method. Simulation results validate the effectiveness of the proposed approach. In [3] presents a new tracking control methodology for general nonlinear systems, which avoids linearization and nonlinear cancellation. The approach enables exact trajectory tracking with low computational requirements, making it suitable for real-time control of nonlinear systems.

On one hand, data-driven control (DDC) algorithms have been rapidly advancing. Compared to traditional control methods, the Model Free Adaptive Control (MFAC) algorithm demonstrates greater efficacy [4] and is applicable to nonlinear systems [5]. Moreover, MFAC has found extensive applications in the field of autonomous driving [6] and has made significant contributions to the field of physics [7]. One of the major advantages of data-driven methods is their ability to circumvent the need for constructing complex mathematical models [8]. The issue of inference in Nonlinear systems has also been explored [9].

For a considerable period, the classic data-driven control algorithm was the Proportional-Integral-Derivative (PID) control. Subsequently, more advanced data-driven control algorithms, such as Iterative Feedback Control (IFT) and Iterative Learning Control (ILC), were proposed. These data-driven methods are particularly valuable in situations where accurate mathematical models are difficult to establish due to the complexity of the system or unpredictable system behavior.

2. PROBLEM DESCRIPTION

Utilizing data-driven methods to address nonlinear systems with interference has gained popularity as it enables better adaptation to challenges arising from random interference, such as complex system models and unpredictable system behavior. Traditional control strategies for Nonlinear systems suffer from certain drawbacks. Firstly, accurately establishing the mathematical model of the entire nonlinear systems is challenging. Secondly, traditional model-driven approaches face difficulties in achieving tracking control for complex nonlinear systems when the states of complex system, including interference, differ. Moreover, nonlinear systems are susceptible to various random interferences, making it even more challenging to establish an accurate system model. In this case, it can be regarded as a difficult to model nonlinear system. This article is motivated by the aim to design a data-driven collaboration approach specifically tailored for nonlinear systems.

3. DESIGN OF CONTROL LAW

3.1. Controlled system

There are the following nonlinear systems:

$$y(k+1,t)=f(y(k,t),...,y(k-n_1,t),\ u(k,t),...,u(k-n_2,t)) \tag{1}$$

Where $y(k,t)$ and $u(k,t)$ represent the input and output signals the system at the k sampling time in the t-th iteration. n_1 and n_2 dare two unknown positive integers that vary with the system. $f(...)$ denotes an unknown scalar function of the system.

3.2. The control law

The system satisfies the generalized Lipschitz condition along the iteration axis, which means that for all $k\in 0,1,...,T$ and for all $i\in 0,1,2,...$, if $|\Delta u_i(k,t)\neq 0|$, then the following equation must hold:

$$|\Delta y(k+1,t)|\leqslant b|\Delta u(k,t)| \tag{2}$$

where $\Delta y(k+1,t)=y(k+1,t)-y(k+1,t-1)$ represent input in the k-th time and $\Delta u(k+1,t)=u(k+1,t)-u(k+1,t-1)$ represent the changes between two consecutive outputs along the iteration axis. The constant b is a non-zero constant.

It can be inferred that for the above equation, the following data models can be derived:

$$\Delta y(k+1,t)=\phi(k,t)\Delta u(k,t) \tag{3}$$

Derive from the above two equations:

$$\begin{aligned}\Delta y(k+1,t)=&f(y(k,t),...,y(k-n_1,t),u(k,t),...,u(k-n_2,t))\\&-f(y(k,t-1),...,y(k-n_1,t-1),u(k,t-1),...,u(k-n_2,t-1))\end{aligned} \tag{4}$$

Can be inferred:

$$
\begin{aligned}
\Delta y(k+1,t) = & f(y(k,t),...,y(k-n_1,t),u(k,t),...,u(k-n_2,t)) \\
& - f(y(k,t),y(k-n_1,t),u(k,t-1),...,u(k-n_2,t)) \\
& + f(y(k,t),y(k-n_1,t),u(k,t-1),...,u(k-n_2,t)) \\
& - f(y(k,t-1),...,y(k-n_1,t-1),u(k,t-1),...,u(k-n_2,t-1))
\end{aligned} \tag{5}
$$

Let ϖ be defined as follows:

$$
\begin{aligned}
\varpi = & f(y(k,t),...,y(k-n_1,t),u(k,t-1),...,u(k-n_2,t)) \\
& - f(y(k,t-1),..,y(k-n_1,t-1),u(k,t-1),..,u(k-n_2,t-1))
\end{aligned} \tag{6}
$$

Combining the mean value theorem of differentials, this paper can rewrite equation as:

$$
\Delta y(k+1,t) = \frac{\partial f^*}{\partial u(k,t)}(u(k,t) - u(k,t-1)) + \varpi(k,t) \tag{7}
$$

Here, $\frac{\partial f^*}{\partial u(k,t)}$ represents the value between two points and can be expressed as:

$$
\varpi(k,t) = \varrho(k,t)\Delta u(k,t) \tag{8}
$$

Let $\varrho^*(k,t)$ be the solution to the above equation:

$$
\phi(k,t) = \frac{\partial f^*}{\partial u(k,t)} + \varrho^*(k,t) \tag{9}
$$

The criterion function is shown as follows:

$$
\begin{aligned}
J(u(k,t)) = & |y_d(k+1) - y(k+1) - \phi(k,t)(u(k,t) - u(k,t-1))|^2 \\
& + \lambda|u(k,t) - u(k,t-1)|^2
\end{aligned} \tag{10}
$$

And the control input is as follow:

$$
u(k,t) = u(k,t-1) + \frac{\rho\phi(k,t)}{\lambda + |\phi(k,t)|^2} E(k+1,t-1) \tag{11}
$$

And $E(k+1,t) = \sum_{k=0}^{M}(y(k+1,t) - y(k+1,t))$.Pseudo partial derivative is as follow:

$$
\begin{aligned}
\hat{\phi}(k,t) = & \hat{\phi}(k,t-1) + \frac{\eta\Delta u(k,t-1)}{\psi + |\Delta u(k,t-1)|^2} \times \\
& \left(\Delta y(k+1,t-1) - \hat{\phi}(k,t-1)\Delta u(k,t-1)\right)
\end{aligned} \tag{12}
$$

4. STABILITY ANALYSIS

1.The stability analysis of the system can be divided into three aspects:

2.The boundedness of the estimated pseudo Jacobian matrix $\hat{\phi}_i(k,t)$ of system for all $k \in 1,2,3,...,T$ and $t \in 1,2,3,...$.

For all $k \in 1,2,3,...,T$, when the iteration count t tends to infinity, the following equation holds:

$$E(k+1,t) = \sum_{k=0}^{M}(y(k+1,t) - y(k+1,t)) = 0 \tag{13}$$

3.The control inputs and outputs $u_i(k,t)$ and $y_i(k,t)$ are bounded for all $k \in 1,2,3,...,T$ and $t \in 1,2,3,...$

Pseudo partial derivatives can be represented as:

$$\begin{aligned} \Delta\phi(k,t) &= \phi(k,t) - \phi(k,t-1) \\ \tilde{\phi}(k,t) &= \phi(k,t) - \hat{\phi}(k,t) \end{aligned} \tag{14}$$

By combining several equations, it can be inferred that:

$$\tilde{\phi}(k,t) = \left[1 - \frac{\eta\Delta u(k,t-1)^2}{\psi + |\Delta u(k,t-1)|^2}\right]\tilde{\phi}(k,t-1) - \Delta\phi(k,t) \tag{15}$$

By taking the absolute value on both sides, the following equation can be deduced.

$$\begin{aligned} |\tilde{\phi}(k,t)| = {} & |\left[1 - \frac{\eta\Delta u(k,t-1)^2}{\psi + |\Delta u(k,t-1)|^2}\right]| \\ & |\tilde{\phi}(k,t-1)| - |\Delta\phi(k,t)| \\ |\tilde{\phi}(k,t)| \leqslant {} & |\left[1 - \frac{\eta\Delta u(k,t-1)^2}{\psi + |\Delta u(k,t-1)|^2}\right]| \\ & |\tilde{\phi}(k,t-1)| + |\Delta\phi(k,t)| \end{aligned} \tag{16}$$

Establish an inequality.

$$\begin{aligned} |\tilde{\phi}(k,t)| &\leqslant \varrho|\tilde{\phi}(k,t-1)| + 2\chi \\ &\leqslant \varrho^{i-1}|\tilde{\phi}(k,t-1)| + \frac{2\chi}{1-\varrho} \end{aligned} \tag{17}$$

Pseudo partial derivative convergence.

$$e(k,t) = y(k,t) - y(k,t-1) \tag{18}$$

The output error can be expressed as:

$$e(k,t) = \left[1 - \frac{\eta\hat{\phi}(k-1,t)}{\psi + |\hat{\phi}(k-1,t)|^2}\right]e(k,t-1) \tag{19}$$

By combining the derivation process of pseudo partial derivatives, it can be concluded that:

$$\begin{aligned}\frac{\eta\hat{\phi}(k-1,t)\phi(k-1,t)}{\psi+|\hat{\phi}(k-1,t)|^2} &\leqslant \frac{\eta\hat{\phi}(k-1,t)\chi}{\psi+|\hat{\phi}(k-1,t)|^2} \\ &\leqslant \frac{\eta\hat{\phi}(k-1,t)\chi}{2\sqrt{\psi}\,\hat{\phi}(k-1,t)} \\ &< \frac{\eta\chi}{2\sqrt{\min(\psi)}}\end{aligned} \tag{20}$$

The output error is represented as:

$$\begin{aligned}|e(k,t)| &= \left[1-\frac{\eta\hat{\phi}(k-1,t)}{\psi+|\hat{\phi}(k-1,t)|^2}\right]|e(k,t-1)| \\ &\leqslant \frac{\eta\chi}{2\sqrt{\min(\psi)}}|e(k,t-1)| \leqslant |e(k,t-1)| \\ &\leqslant \left(\frac{\eta\chi}{2\sqrt{\min(\psi)}}\right)^{t-2}|e(k,1)| \leqslant |e(k,1)|\end{aligned} \tag{21}$$

Output error convergence.

5. NUMERICAL SIMULATION

The simulation part of this paper focuses on the study of nonlinear systems. From the perspective of model-free adaptive control, the mathematical models of the nonlinear system are unknown. However, for the convenience of simulation research, this paper provides mathematical models for the systems. It is important to note that in practical applications, the mathematical models of the system are not necessary, and only input-output (IO) data is required.

The system model for the system is as follows:

$$y(k,t+1) = \frac{y(k,t)}{1+y^2(k,t)} + u^3(k,t) \tag{22}$$

Tracking performance under 30 iterations of learning:

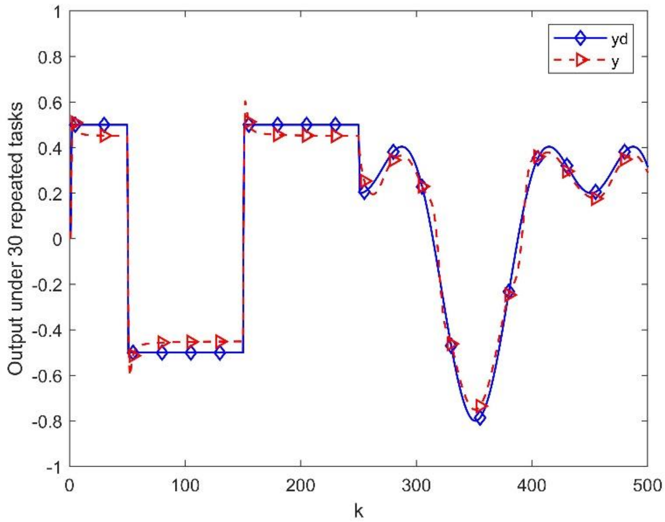

Figure 1 Tracking output under 30 iterations

Tracking performance under 100 iterations of learning:

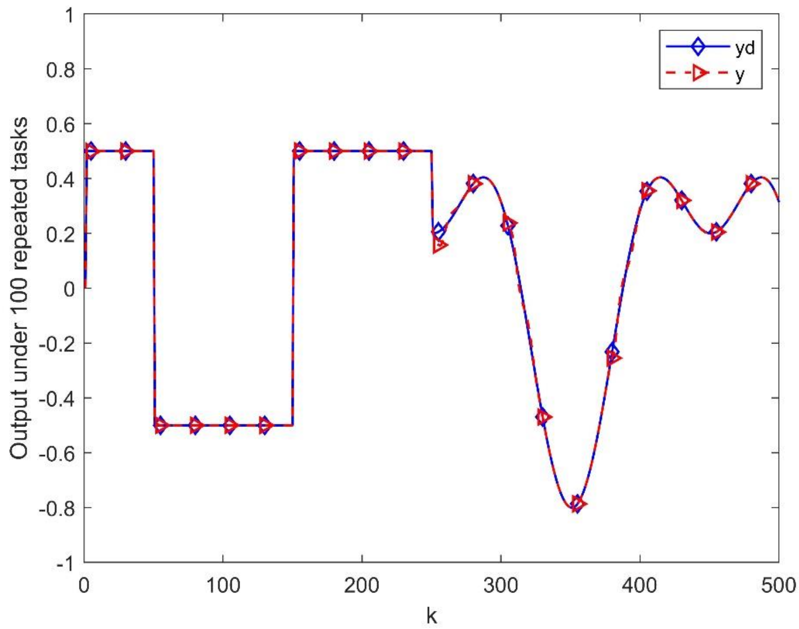

Figure 2 Tracking output under 100 iterations

From the Figure 2, it can be seen that the control effect has significantly improved with the increase of iteration times.

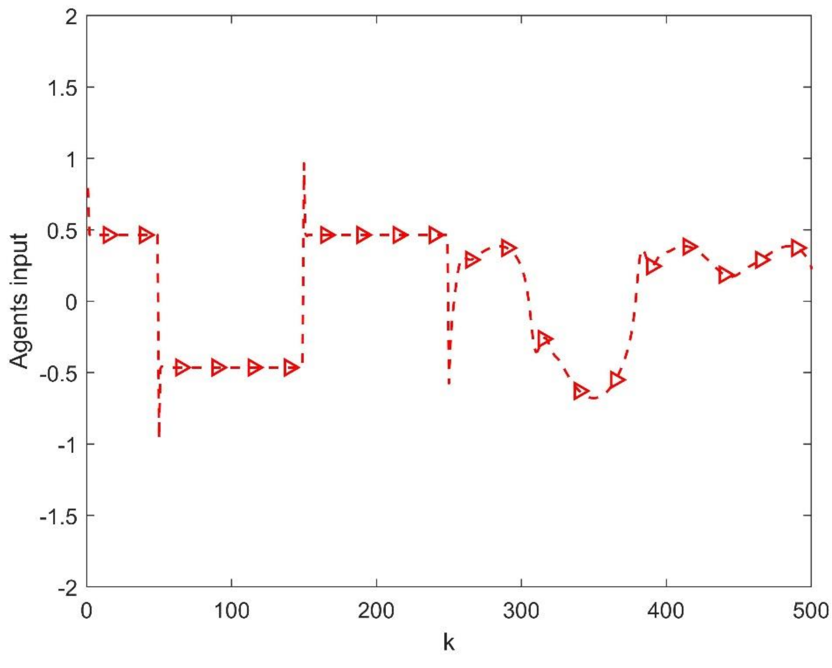

Figure 3. Control input

Control input as shown in the figure. From the Figure 3, it can be seen that the control input is stable and the trend is similar to that of the control input.

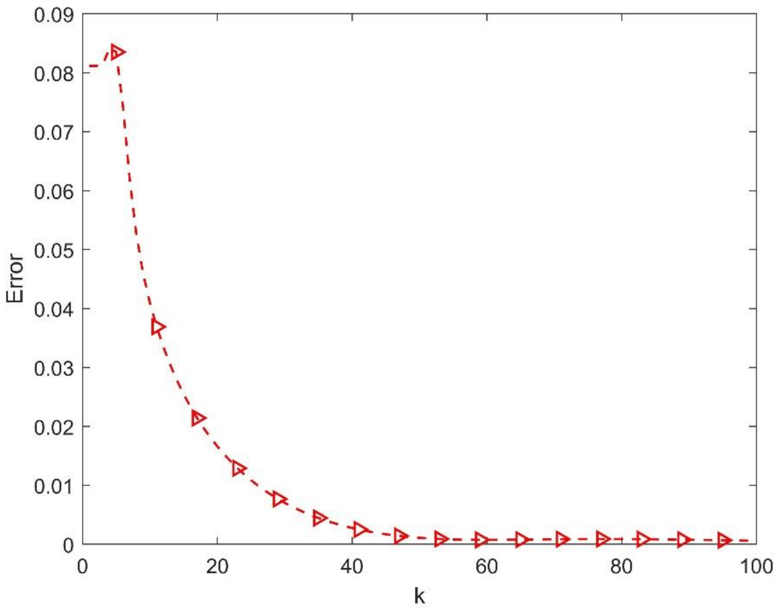

Figure 4 Error convergence chart

The error convergence image is shown in the Figure 4. The error shows a significant downward trend with the increase of iteration times, proving that the algorithm designed in this paper has a good tracking effect on nonlinear systems

6. CONCLUSION

In summary, this paper focuses on the tracking control of nonlinear multi-agent systems. It adopts the iterative domain approach, abandoning the traditional time-domain perspective, to achieve precise tracking control of nonlinear systems. Additionally, a data-driven method is employed to establish a mathematical model, avoiding the challenges posed by complex nonlinear systems or systems that are difficult to model. The reliability of this approach is validated through stability analysis, and numerical simulations are conducted to demonstrate the excellence of the proposed method.

CONFLICTS OF INTEREST

The authors declare that they have no conflict of interest.

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