

# A review of Vehicle Routing Problem with Selective Pickup Point and Delivery

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## ABSTRACT

The problem of picking up and delivery vehicle routing is that one delivery point can correspond to multiple pickup points, and one of these pickup points needs to be selected for pickup before delivery. With the reform of the retail industry, the omnichannel retail model has gradually become popular, which provides consumers with a variety of choice of pick-up points in collaboration with online e-commerce and offline stores. This paper gives a systematic overview of the relevant operational research model and its solution for the pick-up and delivery vehicle routing problem considering the selection of pick-up points, hoping to encourage logistics community and transportation science to do more research in this emerging field.

## KEYWORDS

Vehicle Routing; Multiple pickup points; Selective pickup delivery problem; Time Windows

## 1. INTRODUCTION

The development of the internet and e-commerce has provided diversified shopping channels and methods, which has not only changed people's consumption behaviors but also brought about reforms in business models. The traditional single-channel retail model can no longer satisfy people's shopping demand for multi-channel or even omni-channel shopping. With the rapid development of new retail, various new forms of consumption with novel business models have begun to emerge. Therefore, both offline retailers and online e-commerce platforms are reintegrating and collaborating across multiple sales channels, giving rise to the Omni-channel retail model. Retail enterprises can provide consumers with comprehensive services through physical channels, e-commerce channels, mobile e-commerce, and other means, such as offline stores, social media, official websites, and so on[1].

In traditional multi-channel retailing, each channel operates independently with separate inventory, picking, organization, and systems. This often leads to a lack of information sharing among channels when responding to the diverse needs of consumers, resulting in higher distribution costs. However, omni-channel retailers, through information sharing, enable logistics personnel to choose to pick up goods from either e-commerce warehouses or offline retail stores to meet consumer delivery demand. Additionally, warehouses have evolved beyond their traditional role as mere storage facilities and now serve as integrated hubs for storage, product display, and customer pick-up services. Many online e-commerce brands have expanded into offline physical stores.

Amid this transformational trend, retail enterprises are increasingly diversifying their logistics needs. Among them, the timeliness of logistics is a key driver of consumer satisfaction and loyalty in an omni-channel environment[2]. The efficiency and service quality of logistics delivery directly impact the consumer experience. As market competition intensifies and consumption continues to upgrade, consumers have higher expectations for the timeliness and service quality of last-mile logistics

delivery. If delivery routes are not properly planned during this period, it can lead to issues such as repeated delivery routes, extended delivery times, and consequently, increased transportation and service penalty costs for logistics enterprises, as well as decreased consumer satisfaction. Therefore, optimizing the terminal delivery process in the omni-channel retail model, reducing logistics costs, and enhancing the consumer experience are crucial and necessary issues faced by both retail and logistics enterprises[3].

In addition to the delivery scenarios for end consumers in the omni-channel retail model, Takada, Y[4] and others have proposed store substitution shopping services. The rational arrangement of multiple pickup points and the routing of delivery vehicles are crucial to reducing distribution costs. Therefore, conducting research on the problem of vehicle routing with pickup and delivery considering the selection of pickup points can provide important theoretical support and practical guidance for retail enterprises, third-party logistics companies, and other relevant enterprises facing scenarios where pickup point selection is involved in delivery, enabling them to reduce logistics costs and enhance logistics service levels.

The optimization of the vehicle routing problem with pickup and delivery considering the selection of pickup points can aid in making scientific decisions regarding delivery vehicle routes, minimizing blindness in logistics distribution vehicle routing planning. This provides theoretical support and decision-making guidance for reducing logistics distribution costs, enhancing logistics delivery experiences, and improving logistics service levels. It holds significant guiding significance and practical value for the development of the omni-channel retail model, as well as for retail enterprises and third-party logistics companies. Additionally, it offers reference ideas for vehicle routing planning problems in similar delivery scenarios, such as store substitution shopping services, medical supply distribution, and inventory redistribution in chain supermarkets.

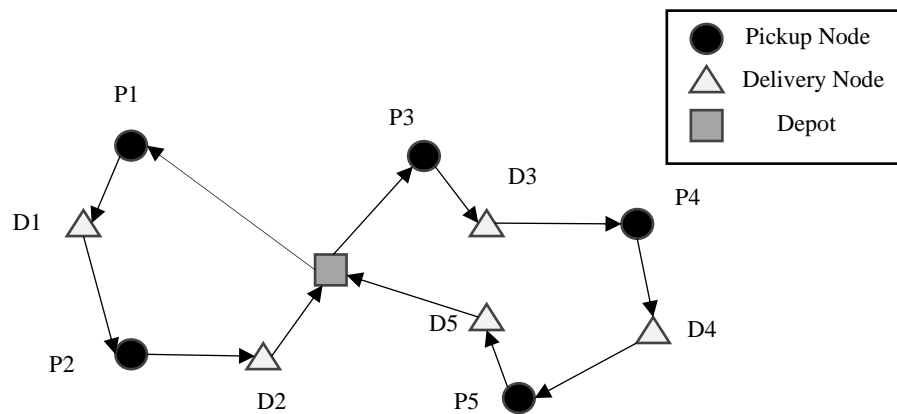
Despite the extensive literature on pickup and delivery problems, research on vehicle routing problems with pickup and delivery that consider the selection of pickup points remains incomplete. There are still many gaps in the algorithms used, and most of them have not been effectively applied in practical scenarios. Therefore, we believe it is timely to systematically overview the vehicle routing problem with pickup and delivery considering the selection of pickup points and propose relevant mathematical models. Our focus is not only on the literature related to vehicle routing with pickup and delivery but also on optimization methods in other transportation fields with similar characteristics. By introducing and formally defining the vehicle routing problem with pickup and delivery considering the selection of pickup points, we aim to encourage more research in this emerging field of logistics distribution brought by the evolving omni-channel retail model within the transportation science and logistics community. Such research is crucial for retail enterprises and third-party logistics companies to optimize their delivery operations, reduce costs, and enhance customer satisfaction. By addressing the gaps in existing algorithms and exploring new optimization techniques, we can contribute to the development of more efficient and effective logistics systems that cater to the needs of the modern retail landscape. Additionally, by drawing parallels with optimization methods in other transportation fields, we can gain insights and leverage best practices to further advance the research in this area.

The structure of the remaining part of this paper is as follows. In Section 2, we will explain and describe the overview of the problem, objectives, and constraints. In Section 3, we will investigate the Vehicle Routing Problem with Pickup and Delivery considering Time Windows and review the existing literature. Section 4 will focus on the Vehicle Routing Problem with Pickup and Delivery considering Selective Pickup Points. In Section 5, we will discuss the Vehicle Routing Problem with Pickup and Delivery involving Split Pickups. Finally, in Section 6, we will summarize our key insights and discuss potential future research directions. By organizing the content in this manner, we aim to provide a comprehensive overview of the topic and contribute to the advancement of research in this field.

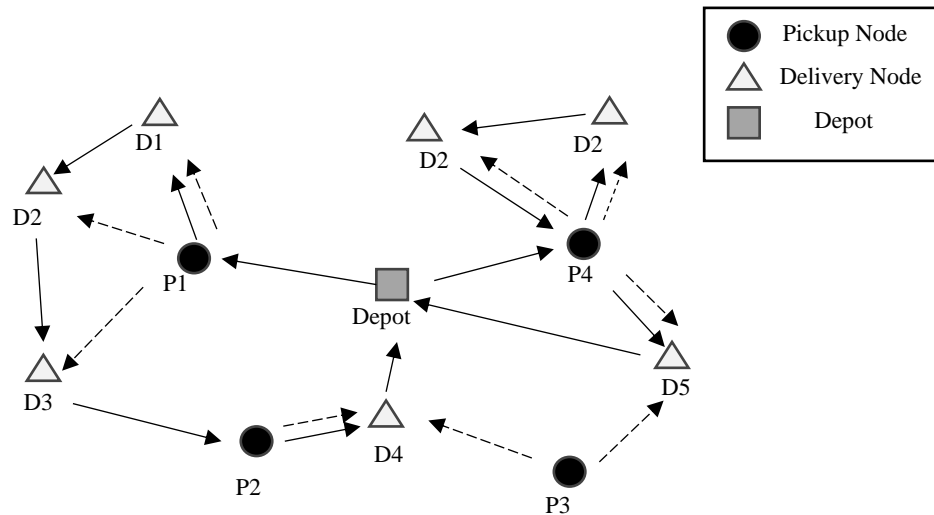
## 2. PROBLEM CHARACTERISTICS

This paper investigates a variant of the Vehicle Routing Problem, specifically the Vehicle Routing Problem with Selective Pickup Point and Delivery (VRPSPPD). In the VRPSPPD, there are three types of nodes: pickup points, delivery points, and depots. The vehicles must visit these nodes within specific time windows. Each delivery point corresponds to a delivery demand, and the vehicle needs to unload the corresponding amount of goods when visiting the delivery point. When visiting a pickup point, the vehicle loads the corresponding amount of goods. Each delivery point can be supplied by multiple pickup points, and any one of these pickup points can fulfill the supply demand of the current delivery point, with the selection of only one point required. Additionally, each pickup point can supply goods to a limited number of delivery points, with the inventory level of the pickup point representing the sum of the demand from all its corresponding delivery points. Each vehicle has a maximum loading capacity, and the loaded quantity during any node visit must not exceed this capacity. At the beginning of the task, the vehicles depart from the depot, complete the delivery to all delivery points, and finally return to the depot. Each pickup point can be visited multiple times by the same or different vehicles, while each delivery point can only be visited once. The objective is to minimize the total cost of the vehicle routing while satisfying all delivery demands.

Figure 1 below shows the basic VRPPD distribution network, where the pick-up point and the delivery point are one-to-one. Figure 2 below is the distribution network of VRPSPPD, where the dotted arrows represent the corresponding relationship between the pick-up point and the delivery point. It can be seen that there is a pick-up point supplying multiple delivery points, such as pick-up points 1, 3 and 4; The solid line represents the optimal pick-up point and the optimal path. It can be seen that the pick-up point has been visited several times in the distribution network, for example, pickup point 4.



**Figure 1.** Basic VRPPD distribution network



**Figure 2.** VRPSPPD Distribution network

## 2.1. Constituent Element

The key components of the Vehicle Routing Problem with Pickup and Delivery (VRPPD) include delivery vehicles, depots, pickup points, customer demand points (delivery points), road networks, constraints, and optimization objectives. The specific meanings of these components are as follows:

**(1) Delivery vehicles:** These are the vehicles used to carry and transport goods during the pickup and delivery process. Considerations for these vehicles include their maximum loading capacity, cost per kilometer, maximum allowable travel distance or duration, and the different types of vehicles available.

**(2) Pickup points:** In the VRPPD, pickup points refer to locations where vehicles need to collect goods, such as warehouses, retail outlets, or physical stores. Pickup points are typically associated with customer demands and can fulfill the needs of multiple customers.

**(3) Customer points:** These represent the service recipients in the VRPPD, i.e., the destinations where goods are to be delivered. Each customer point has a corresponding demand, which can be the amount of goods to be delivered or collected from that point. Meeting the delivery time windows agreed upon with customers can enhance customer satisfaction.

**(4) Depots:** In the vehicle routing problem, depots are typically the starting and ending points of a route. These depots house identical or different types of vehicles. While most studies on vehicle routing problems focus on a single depot, in real-world scenarios, there may be multiple depots located in different locations, necessitating not only the planning of vehicle routes but also the determination of which depot and corresponding vehicles will service which customer points.

**(5) Delivery routes:** These are typically represented as weighted graphs consisting of nodes and arcs. Nodes represent depots, pickup points, and customer points, while arcs indicate the travel sequence and direction of vehicles. Each arc is assigned a cost weight, which can represent the distance between two points, vehicle travel time, vehicle operating cost, or other metrics.

**(6) Constraints:** These include limitations such as the total demand of each customer point on a route not exceeding the maximum capacity of the vehicle, the travel distance or duration of a vehicle not exceeding its maximum allowable limit, the service time of customer points falling within a specified time window, as well as other constraints related to vehicle types, customer service priorities, and other factors.

## 2.2. Optimization Objectives

Based on the variation of practical problems, the optimization objectives of the Vehicle Routing Problem with Pickup and Delivery (VRPPD) can indeed be categorized into two main groups: single-objective optimization and multi-objective optimization. Detailed explanations of these two types of objectives are provided below:

### 2.2.1. Single-Objective Optimization:

**Minimizing Vehicle Travel Distance:** This is one of the most common optimization objectives, aiming to reduce the total travel mileage of vehicles, thereby lowering transportation costs and time.

**Minimizing the Number of Vehicles:** In certain situations, reducing the number of vehicles utilized may be a critical objective, especially when resources are limited or vehicle operating costs are high.

**Minimizing Vehicle Transportation Costs:** In addition to direct travel costs, other vehicle-related costs such as vehicle depreciation, fuel consumption, and driver salaries may also be considered.

**Maximizing Logistics Revenue:** From an economic perspective, this objective considers how to maximize total revenue by optimizing vehicle routes while satisfying customer demands.

**Maximizing Customer Satisfaction:** Customer satisfaction may involve multiple aspects, such as punctual delivery, cargo integrity, and service attitude. In VRPPD, customer satisfaction is often enhanced by ensuring timely delivery within the customer's specified time window.

### 2.2.2. Multi-Objective Optimization:

Multi-objective optimization refers to the need to consider multiple optimization objectives simultaneously when addressing VRPPD, as these objectives may conflict or constrain each other. For instance:

**Minimizing Total Vehicle Travel Distance and Total Vehicle Operating Costs:** These two objectives are often interrelated, as increased travel distance often leads to higher costs. However, in some cases, other cost factors such as tolls and highway fees may exist, making these two objectives not entirely aligned.

**Minimizing the Number of Vehicles and Carbon Emissions:** Reducing the number of vehicles can lower operating costs, but it may also increase the loading capacity of each vehicle, potentially leading to higher carbon emissions. Therefore, these two objectives require comprehensive consideration to achieve sustainable development.

**Balancing Customer Service Levels with Minimized Transportation Costs:** This may involve minimizing transportation costs while ensuring customer satisfaction in terms of punctuality, cargo integrity, and other aspects.

In practical scenarios, VRPPD often involves multiple stakeholders, including customers, enterprises, and the environment. Therefore, it is typically a multi-objective optimization problem. Solving such problems often requires the application of complex algorithms and techniques, such as multi-objective genetic algorithms and particle swarm optimization, to find optimal solutions or solution sets that balance various objectives.

## 2.3. Constraints

In the context of vehicle routing problems, vehicles must adhere to specific constraints regarding their starting and ending positions. Specifically, all dispatched vehicles must depart from a preset depot, execute their designated delivery tasks, and subsequently return to the initial depot. Additionally, to ensure efficient task completion, each delivery point can only be visited once by a single vehicle throughout the entire distribution process. This approach helps avoid unnecessary path repetitions and resource wastage. During the planning phase, it is crucial to ensure that the number of visits by

any vehicle to a specific pickup point does not exceed the total number of deliveries associated with that pickup point. Simultaneously, the number of visits by a single vehicle to a particular pickup point is also strictly limited, preventing it from exceeding the total number of deliveries corresponding to that pickup point.

Within the distribution network, it is imperative to ensure that each delivery point is supplied by exactly one pickup point from its entire set of potential pickup locations. This constraint guarantees the fulfillment of delivery points' supply demands while mitigating resource wastage and conflicts. Furthermore, once a vehicle selects a specific pickup point to supply a delivery point, a feasible path must exist from another node to that pickup point, enabling the vehicle to arrive and complete the pickup task smoothly. During the process of picking up and delivering goods, the selected pickup node and its corresponding delivery point must be visited by the same vehicle. This approach reduces the number of vehicles required and shortens the travel distance, thereby enhancing overall delivery efficiency. Additionally, the selected pickup point must be visited prior to the delivery node to ensure the timeliness and effectiveness of the delivery.

It is noteworthy that all pickup and delivery points must adhere to preset time window constraints during the distribution process. This means that vehicles must arrive and complete their tasks within the specified time frame. Moreover, the loading capacity of vehicles is a critical factor to consider during planning. The loading volume of any vehicle at any node must not exceed its maximum capacity, preventing safety issues caused by overloading and ensuring efficient delivery. Finally, the number of vehicles departing from the depot is strictly limited, not exceeding the initially set vehicle count. This constraint helps control operational costs and ensures the sustainability of the entire distribution process.

## **2.4. Optimization Method**

The Vehicle Routing Problem with Pickup and Delivery (VRPPD) is one of the most extensively studied and critically important issues in the field of routing planning. As VRPPD continues to find applications in practice, new and more complex problem models are constantly emerging alongside novel application scenarios. Simultaneously, the difficulty in solving these problems is also escalating. The VRPPD is recognized as an NP-hard problem, which has prompted numerous scholars to make theoretical innovations in solving methods. These methods can be broadly classified into exact algorithms and heuristic algorithms.

Exact algorithms, as their name suggests, aim to find the optimal solution to the VRPPD. They typically involve mathematical modeling and the application of advanced optimization techniques such as linear programming, integer programming, or dynamic programming. While exact algorithms can provide the most accurate solutions, their computational complexity often increases exponentially with the size of the problem, limiting their practical application to smaller-scale instances.

On the other hand, heuristic algorithms are designed to find good-enough solutions to the VRPPD within a reasonable amount of time. These algorithms often rely on problem-specific heuristics or meta-heuristics such as genetic algorithms, simulated annealing, or ant colony optimization. By exploring the search space efficiently, heuristic algorithms can provide satisfactory solutions even for large-scale problems, although they may not always guarantee the optimality of the solution.

The diversity and complexity of VRPPD scenarios have led to the development of various hybrid algorithms that combine the strengths of both exact and heuristic methods. These hybrid approaches often leverage the computational efficiency of heuristic algorithms to generate initial solutions and then employ exact algorithms to further optimize these solutions.

In summary, the VRPPD remains a challenging but crucial problem in the field of routing planning. The continuous emergence of new problem models and scenarios, coupled with the increasing difficulty in solving them, has prompted scholars to explore innovative solutions. The development

of both exact and heuristic algorithms, as well as hybrid approaches, has contributed significantly to the advancement of VRPPD research and its practical applications.

### **3. VRPPD WITH SELECTIVE PICKUP**

In practical delivery scenarios, the demand of a delivery point may be fulfilled by only one pickup point, or there may be multiple pickup points that can satisfy the demand simultaneously. Consequently, Berbeglia et al. (2007)[5] classified the vehicle routing problem with pickup and delivery (VRPPD) into three types based on the correspondence between pickup and delivery points: one-to-one (1-1-VRPPD), many-to-many (M-M-VRPPD), and one-to-many-to-one (1-M-1-VRPPD). In the one-to-one scenario, each commodity has a designated single pickup point and a single delivery point, meaning that a single pickup point supplies a single delivery point for a specific commodity, such as in express delivery services and door-to-door transportation. In the many-to-many scenario, there are multiple pickup points and multiple delivery points for each commodity. In the 1-M-1-VRPPD case, some goods that need to be delivered are transported from the warehouse to delivery nodes, while other goods provided by these delivery nodes are simultaneously brought back to the warehouse.

#### **3.1. One-to-One (1-1-VRPPD)**

In the one-to-one selective access pickup problem, the revenue is related to the number of pickup and delivery demands that can be fulfilled. Each satisfied pair of pickup and delivery demands generates corresponding revenue. Therefore, the key to solving this type of problem lies in selecting the demands that maximize the overall total revenue while considering the constraints on path length or delivery time. Additionally, minimizing the total cost is also an essential consideration.

Li and Chen (2016)[6] studied the scenario where carriers cooperate to fulfill some requests and selectively serve a portion of requests from other carriers to maximize revenue. They constructed a model with the objective of maximizing net revenue. Riedler and Raidl (2018)[7] explored the ride-hailing problem, considering the maximization of revenue and the inconvenience of not serving all customer requests. They developed a mixed-integer linear programming model and designed a logic-based Benders decomposition and branch-and-check algorithm to solve the problem.

Baniamerian et al. (2018) [8] investigated the scenario where pickup vehicles depart from a cross-dock to collect goods from multiple optional suppliers, return to the dock for unloading, and outbound vehicles then load and deliver these goods to the corresponding customers. The ultimate goal is to maximize net revenue, which is obtained by subtracting costs from total revenue. Chami et al. (2019)[9] proposed a selective pickup and delivery vehicle routing problem that considers both maximizing revenue and minimizing cost. They designed a lexicographic method to solve it. Qiu and Feuerriegel (2016)[10] studied the pickup and delivery vehicle routing problem with paired demand constraints, deliverable time constraints, loading constraints, heterogeneous vehicles, and simultaneous pickup and delivery features. They employed graph search algorithms and maximum set partitioning formulas for solving the problem. Peng et al. (2019)[1] introduced transfer points to the SPDPTWPD model, allowing goods to be transferred from one vehicle to another at a transfer point, thus exploring better solutions. Bruni and Toan (2021)[11] studied a multi-vehicle pickup and delivery vehicle routing problem with revenue under uncertain travel times. In this problem, the travel time between two nodes is a continuous random variable. Due to the uncertainty in arrival times at certain nodes, vehicles are allowed to incur penalty costs for violating time windows.

#### **3.2. Many-to-Many (M-M-VRPPD)**

In the M-M-VRPPD scenario, a delivery point may correspond to multiple pickup points in different locations. Therefore, before delivering to the demand point, a pickup point needs to be selected for

picking up goods and subsequent delivery. Based on the number of pickup points that need to be visited for delivery, the problem can be classified into M-M-VRPPD that visits all pickup points and M-M-VRPPD that visits a subset of pickup points.

Anily and Hassin (1992) [12] solved the multi-commodity pickup and delivery problem with multiple pairs using a polynomial approximation algorithm. In this problem, a node can simultaneously make a delivery request and provide at most one type of commodity for delivery. The vehicles return to the distribution center after completing all pickup and delivery demands. Hernández-Pérez and Salazar-González (2004) [13] proposed a branch-and-cut algorithm to solve the one-commodity pickup and delivery traveling salesman problem (1-PDTSP), where any commodity picked up from a pickup customer can be delivered to any delivery demand point, without constraints on the source of delivery points. For the same problem model, Hernández-Pérez and Salazar-González (2004) [14] also proposed a greedy algorithm improved with a K-best criterion. Subsequently, the same authors further combined a greedy random search process with a meta-heuristic algorithm based on variable neighborhood descent, achieving new breakthroughs in the results of solving example instances. Addressing the same issue, Zhao et al. (2009) [15] introduced a hybrid genetic algorithm for solving the problem and demonstrated its superior convergence properties. Mladenović et al. (2012) [16] proposed a variable neighborhood search algorithm for solving the problem. Shi XY et al. (2009) [17] considered that the unrestricted maximum driving distance of vehicles in the 1-PDTSP problem is unrealistic. Therefore, they first proposed a 1-PDTSP problem considering multi-vehicle distribution. Ai T and Kachit V (2009) [18] employed a particle swarm optimization algorithm to solve the simultaneous pickup and delivery vehicle routing problem and obtained new optimal solutions for some known instances. Hernández-Pérez and Salazar-González (2014) [19] also extended the single-commodity problem to multiple commodities, applying a branch-and-cut algorithm to solve the mixed-integer programming model of the multi-commodity traveling salesman problem (M-PDTSP). They also proposed some effective inequalities to improve the algorithm's performance.

The M-M-VRPPD problem also exists in the sharing economy, such as the redeployment problem of shared bicycles. Qiao J et al. [20] proposed six methods for constructing initial solutions and two heuristic algorithms. Their results identified factors that influence the final solution of the shared bicycle rebalancing problem under heuristic algorithms. They pointed out that the high quality and diversity of initial solutions have a significant impact on finding high-quality final solutions.

### **3.3. One-to-Many-to-One (1-M-1-VRPPD)**

Gribkovskaia et al. (2008) [1] investigated a scenario where vehicles fulfill delivery demands while optionally picking up goods from certain nodes. They termed this the Single Vehicle Routing Problem with Deliveries and Selective Pickups (SVRPDSP). Their objective function aimed to minimize the net cost, which is the total travel cost minus any additional income. To solve this problem, they employed an improved simulated annealing (SA) approach. Gutierrez-Jarpa et al. (2009) [21] used a branch-and-cut algorithm to tackle the SVRPDSP. Subsequently, Gabriel (2010) [22] extended the SVRPDSP model by incorporating time windows, resulting in the VRPDSPTW model. They employed a branch-and-price algorithm for solving it. Assis and Maravilha (2013) [23] examined the Vehicle Routing Problem with Fixed Deliveries and Selective Pickups. Their study considered two objectives: minimizing the vehicle's travel cost and minimizing the number of nodes where no pickup was made. To address this, they designed a multi-objective iterative local search algorithm

Coelho and Munhoz (2012) [24] studied the solution approach for the SVRPDSP, developing a hybrid heuristic algorithm that combines an exact algorithm for generating initial solutions with a generalized variable neighborhood search. Bruck et al. (2013) [25] studied the multiple-vehicle version of the problem, termed the Multiple Vehicle Routing Problem with Deliveries and Selective Pickups (MVRPDSP). They established a mixed-integer programming model for the MVRPDSP, aiming to minimize the net cost, and proposed a hybrid algorithm that combines clustering with an

exact solution approach. Coelho et al. (2015)[26] introduced a four-neighborhood VNS to solve the SVRPDSP.

Due to the presence of nodes with dual demands for both pickup and delivery in the one-to-one selective pickup and delivery vehicle routing problem, scholars often rely on virtual nodes for solving. However, Bruck and Iori (2017) [27] directly constructed a mathematical model for the VRPDSPD original network without using virtual nodes and developed an exact heuristic algorithm to find optimal solutions for all known benchmark instances within a reasonable computation time.

#### **4. VRPPD WITH SPLIT PICKUPS**

The vehicle routing problem with split pickups and deliveries concerns the scenario where multiple vehicles can pick up goods at a single delivery point to better serve the demands of that point. This approach relaxes the constraint that a pickup point can only be visited once by a single vehicle, thereby enhancing the flexibility of the delivery process. Current research in this area is limited, and most of the existing studies focus on the splitting behavior of pickup nodes. Lee et al. (2006)[28] investigated the vehicle routing problem with split pickups and deliveries (VRPPD), considering a specific scenario where multiple pickup points and a depot exist in the network. Each pickup point supplies a certain quantity of products to the depot, and these products can be picked up in batches. The objective is to devise a cost-effective transportation plan that ensures all the products supplied by the customer points are delivered to the depot. Based on the characteristics of the model, the authors employed a dynamic programming algorithm for solution.

Nowak et al. (2008)[29] studied the VRPPD with splitting behavior and developed a heuristic algorithm for solving it. Their analysis validated that allowing split pickups can reduce transportation costs and the number of vehicles required for the vehicle routing problem with pickups and deliveries. Subsequently, Nowak et al. (2009)[30] explored the relationship between the benefits of allowing split pickups and factors such as the average pickup volume per customer, the ratio of pickup points to delivery points, and the clustering degree of pickup and delivery points, using real-world examples. In a subsequent study, Nowak et al. (2012)[31] introduced a constraint on the preferential access order of nodes to the previous model, mandating that all pickup points must be visited before any delivery point, and employed an exact algorithm for solution. Andersson et al. (2011)[32] also addressed the routing problem for marine vessels, studying the ship routing problem with split pickups, deliveries, and time windows, and establishing an arc-flow model.

In summary, the vehicle routing problem with split pickups and deliveries represents a research area with practical application value. It not only enhances delivery efficiency but also saves costs for transportation enterprises. However, due to the complexity of this problem, current research is still inadequate, and there remains significant room for future exploration.

#### **5. VRPPD WITH MULTI VISIT**

Allowing multiple pickups by a single vehicle refers to the situation where the same vehicle visits a pickup point more than once. Although this scenario is common in real life, there are relatively few studies on it in the literature. Some notable studies on allowing multiple visits to a pickup point by a single vehicle include those by Takada et al.[4] and Azadian[33]. Currently, most cases of a single vehicle visiting the same node multiple times are caused by the splitting of node demand or the allowance of reloading in the distribution network. Demand splitting involves dividing a single demand into multiple demands, which are then transported by the same or different vehicles. These situations lead to multiple visits by the same vehicle to a node and distribute the node's demand across multiple paths. Similar situations also exist in bicycle redeployment networks, where a station may be visited multiple times.

Salazar-González and Santos-Hernández (2015)[34] proposed the single-vehicle single-commodity pickup and delivery traveling salesman problem with splittable demands. Reloading and demand splitting allowed customers to be visited multiple times. They used a branch-and-cut algorithm based on Benders decomposition to solve instances with up to 50 customers and found that in the optimal case, each customer was visited up to two or three times in a route. Hernández-Pérez and Salazar-González (2018) [35] studied the same problem and designed a meta-heuristic algorithm to solve instances with up to 500 customers. For the same problem, Hernández-Pérez and Salazar-González (2022)[34] employed a new branch-and-cut algorithm for solving. Erdoğan and Battarra et al. (2015)[36] examined the single-vehicle, multiple-visit, reloading-allowed, loading-constrained, single-commodity BRP. This study aimed to determine the number of bicycles loaded by each vehicle at each station and the vehicle routing that minimized the total travel cost. Cruz and Subramanian (2017)[37] studied a BRP with multiple visits allowed, reloading allowed, loading constraints, a single vehicle, and a single commodity. In this problem, vehicles were allowed to transport bicycles from some stations to temporary storage points, and later, the same vehicle would visit those points again for pickups. They designed an iterative local search algorithm to solve the problem.

Bulhões and Subramanian (2018) [38] investigated a BRP with maximum path time, service time proportional to the number of bicycles loaded and unloaded by the vehicle, a maximum of  $N$  visits by the same vehicle to a station, and the constraint that a station cannot be visited by different vehicles. Their objective was to minimize the total vehicle travel cost. They developed a branch-and-cut algorithm and an iterative local search meta-heuristic to solve the problem. Casazza and Ceselli (2021) [39] studied a single-commodity, multi-vehicle, split pickup and delivery VRPPD, where each node could be visited multiple times. They proposed a new formulation that decomposed routes into simpler substructure sequences called clusters, alleviating the combinatorial explosion of feasible solutions. Xu et al. (2017)[38] examined the unpaired pickup and delivery vehicle routing problem with multi-visit (USDSPDVRP), which features a single depot, a single vehicle type with multiple vehicles, multiple commodities, multiple visits to nodes, unmatched demands, and splittable demands. Their objective was to minimize the total transportation distance. They used a tabu search algorithm to solve instances with up to five commodities and 10 customers.

## 6. SUMMARY

With the growth of logistics operations, the transportation and delivery tasks of logistics enterprises have become routine duties. The thriving development of e-commerce has not only driven the innovation of business models but also profoundly transformed the logistics distribution paradigm. Retail enterprises now offer a diverse range of services to consumers through various channels, including physical stores, e-commerce platforms, mobile e-commerce, and even omni-channel sales. This transformation has led to the emergence of multiple optional pickup nodes during the logistics distribution process. Delivery personnel can choose to pick up goods from e-commerce warehouses or offline stores closer to consumers, based on their needs, to meet consumer demands for product delivery. This phenomenon manifests in practical distribution as a complex scenario where a single delivery demand corresponds to multiple selectable pickup points.

The practice of logistics distribution with multiple pickup points poses numerous new challenges, such as determining which pickup point to choose, scheduling the time windows between pickup points and demand points, and handling multiple vehicles making multiple pickups. These complexities make the problem more intricate than traditional vehicle routing problems for pickup and delivery. For researchers exploring vehicle routing optimization, this undoubtedly poses a significant challenge. Nevertheless, these challenges present rich research opportunities in the fields of transportation science and logistics. We hope that through the introduction and review presented in this paper, we can contribute to future research in this vibrant and promising emerging field of logistics distribution.

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