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Analysis of a Chemotaxis Model with Nonlinear Production Terms in Wheat Pest Control

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ABSTRACT

Chemotaxis refers to the influence of chemical substances in the environment on the movement of motile species and serves as an important mechanism of cellular communication. Cells communicate with each other by secreting chemical substances, which in turn determine their movement and differentiation. In order to better understand the growth patterns and aggregation distribution of biological populations, an increasing number of scientists have begun to study chemotaxis and describe the observed phenomena by establishing mathematical models. Among these models, the most classical one is the Keller–Segel chemotaxis model. The first part of this paper mainly introduces the development process and current research status of chemotaxis models related to the content of this study, and provides a general overview of the main topics investigated in this paper. The second part of this paper, namely Chapter 2, studies the following chemotaxis mode (which is a parabolic–parabolic–elliptic system with nonlinear production terms and a Logistic-type source). We analyze the global boundedness of its solutions, where is a bounded domain and the system is subject to Neumann boundary conditions. Here, the nonlinear production terms of the attractive and repulsive chemical substances are described by and respectively. Moreover, the Logistic source in this model satisfies Finally, it is concluded that if that is, when either the Logistic source or the repulsive term dominates the attractive term, the solution is globally bounded. Moreover, in the three balanced cases or the boundedness of the solution depends on the magnitude of the corresponding coefficients. The third part of this paper mainly summarizes the main results of the study and provides a preliminary outlook on future research on chemotaxis models with nonlinear production terms.

KEYWORDS

Logistic source; Chemotaxis; Nonlinear production term; Boundedness

1. INTRODUCTION

Wheat is a vital cereal crop and one of the primary sources of dietary protein for humans. Globally, it ranks as the second most produced crop, following only maize, and serves as a major staple food in China [1]. Consequently, the development of the wheat industry is closely linked to national food security and social stability. Ensuring the safe and sustainable production of wheat is a critical component of global food security [2]. Approximately 70/100 of the world's wheat consumption is used directly for food, while the remainder supports various industries, including brewing, animal feed, pharmaceuticals, and food flavoring [3]. As one of the world's leading wheat producers, China accounts for about one-fifth of global wheat output, playing a significant role in the international grain market [4].

In recent years, climate change, shifts in farming practices, and the emergence of pesticide-resistant pests have led to an increasing variety and frequency of wheat diseases and pests [5]. This trend has severely hindered improvements in both wheat yield and quality, becoming a major bottleneck in modern agricultural production [6]. Among these pests, wheat aphids are some of the most common and widely distributed worldwide. They feed on the sap of wheat leaves, stems, and young ears using their piercing-sucking mouthparts, directly damaging the plant's nutrient transport system [7-8]. This results in leaf curling, chlorosis, suppressed photosynthesis, and stunted plant growth. The wheat midge poses a significant threat to wheat ears by extracting the milky content from developing grains, which causes grain shriveling, poor development, or complete desiccation. Another major pest in wheat fields is the armyworm, which can rapidly consume large portions of wheat foliage, leading to leaf damage, withering, and impaired plant health [9-11].

Over the years, various measures have been employed to control wheat diseases and pests in the field [12]. Agricultural practices include crop rotation, enhancement of crop resistance, soil management, and fertilization [13]. Biological control methods involve the use of natural enemies and the application of biopesticides. Chemical control focuses on the rational use of chemical agents, as well as the timing and methods of pesticide application. Physical control strategies include the use of traps and repellent devices. While these approaches can effectively suppress pest populations in the short term, they often demand substantial human and material resources from an economic perspective and may compromise regional ecological stability from an ecosystem standpoint [14-16]. Therefore, there is an urgent need to introduce interdisciplinary technologies. The integration of biomathematics provides a scientific foundation for addressing this issue in a more systematic and sustainable manner [17].

Chemotaxis refers to the influence of chemical substances in the environment on the movement of motile species. This may lead to strictly directed motion or partially directed motion combined with random reorientation. Positive chemotaxis refers to the movement of motile species toward regions with relatively higher concentrations of a chemical substance, whereas negative chemotaxis refers to movement toward regions with relatively lower concentrations. Substances that induce positive chemotactic movement are called chemoattractants, while those that cause negative chemotactic movement are referred to as chemorepellents. For cellular organisms, chemotaxis serves as an important means of communication. Cells interact with one another by secreting chemical substances, which in turn determine how they move and develop. For example, food itself can emit pleasant odors that attract insects to approach it, whereas insect repellents release irritating and unpleasant odors, causing insects to move away from the source. Chemotactic phenomena are almost ubiquitous in the activities of biological populations. From a microscopic perspective, they occur in cellular organisms and viruses, while from a macroscopic perspective they can be observed in animals such as spiders and ladybirds. Moreover, chemotaxis also plays an important role in biological evolution, such as in the formation of the digestive and respiratory systems, the development of bones, and the formation of skin tissue cells [18–23]. Therefore, the study of chemotaxis is of great significance in biology. These phenomena have motivated many scientists to investigate chemotaxis and to describe the observed processes using mathematical models.

Studying biological phenomena through partial differential equations has long occupied an important position in modern mathematics. By establishing mathematical models for the biological phenomena under investigation, researchers can indirectly infer biological processes by analyzing the properties and characteristics of the solutions to these models. Among these models, the Keller–Segel model was the first mathematical model established to study chemotactic phenomena [24-26]. Its basic form can be written as

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v), & (x,t)\in\Omega\times(0,T), \\ \tau v_t = \Delta v - \alpha v + \beta u, & (x,t)\in\Omega\times(0,T), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0, & (x,t)\in\partial\Omega\times(0,T), \\ u(x,0)=u_0(x), \quad v(x,0)=v_0(x), \quad x\in\Omega, \end{cases} \tag{1}$$

Where the chemotactic sensitivity is represented by χ. In (1), when $\chi > 0$, the secreted chemical substance has an attractive effect on biological cells; when $\chi < 0$, the effect is repulsive. For the second equation of (1), when $\tau = 0$, the equation becomes an elliptic equation. In this case, the model describes the situation in which the diffusion coefficient tends to infinity, so that the growth rate of the chemical substance can be neglected. So far, many scientists have developed and extended this model. When $\tau = 1$, the model becomes a parabolic–parabolic chemotaxis model. For this model, the following result has been obtained. Osaki and Yagi [27] proved that when $N = 1$, for any nonnegative initial data $u_0(x)\in C^0(\overline{\Omega})$, $v_0(x)\in W^{1,q}(\overline{\Omega})(q > N)$ the solution is always globally bounded. Toshitaka Nagai, Takashi Senba, and Kohji Yoshida [28] proved that when $N = 2$, if $\int_\Omega u_0 dx < \dfrac{4\pi}{\alpha\chi}$, the solution is globally bounded. Meanwhile, in the radially symmetric case, they also showed that the global boundedness of solutions can be guaranteed provided that $\int_\Omega u_0 dx < \dfrac{8\pi}{\alpha\chi}$.

For the above Keller–Segel model, many interesting results have been obtained by researchers in recent years [29-35]. In particular, Juan Tello and Michael Winkler [36] proved that for the chemotaxis system

$$\begin{cases} u_t = \Delta u - \chi\nabla.(u\nabla v) + f(u) & \Omega\times(0,T) \\ 0 = \Delta v + \alpha u^k - \beta v & \Omega\times(0,T) \end{cases} \tag{2}$$

When $f(u)\le u(a-bu^s)$, $k = s = 1$, and $\dfrac{N-2}{N}\chi < b$ hold, the solution is globally bounded. For the more general case of model (2), namely when $k,s > 0$ holds, Yong Wang and Tao Xiang [37] obtained the following result

$$if\ s > k,\ or\ if\ both\, s = k\ and\, \frac{kN-2}{kN}\chi < b \tag{3}$$

Then the solution of the chemotaxis model is globally bounded. For the attraction–repulsion system

$$\begin{cases} u_t = \Delta u - \chi\nabla.(u\nabla v) + \xi\nabla.(u\nabla w) + f(u) & \Omega\times(0,T), \\ 0 = \Delta v + \alpha u - \beta v & \Omega\times(0,T), \\ 0 = \Delta w + \gamma u - \delta w & \Omega\times(0,T), \end{cases} \tag{4}$$

With linear production terms, when $f(u)\le u(a-bu)$ holds, Zhang and Li [38] proved that the solution of model (4) admits a globally bounded classical solution for any nonnegative initial data $u_0(x)\in C(\Omega)$, provided that one of the following conditions holds:

$$(a_1)\ \ \alpha\chi-\gamma\xi\le b;\ \ (b_1)\, n\le 2;\ \ (c_1)\ \frac{n-2}{n}(\alpha\chi-\gamma\xi)<b\ \ n\ge 3 \tag{5}$$

Recently, Hong et al. studied the following model

$$\begin{cases} u_t=\Delta u-\chi\nabla.(u\nabla v)+\xi\nabla.(u\nabla w)+f(u), & (x,t)\in\Omega\times(0,T),\\ 0=\Delta v+\alpha u^k-\beta v, & (x,t)\in\Omega\times(0,T),\\ 0=\Delta w+\gamma u^l-\sigma w, & (x,t)\in\Omega\times(0,T),\\ \dfrac{\partial u}{\partial v}=\dfrac{\partial v}{\partial v}=\dfrac{\partial w}{\partial v}=0, & (x,t)\in\partial\Omega\times(0,T),\\ u(x,0)=u_0(x), & x\in\Omega. \end{cases} \tag{6}$$

Based on the Keller–Segel model. This model is a parabolic–elliptic–elliptic attraction–repulsion chemotaxis system with nonlinear production terms and a Logistic growth source. Their main results are summarized in the following theorem:

Theorem Let $u_0(x)\in C(\Omega)$ be nonnegative initial data.

If $k<\max\{l,s,\frac{2}{N}\}$ holds, then the system (6) admits a globally bounded classical solution.

Assume that $k=\max\{l,s\}\ge\frac{2}{N}$ holds. Then the system (6) admits a globally bounded classical solution, provided that one of the following conditions is satisfied:

$(a)\ k=l=s,\ \ \frac{(kN-2)}{kN}(\alpha\chi-\gamma\xi)<b;$

$(b)\ \ k=l>s, \alpha\chi-\gamma\xi<0;$

$(c)\ \ k=s>l, \frac{(kN-2)}{kN}\alpha\chi<b.$

Remark 1. In system (6), the behavior of the solution is determined by the interaction among four mechanisms: random diffusion, the repulsion term, the attraction term, and the Logistic source. Except for the attraction term, the other three mechanisms are all favorable for the global boundedness of the solution. Theorem 1 illustrates how the nonlinear exponents $k,l,s>0$, (which represent the nonlinear production terms of v and w, and the Logistic source of u, respectively) influence the behavior of the solution. More precisely, if the attraction term is dominated by one of the mechanisms among the Logistic source, random diffusion, and the repulsion term, that is, when $k<\max\{l,s,\frac{2}{N}\}$, then the solution is globally bounded. In the three balanced cases $k=s>l, k=l>s$, or $k=s=l$, the global boundedness of the solution depends on the magnitude of the associated coefficients.

Remark 2. We now compare the obtained results with those for system (4) with linear production terms. Clearly, system (4) is a special case of model (6) when $k=s=l=1$ holds. Theorem 1 includes condition (5) in [38, Theorem 1.1]. It should be pointed out that condition (a_1) itself is already contained in the other conditions (b_1) and (c_1). Moreover, when $\xi=l=0$ holds, one can obtain the global boundedness condition (3) for the chemotaxis system (2) with nonlinear production term αu^k.

Remark 3. It can also be observed that the boundedness criteria for the attraction–repulsion chemotaxis system (6) with nonlinear production terms are parallel to those for the corresponding attraction–repulsion chemotaxis model with nonlinear diffusion and linear production terms [39].

Based on the work of Hong et al., this paper further investigates the Keller–Segel model describing a parabolic–parabolic–elliptic attraction–repulsion chemotaxis system with nonlinear production terms and a Logistic growth source.

The first part of this paper introduces the research background and significance of chemotaxis models and provides an overview of their development. In the second part, we investigate a parabolic–parabolic–elliptic chemotaxis model with nonlinear production terms and a Logistic-type source, namely

$$\begin{cases} u_t = \Delta u - \chi \nabla.(u \nabla v) + \xi \nabla.(u \nabla w) + f(u), & (x,t) \in \Omega \times (0,T), \\ v_t = \Delta v + \alpha u^k - \beta v, & (x,t) \in \Omega \times (0,T), \\ 0 = \Delta w + \gamma u^l - \sigma w, & (x,t) \in \Omega \times (0,T), \\ \dfrac{\partial u}{\partial v} = \dfrac{\partial v}{\partial v} = \dfrac{\partial w}{\partial v} = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases}$$

By applying relevant theories of partial differential equations, we analyze the solutions of the above attraction–repulsion chemotaxis model and prove the global boundedness of solutions under different balance conditions.

2. PARABOLIC–PARABOLIC–ELLIPTIC MODEL WITH NONLINEAR PRODUCTION TERM AND LOGISTIC SOURCE

2.1. Problem Introduction

The main model studied in this paper is the following parabolic–parabolic–elliptic chemotaxis system with nonlinear production terms and a Logistic-type source:

$$\begin{cases} u_t = \Delta u - \chi \nabla.(u \nabla v) + \xi \nabla.(u \nabla w) + f(u), & (x,t) \in \Omega \times (0,T), \\ v_t = \Delta v + \alpha u^k - \beta v, & (x,t) \in \Omega \times (0,T), \\ 0 = \Delta w + \gamma u^l - \sigma w, & (x,t) \in \Omega \times (0,T), \\ \dfrac{\partial u}{\partial v} = \dfrac{\partial v}{\partial v} = \dfrac{\partial w}{\partial v} = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{7}$$

Here, $\Omega \subset R^N$ $(N \geq 1)$ is a bounded domain with smooth boundary, and ν denotes the outward normal vector. The Logistic source $f \in C^2[0,\infty)$ satisfies $f(u) \leq u(a - bu^s)$ and $f(0) \geq 0$, and the parameter $\alpha, \beta, \gamma, \delta, \chi, \xi, a, b, k, l > 0$. In model (7), the cell density is denoted by u, while the concentrations of two different chemical signals produced by the cells are denoted by v and w, respectively. These two signals exert attractive and repulsive effects on u, respectively, and $\chi, \xi > 0$ represents the chemotactic sensitivity. It should be mentioned that, in contrast to the linear production term u, the nonlinear production term u^{α} is used to model aggregation patterns formed by the chemotactic behavior of certain bacteria (see [40, Chapter 5] and [41-44]). Accordingly, the production terms of the signals v and w in the model are both nonlinear and take the forms αu^{k} and γu^{l}, respectively. As we will see, this has a substantial influence on the behavior of the solutions.

2.2. Main Theorem of the Model

The main results of this paper are summarized in the following theorem:

Theorem 1. Let $u_0(x) \in C(\Omega)$ be nonnegative initial data.

If $k < \max\{l, s\}$ holds, then system (7) admits a unique globally bounded classical solution.

Assume that $k = \max\{l, s\}$ holds. If there exists a positive constant θ_0, b_0, b_1 such that one of the following conditions is satisfied:

(a) $k = l = s,$ and b, or $\gamma\xi$ is sufficiently large such that $b + \theta_0 \gamma \xi > b_0$;

(b) $k = l > s,$ and $\gamma\xi$ is sufficiently large such that $\gamma\xi > b_1$;

(c) $k = s > l,$ and b is sufficiently large such that $b > b_0$, then system (7) admits a unique globally bounded classical solution.

The behavior of solutions to system (7) is determined by the interaction among four mechanisms: random diffusion, the repulsion term, the attraction term, and the Logistic source. Except for the attraction term, the other mechanisms are favorable for the global boundedness of solutions. Theorem 1 illustrates how the nonlinear exponents $k, l, s > 0$ (which represent the nonlinear production terms of v and w, and the Logistic source of u, respectively) influence the behavior of solutions. More precisely, if the attraction term is controlled by either the Logistic source or the repulsion mechanism, that is, $k < \max\{l, s\}$, then the solution is globally bounded. In the three balanced cases $k = s > l, k = l > s$ or $k = s = l$, the global boundedness of solutions depends on the magnitudes of the corresponding coefficients involved.

Now we compare our results with those of Hong for the parabolic–elliptic–elliptic system (6) with nonlinear production terms. Clearly, Hong's results are more explicit and precise, since the constant b is given by a specific numerical value. In contrast, in the present paper the second equation is a parabolic equation, which cannot be directly substituted into the estimate as in Hong's work where the corresponding equation is elliptic. Therefore, in this paper we need to employ the maximal Sobolev regularity to obtain the required estimates. However, the constants appearing in the corresponding regularity estimates cannot be made explicit. As a consequence, the parameter b can only be required to be sufficiently large. Although the resulting conclusion is therefore less precise, this limitation arises from the restrictions of the method currently available. Determining an explicit value for b will be an important direction for future research.

We can also observe that the boundedness criterion for the attraction–repulsion chemotaxis system (7) with nonlinear production terms is parallel to the corresponding boundedness criterion for the attraction–repulsion chemotaxis model with nonlinear diffusion and linear production terms studied in [39].

3. PRELIMINARIES

3.1. Young's Inequality and Hölder's Inequality

In the proof of the theorem, we make use of the following commonly used inequalities:

Young's inequality: Let $a>0,\ b>0,\ p>1,\ q>1,$ and $\frac{1}{p}+\frac{1}{q}=1$. Then $ab\le\frac{a^p}{p}+\frac{b^q}{q}$. In particular, When $p=q=2,$ the above inequality is also called the Cauchy inequality.

Let $\varepsilon>0$. By substituting a and b in the above inequality with $\varepsilon^{\frac{1}{p}}a$ and $\varepsilon^{-\frac{1}{p}}b$, respectively, we obtain the Young's inequality with ε:

$$ab\le\frac{\varepsilon a^p}{p}+\frac{\varepsilon^{-\frac{q}{p}}b^q}{q}\le\varepsilon a^p+\varepsilon^{-\frac{q}{p}}b^q.$$

In particular, when $p=q=2,$ the above inequality becomes

$$ab\le\frac{\varepsilon a^2}{2}+\frac{b^2}{2\varepsilon},$$

Which is called the Cauchy inequality with ε.

Hölder's inequality: Let $p>1,$ $q>1,$ and $\frac{1}{p}+\frac{1}{q}=1$. If $f\in L^p(\Omega)$ and $g\in L^q(\Omega),$ then $f,g\in L^1(\Omega),$ and

$$\int_\Omega|f(x)g(x)|\le\|f\|_{L^p(\Omega)}\|g\|_{L^q(\Omega)}.$$

In particular, when $p=q=2,$ the above inequality becomes

$$\int_\Omega|f(x)g(x)|\le\|f\|_{L^p(\Omega)}\|g\|_{L^q(\Omega)},$$

Which is called the Schwarz inequality.

3.2. Fundamental Theorems and Related Lemmas

ODE Comparison Principle [45, Lemma 3.4]: Consider the ordinary differential equation

$$\begin{cases}\dfrac{du}{dt}=f(t,u),\\ u(t_0)=u_0,\end{cases}$$

Where $t\ge t_0$, $f(t,u)$ is continuous in t, and satisfies a local Lipschitz condition in u. Let the maximal interval of existence for u be $[t_0,T)$, where T may be ∞. If, for any $t\in[t_0,T)$, $v=v(t)$, the following holds:

$$\begin{cases}\dfrac{dv}{dt}\le f(t,v),\\ v(t_0)\le u_0,\end{cases}$$

Then $v(t)\le u(t),\ t\in[t_0,T)$.

In this section, we deal with local solutions as preliminary knowledge. Within the appropriate fixed-point theory framework, the local existence of solutions to system (7) can be obtained using the standard theory of parabolic equations [46, 36]. The following lemma on the local existence of solutions is stated without proof.

Lemma 1. Assume that $\Omega\subset R^N (N\geq 1)$ is a bounded domain with a smooth boundary. Then, for any nonnegative initial data $u_0(x)\in C(\overline{\Omega})$, there exists a nonnegative function

$u,v,w\in C^0(\bar\Omega\times[0,T_{\max}))\cap C^{2,1}(\bar\Omega\times(0,T_{\max}))$ and $T_{\max}\in(0,\infty]$

Which is a classical solution of (7). If $T_{\max}<\infty$, then

$$\lim_{t\to T_{\max}}\|u(\cdot,t)\|_{L^\infty(\Omega)}=\infty.$$

Next, we introduce a key lemma to establish the global existence of solutions, which is a generalization of the result with arbitrary exponent $l>0$ [39, Lemma 2].

Lemma 2. Let (u,v,w) be a solution of system (7) determined by Lemma 1. Then, for any $l,\eta>0,\theta>1$, there exists $c_0=c_0(\eta,\theta,l)>0$ such that

$$\int_\Omega \omega^\theta\leq\eta\int_\Omega u^{l\theta}+c_0,\ t\in(0,T_{\max}) \tag{8}$$

Proof. Integrating the first equation of system (7), we obtain

$$\frac{d}{dt}\int_\Omega u=\int_\Omega u(a-bu^s)\leq a\int_\Omega u-\frac{b}{|\Omega|^s}(\int_\Omega u)^{1+s},\quad t\in(0,T_{\max})$$

By the ODE comparison principle, we have

$$\int_\Omega u\leq\max\{\int_\Omega u_0,(\frac{a}{b})^{\frac{1}{s}}|\Omega|\}:=M,\ t\in(0,T_{\max}). \tag{9}$$

Integrating the third equation of system (7), we obtain

$$\|\omega\|_{L^1(\Omega)}=\frac{\gamma}{\delta}\|u^l\|_{L^1(\Omega)} \tag{10}$$

Multiplying both sides of the third equation in system (7) by $\omega^{\theta-1}$ and integrating over Ω, we obtain

$$\frac{4(\theta-1)}{\theta^2}\int_\Omega\left|\nabla\omega^{\frac{\theta}{2}}\right|^2+\delta\int_\Omega\omega^\theta=\gamma\int_\Omega u^l\omega^{\theta-1}\leq\frac{\theta-1}{\theta}\delta\int_\Omega\omega^\theta+\frac{\gamma^\theta}{\theta\delta^{\theta-1}}\int_\Omega u^{l\theta},\ t\in(0,T_{\max})$$

By Young's inequality, we obtain

$$\|\omega\|_{L^{\theta}(\Omega)} \le \frac{\gamma}{\delta}\|u^l\|_{L^{\theta}(\Omega)}, \ t \in (0, T_{\max}) \tag{11}$$

$$\tfrac{4(\theta-1)}{\theta}\int_{\Omega}\left|\nabla \omega^{\frac{\theta}{2}}\right|^2 \le \frac{\gamma^{\theta}}{\delta^{\theta-1}}\int_{\Omega} u^{l\theta}, \quad t \in (0, T_{\max}) \tag{12}$$

According to the Ehrling lemma, for any $\eta > 0, \theta > 1$, there exists $\overline{C} = \overline{C}(\eta, \ \theta) > 0$ such that

$$\|\phi\|^2_{L^2(\Omega)} \le \eta\|\phi\|^2_{W^{1,2}(\Omega)} + \overline{C}\|\phi\|^2_{L^{\frac{2}{\theta}}(\Omega)}, \ \phi \in W^{1,2}(\Omega).$$

Taking $\phi = \omega^{\frac{\theta}{2}}$, from (10), (11), and (12) we obtain

$$\int_{\Omega}\omega^{\theta} \le \eta\int_{\Omega} u^{l\theta} + C_1\|u^l\|^{\theta}_{L^1} \tag{13}$$

Where $C_1 = C_1(\eta, \theta) > 0$. For $l \in (0,1]$, by Hölder's inequality and (9), we obtain

$$\|u^l\|^{\theta}_{L^1} < C_2 \tag{14}$$

Where $C_2 = C_2(\eta, \theta, l) > 0$. For $l > 1$, by the interpolation inequality, Young's inequality, and (9), we have

$$\|u^l\|^{\theta}_{L^1} \le \|u^l\|^{\theta_{\tau}}_{L^{\theta}}\|u^l\|^{\theta(1-\tau)}_{L1/l} \le \eta\int_{\Omega} u^{l\theta} + C_3 \tag{15}$$

Where $\tau = \dfrac{l-1}{l-\frac{1}{\theta}} \in (0,1)$, and $C_3 = C_3(\eta, \theta, l) > 0$. Combining (13)–(15), we obtain (8).

The following key lemma used in the proof of our main theorem is introduced without proof; it is a variant of the maximal Sobolev regularity.

Lemma 3 ([30], Lemma 2.5). Let $\sigma > 1$ and consider the following parabolic equation

$$\begin{cases} v_t = \Delta v + \alpha u^k - \beta v, & (x,t) \in \Omega \times (0,T), \\ \dfrac{\partial v}{\partial \nu} = 0, & (x,t) \in \Omega \times (0,T), \\ v(x,0) = v_0(x), & x \in \Omega. \end{cases}$$

For each $v_0 \in W^{2,\sigma}(\Omega)(\sigma > N)$ and $u \in L^{\sigma}((0,T); L^{\sigma}(\Omega))$ satisfying $\dfrac{\partial v_0}{\partial \nu}|\partial\Omega = 0$, there exists a unique solution

$$v \in W^{1,\sigma}((0,T); L^{\sigma}(\Omega)) \cap L^{\sigma}((0,T); W^{2,\sigma}(\Omega)).$$

Moreover, there exists $C_\sigma > 0$ such that if $\tau_0 \in [0,T), v(\cdot,\tau_0) \in W^{2,\sigma}(\Omega)(\sigma > n)$ and $\frac{\partial v(\cdot,\tau_0)}{\partial n}|\partial\Omega = 0$, then

$$\int_{\tau_0}^{T}\int_\Omega e^{\frac{\beta}{2}\sigma\tau|\Delta v|^\sigma} \le C_\sigma \alpha^\sigma \int_{\tau_0}^{T}\int_\Omega e^{\frac{\beta}{2}\sigma\tau} u^{k\sigma} + C_\sigma e^{\frac{\beta}{2}\sigma\tau_0}(\|v(\cdot,\tau_0)\|_{L^\sigma(\Omega)}^\sigma + \|\Delta v(\cdot,\tau_0)\|_{L^\sigma(\Omega)}^\sigma).$$

4. GLOBAL BOUNDEDNESS OF SOLUTIONS TO THE MODEL

4.1. Proof of the Global Boundedness of Solutions

The proof of Theorem 1 is given in this section.

Proof of Theorem 1. First, we show that for any $p > 1$, there exists $c = c(p) > 0$ such that

$$\int_\Omega u^p \le c, \quad t \in (0, T_{\max}). \tag{16}$$

For simplicity, we use c and c_ε to denote positive constants corresponding to $c = c(p)$ and $c = c(p,\varepsilon)$, espectively. These constants are independent of t and may vary from line to line.

Multiply both sides of the first equation in system (7) by u^{p-1} and integrate over Ω. Substituting the remaining two equations into the resulting expression yields

$$\begin{aligned}\frac{1}{p}\frac{d}{dt}\int_\Omega u^p \le& -\frac{4(p-1)}{p^2}\int_\Omega \left|\nabla u^{\frac{p}{2}}\right|^2 - \frac{\chi(p-1)}{p}\int_\Omega u^p \Delta v - \frac{\gamma\xi(p-1)}{p}\int_\Omega u^{p+l} + \frac{\delta\xi(p-1)}{p}\int_\Omega u^p \omega \\ &+ a\int_\Omega u^p - b\int_\Omega u^{p+s}, \quad t \in (0, T_{\max}).\end{aligned} \tag{17}$$

By Young's inequality and (8), for any $\varepsilon > 0$, we have

$$\frac{\delta\xi(p-1)}{p}\int_\Omega u^p \omega \le \frac{\varepsilon}{2}\int_\Omega u^{p+l} + c_\varepsilon \int_\Omega \omega^{\frac{p+l}{l}} \le \varepsilon \int_\Omega u^{p+l} + c_\varepsilon, \qquad t \in (0, T_{\max}) \tag{18}$$

Substituting (18) into (17) yields

$$\begin{aligned}\frac{1}{p}\frac{d}{dt}\int_\Omega u^p \le& -\frac{4(p-1)}{p^2}\int_\Omega \left|\nabla u^{\frac{p}{2}}\right|^2 - \frac{\chi(p-1)}{p}\int_\Omega u^p \Delta v - \left(\frac{\gamma\xi(p-1)}{p} - \varepsilon\right)\int_\Omega u^{p+l} \\ &+ a\int_\Omega u^p - b\int_\Omega u^{p+s} + c_\varepsilon, \qquad t \in (0, T_{\max}).\end{aligned} \tag{19}$$

According to Young's inequality, for any $\eta > 0$, we obtain

$$-\frac{\chi(p-1)}{p}\int_\Omega u^p \Delta v \le \chi \int_\Omega u^p |\Delta v| \le \eta \int_\Omega u^{p+k} + c_\eta \chi^{\frac{p+k}{k}} \int_\Omega |\Delta v|^{\frac{p+k}{k}}, \quad t \in (0, T_{\max}). \tag{20}$$

Substituting (20) into (19) and adding $\frac{\beta(p+k)}{2pk}\int_{\Omega} u^{p}$, we obtain

$$\begin{aligned}
&\frac{1}{p}\frac{d}{dt}\int_{\Omega} u^{p}+\frac{\beta(p+k)}{2pk}\int_{\Omega} u^{p} \\
&\le -\frac{4(p-1)}{p^{2}}\int_{\Omega}\left|\nabla u^{\frac{p}{2}}\right|^{2}+\eta\int_{\Omega} u^{p+k}+c_{\eta}\chi^{\frac{p+k}{k}}\int|\Delta v|^{\frac{p+k}{k}}-\left(\frac{\gamma\xi(p-1)}{p}-\varepsilon\right)\int_{\Omega} u^{p+l} \\
&\quad+\left(a+\frac{\beta(p+k)}{2pk}\right)\int_{\Omega} u^{p}-b\int_{\Omega} u^{p+s}, \qquad t\in(0,T_{\max}).
\end{aligned}$$

Applying the variation-of-constants formula, we derive

$$\begin{aligned}
\frac{d}{dt}\left(e^{\frac{\beta(p+k)}{2k}t}\frac{1}{p}\int_{\Omega} u^{p}\right)\le& -\frac{4(p-1)}{p^{2}}e^{\frac{\beta(p+k)}{2k}t}\int_{\Omega}\left|\nabla u^{\frac{p}{2}}\right|^{2}+\eta e^{\frac{\beta(p+k)}{2k}t}\int_{\Omega} u^{p+k}+c_{\eta}\chi^{\frac{p+k}{k}}e^{\frac{\beta(p+k)}{2k}t}\int_{\Omega}|\Delta v|^{\frac{p+k}{k}} \\
&-\left(\frac{\gamma\xi(p-1)}{p}-\varepsilon\right)e^{\frac{\beta(p+k)}{2k}t}\int_{\Omega} u^{p+l}+e^{\frac{\beta(p+k)}{2k}t}\left(a+\frac{\beta(p+k)}{2pk}\right)\int_{\Omega} u^{p} \\
&-be^{\frac{\beta(p+k)}{2k}t}\int_{\Omega} u^{p+s}, \qquad t\in(0,T_{\max}).
\end{aligned}$$

Integrating from τ_0 to t and applying Lemma 3, we obtain

$$\begin{aligned}
\frac{1}{p}\int_{\Omega} u^{p}<& -\frac{4(p-1)}{p^{2}}\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega}\left|\nabla u^{\frac{p}{2}}\right|^{2}+\left(a+\frac{\beta(p+k)}{2pk}\right)\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p}+ \\
&\left(\eta+c_{\eta}(\alpha\chi)^{\frac{p+k}{k}}C_{\frac{p+k}{k}}\right)\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p+k}-\left(\frac{\gamma\xi(p-1)}{p}-\varepsilon\right)\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p+l} \\
&-b\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p+s}+C,\ t\in(0,T_{\max}).
\end{aligned}$$

Case (i) $k<\max\{l,s\}$.

When $k<s$, by Young's inequality we obtain

$$\left(\eta+c_{\eta}(\alpha\chi)^{\frac{p+k}{k}}C_{\frac{p+k}{k}}\right)\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p+k}<\frac{b}{2}\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p+s}+C, \qquad t\in(0,T_{\max}). \tag{21}$$

Taking $\varepsilon=\frac{\gamma\xi(p-1)}{p}$ in (21), we obtain

$$\frac{1}{p}\int_{\Omega} u^{p}\le\left(a+\frac{\beta(p+k)}{2pk}\right)\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p}-\frac{b}{2}\int_{\tau_0}^{t} e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_{\Omega} u^{p+s}+C, \qquad t\in(0,T_{\max}).$$

Hence, inequality (16) is proved.

When $k<l$, by Young's inequality we obtain

$$(\eta+c_\eta(\alpha\chi)^{\frac{p+k}{k}}C_{\frac{p+k}{k}})\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+k}<\frac{\gamma\xi(p-1)}{2p}\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{l+p}+C,\ \ t\in(0,T_{\max}).$$

Taking in (21), we obtain

$$\frac{1}{p}\int_\Omega u^p\le(a+\frac{\beta(p+k)}{2pk})\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^p-(\frac{\gamma\xi(p-1)}{2p}-\varepsilon)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{l+p}$$
$$-b\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+s}+C,\qquad t\in(0,T_{\max}).$$

Taking $\varepsilon=\dfrac{\gamma\xi(p-1)}{2p}>0$ in (21), we obtain

$$\frac{1}{p}\int_\Omega u^p\le(a+\frac{\beta(p+k)}{2pk})\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^p-b\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+s}+C,\qquad t\in(0,T_{\max}).$$

Hence, inequality (21) is proved.

Case (ii) $k=\max\{l,s\}$.

For $k=l$, it follows from (21) that

$$\begin{aligned}\frac{1}{p}\int_\Omega u^p<&-\frac{4(p-1)}{p^2}\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega\left|\nabla u^{\frac{p}{2}}\right|^2+(a+\frac{\beta(p+k)}{2pk})\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^p+\\&(\eta+c_\eta(\alpha\chi)^{\frac{p+k}{k}}C_{\frac{p+k}{k}}-\frac{\gamma\xi(p-1)}{p}+\varepsilon)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+k}\\&-b\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{k+s}+C,\qquad t\in(0,T_{\max}).\end{aligned}\tag{22}$$

Suppose that $k=l=s$. Then inequality (22) becomes

$$\begin{aligned}\frac{\boldsymbol{1}}{p}\int_\Omega u^p<&-\frac{4(p-1)}{p^2}\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega\left|\nabla u^{\frac{p}{2}}\right|^2+\left(a+\frac{\beta(p+k)}{2pk}\right)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^p\\&-\left(b+\frac{\gamma\xi(p-1)}{p}-\eta-c_\eta(\alpha\chi)^{\frac{p+k}{k}}C_{\frac{p+k}{k}}-\varepsilon\right)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+k}+C,t\in(0,T_{\max})\end{aligned}$$

Let $\theta_0 = \dfrac{p-1}{p}$, $b_0 = \max\limits_{\eta>0}\{\eta + c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}}\}$. When $b + \gamma\xi\theta_0 > b_0$, choose ε sufficiently small such that $b + \dfrac{\gamma\xi(p-1)}{p} - \eta - c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}} - \varepsilon > 0$. Furthermore, by $k > 0$ and Young's inequality, we obtain that inequality (16) holds.

Suppose $k = l > s$. From equation (22) and Young's inequality, we obtain

$$\frac{1}{p}\int_\Omega u^p < -\left(\frac{\gamma\xi(p-1)}{p} - \eta - c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}} - \varepsilon\right)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+k} + C, \qquad t \in (0, T_{\max}).$$

Let $b_1 = \dfrac{p}{p-1}\max\limits_{\eta>0}\{\eta + c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}}\}$. When $\gamma\xi > b_1$, choose ε sufficiently small such that $\dfrac{\gamma\xi(p-1)}{p} - \eta - c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}} - \varepsilon > 0$. Furthermore, by $k > 0$ and Young's inequality, we obtain that inequality (16) holds.

Suppose $k = s > l$. Taking $\varepsilon = \dfrac{\gamma\xi(p-1)}{2p}$ in (21), we obtain

$$\begin{aligned}\frac{1}{p}\int_\Omega u^p &< \left(a + \frac{\beta(p+k)}{2pk}\right)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^p \\ &\quad - \left(b - \eta - c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}}\right)\int_{\tau_0}^t e^{-\frac{\beta(p+k)}{2k}(t-s)}\int_\Omega u^{p+k} + C,\ t \in (0, T_{\max}).\end{aligned}$$

When $b > b_0$, we have $b - \eta - c_\eta(\alpha\chi)^{\frac{p+k}{k}} C_{\frac{p+k}{k}} > 0$. Furthermore, since $k > 0$, inequality (16) holds.

Assume that $p > \max\{kN, 1N, 1\}$. Applying the parabolic and elliptic L^P-estimates to the second and third equations in (7), we obtain

$$\|v(\cdot,t)\|_{W^{2,p/k}}, \|\omega(\cdot,t)\|_{W^{2,p/l}} < c, \qquad t \in (0, T_{\max}),$$

Then, by the Sobolev embedding theorem, it follows that

$$\|v(\cdot,t)\|_{C^1(\bar\Omega)}, \|\omega(\cdot,t)\|_{C^1(\bar\Omega)} < c, \qquad t \in (0, T_{\max}).$$

Using the Moser iteration technique [32], we further derive that

$$\|u\|_{L^\infty(\Omega)} \le C$$

holds for all $t \in (0, T_{\max})$. Finally, by Lemma 2.1, we conclude that $T_{\max} = \infty$. This completes the proof of the theorem.

5. SUMMARY

This paper mainly investigates the global boundedness of solutions to a parabolic–parabolic–elliptic chemotaxis model with nonlinear production terms and a Logistic source of the form $u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + \xi \nabla \cdot (u \nabla w) + f(u), v_t = \Delta v + \alpha u^k - \beta v, 0 = \Delta w + \gamma u^l - \delta w$. The model is considered in a smooth bounded domain $\Omega \subset \mathbb{R}^N \left(N \geq 1\right)$ subject to homogeneous Neumann boundary conditions. Here, the nonlinear production terms describing the generation of chemoattractant and chemorepellent are given by αu^k and γu^l, respectively, and the Logistic source term $f \in C^2[0,\infty)$ satisfies $f(u) \leq u(a - bu^s)$ with $s > 0, f(0) \geq 0$. It is shown that if $\max\{l, s\} > k$, that is, either the Logistic source or the repulsion term dominates the attraction term, then the solution is globally bounded. Furthermore, in the three equilibrium cases $k = s > l, k = l > s$ or $k = s = l$, the boundedness of solutions is determined by the corresponding coefficients involved.

Our study is an extension of the work by Hong et al. on a parabolic–elliptic–elliptic chemotaxis model with nonlinear production terms. Specifically, we investigate the global boundedness of solutions to a parabolic–parabolic–elliptic system with nonlinear production terms. We conjectured that our results would be similar to those obtained by Hong, and this conjecture has been verified through rigorous proofs of our theorems. We now compare our results with those of Hong for the parabolic–elliptic–elliptic system (1–6) with nonlinear production terms. Clearly, Hong's results are more precise and explicit; in particular, the parameter b is given in a concrete form. In contrast, in our work, due to the fact that the second equation is parabolic, it cannot be directly substituted into the estimates as in Hong's elliptic case. Instead, we rely on maximal Sobolev regularity to derive the necessary estimates. However, the constants involved in these regularity estimates cannot be explicitly determined, and thus b can only be taken sufficiently large. Consequently, our conclusions are relatively less sharp. Moreover, in the critical cases, the coefficients of the repulsion term and the Logistic source—both playing a "positive" role—are not characterized as finely as in Hong's study of the parabolic–elliptic–elliptic chemotaxis model. Nevertheless, our results still reflect their beneficial effects. In addition, our conclusions do not explicitly capture the "positive" influence of the diffusion term on the global boundedness of solutions, which remains an issue to be addressed in future work.

Finally, studies on chemotaxis can provide theoretical guidance for controlling wheat pests. Based on trends in pesticide application, one can predict the dynamics of pest control, thereby contributing to more effective management of wheat pests.

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